H. Austin Archt. Lith. of E.B. & E.C. Kellogg.

WADSWORTH ATHENEUM,

Hartford, Conn.

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THE

BUILDER'S GUIDE.

A PRACTICAL TREATISE ON

GRECIAN AND ROMAN ARCHITECTURE,

TOGETHER WITH

SPECIMENS OF THE GOTHIC STYLE.

ALSO, PRACTICAL TREATISES ON

GEOMETRY, DECIMAL FRACTIONS, MENSURATION, TRIGONOMETRY,

AND

CARPENTRY AND JOINERY.

EMBRACING ALL NECESSARY DETAILS, AND PARTICULARLY ADAPTED TO THE WANTS OF THE LESS EXPERIENCED.

BY CHESTER HILLS, PRACTICAL ARCHITECT.

REVISED AND IMPROVED WITH ADDITIONS OF

VILLA AND SCHOOL-HOUSE ARCHITECTURE.

BY H. AUSTIN, Architect, *and* HENRY BARNARD, ESQ.

Hartford.

PRINTED AND PUBLISHED BY CASE, TIFFANY AND BURNHAM.

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ADVERTISEMENT.

Although there are numerous Treatises on Architecture already before the public,—yet, from the limited character of many of these works, the cursory manner in which they treat on that which is most essential, and oftentimes the entire omission of what is indispensably requisite to a full development of this, the most important of the Arts, much embarrassment, perplexity and discouragement have arisen.

For the want of a work on a large and comprehensive plan, judiciously arranged, and embracing all the most approved modern modifications of Grecian Architecture, taste, left unguided, has become corrupt, and consequently the progress of the Science in this country seriously impeded.

In preparing this work, the author has availed himself of all possible means to render it such as the exigences of artists and a correction of the public taste imperatively demand.

The Practical as well as the Theoretical Builder, will observe that no pains have been spared to embody in this work every thing which essentially pertains to their profession; and although they are intended to illustrate, by plates and otherwise, architectural models in the most general and comprehensive manner, the details (in which most works are lamentably deficient) are set forth with clearness and precision. In following out the particulars, the author designed to aid *especially* the less experienced Builders; and from the facilities which these his labors will afford, and the low price at which they are offered, he rests assured they will meet the approbation and the patronage of an enlightened Public.

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The Frontispiece is a view of Wadsworth Atheneum, so named, after Daniel Wadsworth, Esq., of Hartford, who gave the lot on which this building was erected at a cost of 30,000 dollars, received from donations of various citizens. It is one hundred feet long, seventy feet deep, and separated into three divisions of equal width, by solid walls, extending from cellar to roof. The northern division, sixty feet by thirty inside, is appropriated to the Young Men's Institute, and contains their Library of near ten thousand volumes in the second story, with rooms below for debating, quarterly meetings, &c. The southern division is of the same dimensions, and is for the use of the Connecticut Historical Society, containing their collections of books, manuscripts, pictures and numerous memorials, relating chiefly to the history of Connecticut, and the United States. The Natural History Society, also have a room in the southern division, with good collections of specimens. In the central division is a Picture Gallery, with an interesting collection of paintings, among which are Trumbull's five large pictures, from revolutionary scenes, and many other interesting specimens of the fine arts, by Sir T. Lawrence, Ræburn, and other foreign and native artists.

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THE BUILDER'S GUIDE.

THE ORIGIN AND PROGRESS OF BUILDING.

BUILDINGS were certainly among the first wants of mankind; and architecture must, undoubtedly, be classed among the earliest antediluvian arts. Scripture informs us that Cain built a city; and soon after the deluge we hear of many cities, and of an attempt to build a tower that should reach the sky; a miracle stopped the progress, and prevented the completion of that bold design.

The first men living in a warm climate, wanted no habitations; every grove afforded shade from the rays of the sun, and shelter from the dews of the night; rain fell but seldom, nor was it ever sufficiently cold to render closer dwellings than groves either desirable or necessary, even in the hours of repose; they fed upon the spontaneous productions of the soil, and lived without care, as without labor. But when the human species increased, and the produce of the earth, however luxuriant, was insufficient to supply the requisite food; when frequent disappointments drew on contention, with all its train of calamities—then separation became necessary, and colonies dispersed to different regions, where frequent rain-storms, and piercing cold, forced the inhabitants to seek for better shelter than trees.

At first, they most likely retired to caverns, formed by nature in rocks, to hollow trunks of trees, or to holes dug by themselves in the earth; but, soon disgusted with the damp and darkness of these habitations, they began to search after more wholesome and comfortable dwellings.

The animal creation pointed out both materials and manners of construction; swallows, rooks, bees, and storks, were the first builders. Man observed their instinctive operations: he admired, he imitated; and, being endued with reasoning faculties, and of a structure suited to mechanical purposes, he soon exceeded his masters in the builder's art.

Rude and unseemly, doubtless, were the first attempts; without experience or tools, the builder collected a few boughs of trees, spread them in a conic shape, and, covering them with rushes, or leaves and clay, formed his hut, sufficient to shelter its hardy inhabitants at night, or in seasons of bad weather. But in the course of time, men naturally grew more expert; they invented tools to shorten and improve labor; fell upon neater, more durable modes of construction; and forms better adapted than the cone, to the purposes for which their huts were intended: they felt the want of convenient habitations, wherein to taste the comforts of privacy, to rest securely, and be effectually screened from troublesome excesses of weather; they wanted room to exercise the arts, to which necessity had given birth; to deposit the grain that agriculture enabled them to raise in abundance; to secure the flocks which frequent disappointments in the chase forced them to collect and domesticate. Thus stimulated, their fancy and hands went arduously to work, and the progress of improvement was rapid.

That the primitive hut was of a conic figure, it is reasonable to conjecture; for of that form do the American aborigines build their wigwams: and from its being simplest of the solid forms, and most easily constructed. And wherever wood was found, they probably built in the manner above described; but, soon as the inhabitants discovered the inclined sides, and the want of upright space in the cone, they changed it for the cube; and, as it is supposed, proceeded in the following manner.

Having, says Vitruvius,* marked out the space to be occupied by the hut, they fixed in the ground several upright trunks of trees to form the sides, filling the intervals between them with branches, closely interwoven, and spread over with clay. The sides thus completed, four beams were laid on the upright trunks, which, being well fastened together at the angles of their junction, kept the sides firm, and likewise served to support the covering, or roof of the building, composed of smaller trees, placed horizontally like joists; upon which were laid several beds of reeds, leaves, and earth or clay.

By degrees, other improvements took place; and means were found to make the fabric lasting, neat and handsome, as well as convenient. The bark and other protuberances were taken from the trees that formed the sides; these trees were raised above the dirt and humidity, on stones; were covered at the top with other stones, and firmly bound round at both ends with ozier, or cords, to secure them from splitting. The spaces between the joists of the roof, were closed up with clay or wax, and the ends of them either smoothed or covered with boards. The different beds of materials that composed the covering, were cut straight at the eaves, and distinguished from each other by different projections. The form of the roof, too, was altered; for being, on account of its flatness, unfit to throw off the rains which sometimes fell in great abundance, it was raised in the middle, on trees disposed like rafters, after the form of a gable roof.

This construction, simple as it appears, probably gave birth to most of the parts that now adorn our buildings; particularly to the orders, which may be considered as the basis of the whole decorative part of architecture; for when structures of wood were set aside, and men began to erect solid, stately edifices of stone, having nothing nearer to imitate, they naturally copied the parts which necessity introduced in the primitive hut; insomuch that the upright trees, with the stones and cordage at each end of them, were the origin of columns, bases and capitals; the beams and joists, gave rise to architraves and friezes, with their

* Markus Pollio Vitruvius, a Roman Architect, the first writer on architecture of whom we have any knowledge. He flourished about fifteen years before the Christian era.

triglyphs and metopes; and the gable roof was the origin of pediments; as the beds of materials, forming the covering, and the rafters supporting them, were of cornices; with their corona, their mutules, modillions, and dentils.

That trees were the originals of columns, seems evident from some very ancient Egyptian ruins still existing, in which are seen columns composed of many small trees tied together with bandages, to form one strong pillar, which before stone was in use, became a necessary operation in a country where no large timber was to be had, and in which the stupendous size of their structures constituted the principal merit. Herodotus describes a stately stone building which stood in the court of the Temple of Minerva, at Sais, the columns of which were made to imitate palm trees.

The form of the bundle pillar above mentioned, though deriving its existence from necessity, is far from disagreeable: it was evidently a beauty in the eyes of the ancient Egyptians, since it was imitated by them in stone; and it seems more natural to suppose that fluted columns owe their origin to the intermediate hollows between the trees composing these pillars, than to the folds of a woman's garment, to which they have but very little resemblance.

Vitruvius, the only remaining ancient writer upon the decorative part of architecture, ascribes almost every invention in that art to the Greeks; as if, till the time of Dorus, it had remained in its infant state, and nothing had till then appeared worth notice; and most, if not all the modern authors, have echoed the same doctrine. Yet, if ancient history be credited,* the Egyptians, Assyrians, Babylonians, and other nations of remote antiquity, had exhibited wonders in the art of building, even before the Grecians were a people.

It must indeed be confessed, though the works of the Asiatic nations were astonishing in point of size and extent, yet in other respects they were of a nature calculated rather to give a high idea of the power and wealth of the founders, than of their skill or taste. We plainly see that all their notions of grandeur were confined to dimension; and all their ideas of elegance or beauty, to richness of materials, or gaudiness of coloring. We observe a barrenness of fancy in their compositions, a simplicity and sameness in their forms, peculiar to primitive inventions; but, even in the early works of the Egyptians, besides their prodigious dimensions, there are evident marks of taste and fancy; it is in them we trace the first ornamental forms in Architecture, and to their builders we are most probably indebted for the invention of columns, bases, capitals, and entablatures. We likewise read of roofs supported by figures of colossal men and animals, in the works of the Egyptians, several ages before the introduction of Persians or Caryatides in the structures of Greece; and of Temples adorned with stately porticos, enriched with columns and sculpture, and built before there were any Temples in Greece.

* See the sacred Scriptures, Homer, Strabo, Diodorus Siculus, Pausanius, Pliny, Justin, and Quintus Curtius.

Hence it may be inferred, that the Grecians were not the inventors of ornamental Architecture, but had that art, as well as their religion and gods, from the Egyptians, or from the Phœnicians, their nearer neighbors—whose skill in the arts is said to have been anterior to theirs, though both were of Egyptian origin.

Diodorus Siculus observes, that the Egyptian priests proved, both by their sacred records, and by other undoubted testimonies, that not only the poets and philosophers of Greece traveled anciently into Egypt to collect their knowledge, but also their architects and sculptors; and that every thing in which the Grecians excelled, and for which they were famous, was originally carried from Egypt into Greece.

The Phœnicians, however, were very early celebrated for their proficiency in the arts of design; and there is no doubt but that the Greeks availed themselves of their inventions.

We are told that Hiram made two capitals for the pillars of Jachin and Boaz in Solomon's Temple, which as far as can be collected from the accounts given of them in several parts of Scripture, very much resembled the Corinthian capitals both in form and proportions, though executed some centuries before Chalimachus is reported by Vitruvius to have invented them at Corinth.

The Cherubim of Hiram, too, or the colossal figures of men and animals in the structures of the Egyptians were prior inventions; and undoubtedly suggested to the Greeks their ideas of Persians and Caryatides.

And though Architecture is certainly indebted to the Grecians for considerable improvements, yet it may with confidence be averred, that they never brought the art to its utmost degree of excellence.

"The art of building," says Leon Baptista Alberti, "sprung up and spent its adolescent state in Asia; after a certain time it flowered in Greece, and finally acquired perfect maturity in Italy, among the Romans." And whether we call to mind the descriptions given by ancient writers of Nineveh, Babylon, Thebes, Memphis, the Egyptian pyramids, the sepulchres of their kings, their temples, and other public monuments; or contemplate among the Roman works, their palaces, amphitheatres, baths, villas, bridges, mausoleums, and numerous other yet existing testimonies of their splendor, it must candidly be confessed that the Grecians have been far excelled by other nations, not only in the magnitude and grandeur of their structures, but likewise in point of fancy, ingenuity, variety, and elegant selection.

AN ESSAY ON THE PRINCIPLES OF DESIGN IN ARCHITECTURE.

THE natural laws upon which Architecture is founded, the principles that govern its designs, and the good sense that directs its application, are serious subjects of inquiry for the student; for from these sources the noble works of antiquity emanated, and by means of such studies our great and intellectual predecessors distinguished themselves, and have become entitled to our gratitude and admiration.

Rules, indeed, like buoys in marine harbors, point out the safest course, and teach the student to avoid the dangers that might otherwise impede his progress. But if he would imitate the circumnavigator, and launch into the ocean of his art beyond the beacons that surround its shore, he must learn to sound the depths of science for himself: for should the artist be satisfied with merely following the track of others, and adopting their models as his sole rule of guidance; content himself with such limited results, his hopes can have little chance of being realized, and he must not expect his labors to receive the approbation of posterity.

Unfortunately for the advancement of Architecture, it is commonly supposed that it is wholly directed by established rules; that the five orders as they are called, are precise ordinances, from which it would be heresy to depart; and that the Architect has little more to do than follow the rules laid down in books, or copy from ancient examples, and, by an easy transposition, make new designs from them, in a way quite as dependent on the fancy as on the judgment—when, in fact, his duty is to explore the treasures with which the vestiges of antiquity and the best works abound, viewing them not as documents and patterns merely, but as invaluable manifestations of mind, in which may be read the very thoughts of their authors, and where may be found the reasonings upon which they acted; thence deducing principles and rules for controlling and directing those exuberances of fancy, with which he who reasonably hopes to become a great architect should be gifted.

Nor is it sufficient that the student should be skilled in superadding the *beau ideal* to suitableness, and to the laws of composition, proportion, light, shadow, and color; this, indeed, will make the artist; but he must be well instructed in the sciences, industrious in tracing causes through effect,—and above all, he must be laborious in applying the results of these to his other studies, always remembering, that neither an acquaintance with the science of building, nor the unassisted qualifications of an artist, will constitute him an architect; but that it is the union of both which alone can give a real claim to distinction.

In Architecture, no style of art is held in reverence in which the character is not correspondent with a great or worthy object, and which does not require the operation of a highly cultivated mind, both as relates to the theory of beauty, and the employment of the principles of science.

Nothing seems so much to have puzzled those who have little information on the subject, as the fact, that there are but three Orders in Architecture—for virtually there are no more; and Architects incur the censure of dullness, because many others have not been invented. If, however, it be considered that the orders are but compositions constituted of two characters—masculine vigor, and femenine delicacy, as in the Doric and Corinthian—and these qualities become united, as it were, in the matronly Ionic or middle Order, we shall no more expect the addition of another order than that of a new color to the primitive ones, or of a new note to the musical scale. The Tuscan and Composite—reckoned among the Roman orders—are merely modifications of the Doric and Corinthian.

The dimensions of columns may naturally be supposed to have engaged the architects of early times: accordingly, upon reference to the best examples, it will be found, that columns were considered to be of proper height and substance when they corresponded with the height and substance of walls; and like them were made to diminish, so as to maintain a proper solidity, without surcharging the lower strata with unnecessary weight—as if a range of pillars were merely portions of a wall, having regular omissions, called intercolumniations. It will be observed, that this outer open walling, if it may be so called, eventually forming the peristyle of the Greek Temple, was not less adequate to support the superstructure, or entablature, and a portion of roof overhanging the cell of the temple, than were the walls to bear the superincumbent roof, of which they were the actual support. Viewing this as a guide by which the columns of such temple might have been governed, we see the cause of a relative proportion between the columns and the building as a whole, and eventually the reason why this decorative feature in Architecture determines the proportion in every other part of the structure, and infer that all should be designed with reference to its magnitude and character.

That the quantity of the entablature should be proportioned to the columns is evident, from the defect that must occur if the latter were not sufficient to support it, and from the offense that would arise to the mind, if the column should have to support a weight to which it was seemingly inadequate. On this principle of reciprocal relativeness, the height of the entablature in the Grecian Orders is proportioned. Thus, it is not governed by the height of the column, but by its thickness or diameter. In Grecian Architecture, therefore, the thickest columns bear the largest entablature, and the slender support the smallest.

In Roman Architecture, this has not always been observed, and still less by the Italian architects. In general, the practice of attaching columns to walls seems to have operated in reducing the height of the entablature; its proportions under these circumstances would necessarily alter, for the Order thence becomes an ornamental feature merely, instead of being an essential portion of the building.

It has been a favorite theory, that the wooden transoms, beam-ends, and rafters of some early buildings, were the precursors of the chief decorations given to the Doric entablature; and perhaps there is no good reason to impugn the theory. That the useful and agreeable, formed the ground work upon which the early architects engrafted that which has become acknowledged

beauty, is very evident; and we cannot do better than follow such admirable examples, and never separate the one from the other.

That timber construction preceded the erection of large buildings in stone, is very probable; and also, that the latter were roofed and decorated with timber, long before the ingenuity of man had contrived the mechanical means of separating from the native rock, and of raising masses of stone, of length and size sufficient to become substitutes for wood. It is, moreover, reasonable to suppose, that in all countries, the means and materials which presented themselves, would influence the style and character of buildings, and that, where, as in the case of materials, they were not in themselves adequate to the end proposed, the ingenuity of the architect would contrive to further the object he had in view. How far the invention of the Arch, a subject of repeated investigation, depended on circumstances requiring the aid of man to substitute construction for quantity, is not so much a matter of speculation, as are the time and place of its invention: but it probably originated where large and ponderous materials had been long employed; and in point of time, after large spaces had been already covered by such means. The progressive nature of Art and Science, warrants the conclusion, and experience proves, that the greatest effort in both, generally takes place soon after the consummation of some great success, the result of which approximates to the objects required. Whether the Greeks were acquainted with the construction of the arch or not, when the Temples in question were erected, is not evident; in those works certainly this contrivance does not appear, and so far as linear arrangement and propriety go, as certainly they would not have employed the arch form in the elevations of those buildings; for they would have felt that it was debased when applied to places which did not require its aid, and where its form could not harmonize with the accompanying objects. Bridges and aqueducts, among the ancients, presented a real demand for arches, and the way was prepared for the attempt, in the stupendous works that are admitted to have preceded the invention. The general application of the Arch allowed the employment of small sized materials, and the Romans finding it convenient in many of their public works to use them in easily portable weights, the arch consequently became a common form in Roman Architecture. After the desolation of Rome, it was for the same reason adopted by her northern conquerors, and thus the Saxon and Gothic styles were made to abound with arches, and the spandril-arched forms which constitute their peculiar characters.

After the local circumstances of climate, materials, and good sense, had combined and done much towards forming the noble Temples of the ancients; the Greeks became solicitous to give importance to their works, but not by means of magnitude alone, created by heaping stone upon stone, as they had seen in the stupendous buildings of Egypt and neighboring countries, and which excited wonder chiefly on account of the manual labor bestowed upon them. They were desirous on the other hand, that the works in question should become admirable in themselves, as manifestations of genius and operations of the mind, to which materials and labor of the hand were merely assistant, just as other materials are aiding to the poet, and without which his thoughts cannot be read by others, or delivered to posterity with advantage. This laudable desire was accomplished during the administration of Pericles, and perfected by the great architect and sculptor, Phidias. One artist, however, ought not to monopolize all these honors, when so much is due to many who preceded, and to many contemporary artists, as well as to the taste and genius of the people, who had feeling to relish and judgment to applaud their admirable exertions.

The best Greek Temples are admitted to afford the most perfect examples in Architecture; and in these we may therefore, first, and perhaps with the greatest advantage, seek the principles upon which the Grecian architects designed their works. It is impossible to view even a model of one of their fine examples, without perceiving that unity of design pervades the whole, and that it is aided by the arrangement, form, and embellishment of the minor parts. The first, simple as it appears, was not the effect of chance, for we know that chance rarely produces excellence in art, and that the commission of a single error is capable of marring it. By this conformity and consequent unity, a principle followed by the Greeks is developed; and whether it originated in the contemplation of the purpose to which the Temple was destined, or whether they considered this semblance of unity to be essential to grandeur, the result is still the same; for we discover in it a source from which we can obtain an effect of sublimity, not dependent on actual magnitude, however much it may be *assisted* by that imposing circumstance; and thence we may learn to design other works, of whatever form, and suitable to whatever purpose we may desire, possessing a quality that we know to be essential in a noble building.

Towards obtaining this unity of effect and character, the combining quality of the roof is obviously necessary in the Greek Temple; it combines in one span the cell, the portico, and the peristyle, without which they would be viewed as parts merely, and to which the steps, or base supporting the whole, greatly contributes.

To complete this unity of effect, only one approach was obvious, under any view of the building; indeed, so carefully was this principle attended to, that on the flanks of the edifice the spaces were ranged in even numbers, so that the column was placed in the middle of its length, and not an intercolumniation; whilst the actual approach was also decidedly indicated by a central opening in the portico, and by the centre-marking character of the pediment.

In the linear composition of these Temples, the architects aimed at obtaining a distinct character in the front and flanks of the building, to prevent the monotony that would otherwise occur. In the portico, the horizontal effect which obviously prevails in those edifices is obliged to yield to the vertical lines formed by the pillars and by the intercolumniations, which also operate as forms, and to the added elevation of the pediment; thus giving to the portico the preponderance in favor of height and vertical character, whilst the flank decidedly assumes a horizontal one.

That the general effect of the linear arrangement of the Grecian Temples, when seen in perspective, should be horizontal, is very obvious: its general form and greatest masses are in proportions, lying in a horizontal position—the roof, the entablature, the aggregate of the columns, the base, and, in this case, the pediment, all concur in declaring the principle of its linear arrangements to be so; and thus viewed, the portico, uniting with the flank, increases that effect, although it possesses in itself the principles of another quality. These great and leading features all conform to the same end, and have proportions relatively agreeable to each other, without reference to the subdivisions that take place within their respective quantities.

It thus appears, that in the composition of the Temple, the arrangement and relative proportion of the masses was an early and chief study. The imposing mass, effected by the aggregate of columns, was placed as if it were the body of the building, to which all others was assistant or subordinate.

As the prevailing masses below and above the columns were of such a character as to seem to bind or girth the whole edifice, the secondary or vertical lines and proportions, as the triglyphs, mutules, &c., submitted as readily to their influence; and, by their judicious use, they obviate the monotony, if it may be so called, of the frequently repeated horizontal lines. It seems to have been a principle with the Architects of the edifices, that all the chief masses should be horizontal, and the subordinate masses vertical, excepting such as partake of a diagonal character, and which merit distinct consideration; also that the leading lines should have, the effect of undisturbed continuity, whilst the secondary lines should be as decidedly, and sometimes even abruptly intercepted.

In the vertical subdivisions of the masses forming the columns, the triglyphs, the metopes, and the mutules, and even the ornaments above them—the acroteria and terminations of the roof—it is evident that great attention was paid to produce the effect of altitude, by conducting the eye from the base upwards along the columns and entablature, in a succession of lines admirably proportioned to each other, and becoming shorter as they approach the summit of the building. Thus the simplicity of the base is not interrupted by longitudinal subdivision; the columns spring from it fluted* and without bases, and form contrasts, the most decided and abrupt, to the horizontal planes on which they stand; thus the eye is necessarily directed upwards—it is pained by an attempt to descend; the diminution of the column assists in leading the eye gently on, and the neck of the pillar, joining in an easy and almost continuous curve with the echinus, is not interrupted until it has passed the abacus which crowns its capital. The triglyph, with the guttæ and mutules, take up the vertical lines, and produce great variety by their forms and quantities; and these lines are still carried on by the enrichments that adorn the termination of the cornice. Thus the eye is irresistibly led upwards through the whole composition.

The diagonals, or third variety of lines employed in the composition of the Grecian Temple, are few in number, chiefly consisting of those formed by the inclined lines of the roof and pediment, by the echinus of the capital, and lastly, by the sloping of the mutules, corresponding as they do with the lines of the roof. But, although these in reality are very few, they are rendered sufficient by an effect equivalent to diagonal inclinations, produced by the laws of perspective, in whatever situation the edifice may be viewed, and which reconciles the vertical with the horizontal effect, by softening the character, which lines, directly opposing each other, necessarily produce.

In addition to this source of diagonal lines, there exists another, which creates them, and by which the Greek Temple is admirably designed to benefit: these are the sloping and curved forms of shadows projected on the several surfaces, imparting a richness, as they seem to revel among the flutings of the pillars, that surprises and delights the observer, and qualifies the otherwise simple forms to harmonize with the sculpture in the building.

On a first view of a Temple, the mind is engrossed by the edifice, as a magnificent whole; on the second, by the relation and harmony of its parts; and at length by the richness produced by its sculpture and its shadows.

But in Greece, design in Architecture was, probably, not limited to the management of the severe details of the art; its ornamental features, which are now considered to belong almost exclusively to the sculptor, appears to have been identified with the duties of the Architect, so far, at least, as relates to their peculiar characteristics and arrangements; and to those points he must have devoted considerable attention. This is evident in the existing remains of the Greek Temples, wherein the sculpture so admirably conforms to the principles which govern the general design, as scarcely to leave a doubt that it is attributable to the same taste and genius, and that it received an equal portion of deep study. The forms of the sculptured embellishments, being as important to the perfection of the design, as any other part of the edifice, are accordingly found to be disposed with the most scrupulous attention towards creating that harmonious effect which is so much admired.

The sculpture in the Greek Temples appears to be of three distinct characters; that of the pediment, that of the metopes, and that of the bassi relievi which decorate the peristyles, as relates both to their lineal composition and to their relief. The composition of the pediment was undoubtedly conformable to its peculiar shape, arranged so as to harmonize with its sloping and overhanging cornices, and so grouped together in a well balanced mass, as to appear a subject worthy the support of the dignified columnar strength beneath it. These figures being entire statues placed in front of the tympanum, produced an effective depth of shadow, well calculated to associate with the solemn recesses of the portico, and without which the tympanum would seem to be but an ornamented wall, too cumbrous for its situation; a defect visible in every pedimented portico that is permitted to be plain or ornamented less independently of relief. In the metopes, this quality of agreement with the surrounding frame is wholly abandoned, and angular or crossing figures are adopted, which do not repeat the upright lines formed by the channels of the triglyphs, but admirably separate them from each other; hence the subjects of the metopes consist chiefly of conflicts between combatants, and which admit of the attitudes necessary for the purpose. In the Roman order, a similar principle of decoration was pursued, and crossing implements of sacrifice were introduced, varied by pateræ, (for the circle is equally separating,) or other suitable devices corresponding with the purpose to which the building was dedicated.

From the peculiar and high relief in which the metopes of the best examples were executed, they most advantageously intercepted the bold shadow projected by the cornice upon its broken surface with an enriching and undulating edge, thus preventing it from appearing to divide the entablature, which it otherwise would do, into neatly equal portions, a little below the middle of the frieze; a situation in which the semblance of a division would be most injurious, as it would disturb that effect of harmony in the relative proportions of the entablature so conspicuous in the Greek examples. The triglyphs themselves are intended to produce this undulating edge of shadow in a degree, and the mutules assist in continuing it; but these being insufficient, and

* That the flutings of the columns, numerous as they are, effectually aid this intention, is obvious, as well as that they enrich the columns, and render them suitable to the sculptured entablature; but the flutings are in other respects essential to the perfection of the design. The repetition of the lines on the surface of the columns prevents them from appearing, under any point of view, as masses too large for the dignity of the whole; and when viewed in perspective, by being thus infinitely multiplied, the appearance of unity is obtained, just as in engraving, the effect of a uniform mass is produced by an accumulation of parallel linas.

the object so beautifully obtained by metopes, when judiciously executed, no experienced eye can be satisfied without them, nor will the connoisseur deem that a just copy of the Doric entablature in which sculptured metopes are not to be found; for, when omitted, that important principle in Architecture becomes violated, which prevents the projected shadows from disturbing the adjusted proportions—a defect which would otherwise occur in the present instance, in one of the leading features of the composition, and in a place of all others, where it is most to be reprehended.

The bassi relievi of the peristyle, although in shade, and placed at a great height, are most useful to the effect of the building; circumstances assuredly the result of mature reflection; for so situated and so delicately marked, they enrich the broad shadow by their varied forms and softly reflected lights, without competing with the enriched entablature and fluted columns, which, contrasted with their repose, seem thence, in the atmosphere of Greece, to glitter with augmented brilliancy.

Upon reviewing all these points in which it appears that sculpture as an accessory is needful to the perfection of the Greek Temple, it is obvious when, in a modern building, intended to exhibit its beauties or imitate its excellencies, sculpture is omitted, the result will be offensive to the taste; the whole will appear neglected; the pediment, if there be one, heavy and obtrusive; the entablature disproportioned, and the walls bare: thus the uninformed spectator will retire from it without regret; and the connoisseur will feel that none of its qualities are perfected; the absence of the intermediate and reconciling, as well as the embellishing agent, is manifest to him, and he views the whole as an unfinished edifice, whilst he deplores that its merits and capabilities of receiving the advantages of sculpture are so neglected.

That the Architects of Greece were not content with mere mathematical precision in the composition of their works, is evident from the deep attention paid to the parts in which a neglect of this might have been deemed excusable. The leading deviation from this precision is exhibited in the difference of intercolumniation at the extremity of the Doric portico, a practice not imitated by the Roman architects, but one that obviates the objection which is justly made to the appearance of weakness at the extreme ends of the portico or colonnade, when the precaution is not resorted to. Notwithstanding the extent of this remedy, they pursued it still further, for observing that all bodies, when viewed so as to be immediately opposed to space, appear to be less than those seen in connection with other substances, they also increased the diameter of the extreme columns, so perfectly in proportion to the optical effect, that the deviation was only discovered by careful measurement. Even here these intelligent and reflecting artists were not content, but overcame another defect in columns arising from a similar cause, when they are formed so as to diminish regularly from the lower to the upper diameter: in this case the column is well known to have a thinner appearance in the middle than such a diminution would be supposed to produce; this defect they however avoided by the entasis or swelling of the column, employed in a degree so exactly suited to the purpose, that until lately the entasis in the best examples of the Greek Doric column was not even suspected.

In the design of the entablature, they were no less careful to satisfy the correct eye; for after having adjusted its magnitude, it appears that they were careful that the epistylium, or architrave, should be in appearance adequate to sustain the weight assumed to be placed upon it by the triglyphs: the height of this portion of the entablature is therefore great when compared with sizes needful for strength merely, had it been executed in timber, as in the early Tuscan temples, or with the examples of the Roman Doric order, the architrave of which is not of sufficient substance to satisfy the mind that it is capable of giving ample support to the parts above it. The massiveness and simplicity of this feature derives additional force from association with the facia of the cornice; they are admirably proportioned, and aid each other in the contrasting duties assigned to them, as well in respect to shadow as to the sculptured embellishments. Indeed it appears on an intimate study of the works of the Greek architects, that the principles which governed the linear arrangement, and that of the shadows, were the same in both, being powerful contrasts opposed to each other, and reconciled by the intervention of a medium capable of preventing the discord that would necessarily occur without it. This in the linear department, is the diagonal line, formed by the mouldings wherever they are found, and in the shadows, by the reflected lights that occur upon the face of them. In considering the profiles of the Greek and Roman Doric order, there seems to be a distinguishing characteristic in each, arising from a difference of proportion in the adjustment of the diagonal lines, and consequently of the middle tints as relates to the shadows; in the former, the diagonal forms being comparatively small, and in the latter, often of quantities surpassing the horizontal and vertical lines connected by them: the effect produced by this arrangement alone, enables the connoisseur, even at a great distance, to declare the order of the building.

This brevity of diagonal or moulding in the Greek arrangement, followed by a proportionate limitation of middle tints, required that its quantity should not be lessened by the shadows that occur when the face is undulating, as in the cimarecta, or in the ogee: the diagonal line was therefore generally preserved, and the form termed an ovolo, resulted from it. In the profiles of the Roman Doric, the diagonals being of considerable length, the middle tint would have prevailed over the lights and shadows in an objectionable degree, if it had not been subdivided and broken by the alternate shades and reflected lights which the quick undulating surfaces, and many filletings produce. Upon principles resulting from this circumstance, the mouldings of the two orders are formed.

On comparing the profiles of the Greek with the Roman Doric order, it appears that the latter is composed of smaller parts than the former, and that in the minutiæ it is more complicated; the consequence is, that the shadows projected are also less simple than those of the Greeks; they endeavored to preserve masses of what the painters term "middle tint," broad quantities of light, relieved by striking depths of shadow and sparkling effects, for which the forms of the mouldings were carefully designed, and for this purpose these were usually generated by the ellipsis, the parabola, or hyperbola; but the mouldings of the Roman orders are almost invariably composed of circles, either simple or compounded, in equal portions from equal radii. This produced similar quantities of middle tint, light, and shadow. The Greeks carefully avoided this sameness, and judiciously and tastefully made the shadows to prevail distinctly; hence, in all their works we find the result of a superior understanding of the principles and effects of the light and shade, which are opposed to each other, and relieved with great skill; whereas, in the Roman

style, being divided and broken, they are certainly less beautiful and less capable of affording the charms of reflecting light than the vestiges of Grecian art, which, by their well studied proportions, merit respect and imitation.

As the principles which direct the forms of the mouldings, and the arrangement of their light and shadow, are the same as those which govern the sculpture, and as they correspond with those forms and effects which we behold in the muscular action of the human figure,—it may be presumed that the ancient architects were intimately acquainted with the principles of the art of sculpture, if they were not always the immediate agents who produced the celebrated works that adorned their Temples. Be this as it may, there can be little doubt that the perfection to which both arrived, resulted from contemplation, and careful study of whatever was deemed suitable in nature, and particularly of that on the characteristics of which the work of art was to be founded. Principles of art may be deduced by study from whatever is most beautiful in nature, and transferred into arrangements that shall preserve its character without reminding us too forcibly of the source whence they were obtained.

In the Ionic and Corinthian examples of the Grecian orders, the result of similar observations and study is decidedly manifest, and the student in architecture will do well to revert to the same sources, to study with assiduity the perfections of nature, and thus qualify himself to impart those inherent principles of beauty to all his works.

As neither the Greek nor the Roman architects were negligent of the beauties of vegetable nature, their edifices abounded with imitations of them, so admirably adapted to the purposes to which they were applied, that they are viewed by the artist, not as copies, but as original inventions. The Greeks, who studied relativeness of form with the greatest care, adopted, as prototypes, for such ornaments, those ligneous plants which best permitted an arrangement of graceful lines, and which they could use as a medium for combining, as it were, one part of the design with another, or for leading the eye of the spectator by the course most advantageous to the general design. In the sculpture of these, they observed the same principle of relief, and of light and shade, as where the human figure was employed. In this species of ornament, among the Greeks, the stem usually prevailed over the quantity of foliage;* whereas, in the Roman decorations the stem was made subservient to its luxuriance; and the Roman examples prove how capable those artists were to use these means of decoration most amply, without seeming to overcharge the orders in which they were adopted.

The artist of both countries employed the circular forms of the flowers for the same purposes, that of separating one part of the design from the other, as observed of the metopes, and of attracting the eye of the spectator to suitable points of repose, where, from a multiplicity of angles, a sort of confusion would otherwise occur, as in the coffers of empanneled buildings, or in the soffits between the modillions of the Corinthian and Composite orders; and in these flowers, the diagonal lines are frequently manifested, obviously for the purpose of a better reconcilement of the parts. Thus, in all the features of the most perfect edifices, it will be seen that certain principles of design have been kept in view; that conformity of the parts to the whole, and relativeness to each other, were combined with elegance and grace; that the changes incident to the laws of perspective were consulted in the design; and that the operations of light and the effects of shadow were no less studied and systematically arranged by the great artists, both of Greece and Rome; and there can be no doubt that, in their admirable works, they sought to avail themselves of such materials for building as were capable of displaying, with the greatest advantage, the results of their studies; and that they considered brightness of tint and smoothness of surface essential to those objects, and employed them as the means of augmenting the beauties which could not be effected with coarser materials. However admirably the Greek temples were designed, both in form and in proportions, and however judiciously prepared to require force of effect by well arranged opposition of light and shade, even under the favorable circumstance of the brilliant illumination and the clear atmosphere of Greece, such exquisitely white materials as the marble in which they were executed, was necessary to the perfection of the chiaroscuro of the composition; for without enquiring if whiteness and polish be sources of abstract beauty, it is evident that the Greek architects must have failed to obtain the perfect results after which they were so arduously seeking, if they had been obliged to employ a dark or coarser material. They assuredly considered the bright complexion of the stone as applicable to their intention, just as the painter does the white color on his pallette, and thence arranged the forms, to obtain striking and pictorial effects of light, modifications of tint, and depth of shade, which it would have been impossible to acquire, with similar effect, by a dull material. The chaste and bright hue of these buildings, when entire, and illuminated by a mid-day sun, having the force of this brilliancy increased by contrast with their solemn shadows, must have inspired an admiration which cannot be adequately produced by the same arrangements of detail in colder regions, and where the complexion of the stone is not favorable to the purpose. These circumstances should receive full consideration from the architectural student, who will thence, perhaps, on this, as well as on many other accounts, see how ineffectual is mere plagiarism towards inspiring a similar sentiment, and learn to imitate the judicious horticulturist, who fully considers the consequences of transplanting a tree from its congenial soil and climate to another unsuited to support the perfection of its nature. Indeed, the young architect who seeks the principles of his art, and studies at the same sources from which they were obtained by the ancient masters, will not only find his course directed and his labors lightened by the examples which are yet open to his observation; but, if he be capable of duly appreciating the excellence of their works, and the principles on which they were produced, he will abstain from the too common practice of selecting parts from various works, and of combining them, to make new designs; scorning the easy process of mere plagiarism, he will think for himself, and endeavor to rescue the art he pursues, from the aspersions that are too often and too freely cast upon it by those uninformed of its merits or its powers.

It is evident from the great size of some of their temples, that the Greeks admitted the circumstance of magnitude to be a source of dignity in architecture, seeing that however small the cell was required to be, the contour was occasionally increased to magnificence by its portico and peristyles, as appears in the plans of temples, and from the testimony of Vitruvius and other writers. But, however fully impressed with the importance of magnitude towards creating notions of sublimity, they seem to have imposed such limits to the size of their temples as rendered them capable of being embraced at one view by the eye of the spectator, when situated at so inconsiderable a distance that

their component parts could be sufficiently distinguished and admired; as if they feared to present a cause for comparison, on the single account of vastness, with other and larger works, or with the stupendous productions of nature around them. Had the Greek artist sought to impress the mind chiefly by such means, they would surely have made the approach to the temple on the side, that so they might have presented the greatest quantity of the building to the eye of the persons who approached it. As they selected the end of the edifice for this purpose, it may be presumed that the effect of magnitude was considered by them to be of secondary importance. This circumstance, and the perspicuity evident in all their designs, seem to prove that those architects were anxious to impress the spectator, at one and the same time, with the greatest force of which the work was capable, and to prevent this from being weakened by speculations arising from intricate forms, or doubtful extent, at a moment when they desired to engage the understanding by qualities of refinement worthy of the edifice, more rare, and less easy of attainment; and also that they sought to inspire the spectator with sentiments of admiration for their sacred buildings, by means more adequate to exhibit the powers of intellect, when directed to such objects, by manifesting the infinite superiority of the energies of the mind over the operations of mere labor.

On an examination of the vestiges of the best Grecian architecture, it will be obvious that the artists were less anxious to introduce novelties, than to perfect certain effects that had already obtained the approbation of the judicious, as well as to remove those appearances of imperfection which arise from optical or other causes, even at the expense of mathematical accuracy; and hence the little difference exhibited in the detail of the Order, as represented in the profiles of those of the Temple of Theseus and of the Parthenon, wherein the parts are curiously corresponding, but which are, perhaps, most perfect in the latter example; but towards accuracy in their conclusion, it is proper to consider the relative magnitude of these buildings, the column of the Parthenon being nearly twice the diameter of that of the Temple of Theseus; and it is quite possible that so considered, each may be perfect in its proportions and detail.

Towards an analysis of the Doric Order, and the means of exhibiting the steps of its progressive improvement up to the period of its decline, it would be of great advantage if the dates of the various buildings were correctly ascertained, as it would thence be easy to deduce what the Greeks considered improvements of the Order; and this information would guard the modern architect against retrograding in his art, or, if he must implicitly adopt, at least prevent him from disgracefully selecting an inferior example, when one more perfect was before him. An ingenious writer has endeavored to supply this deficiency in the history of the Doric style of art, by conclusions drawn from the vestiges themselves, unaccompanied, as they chiefly are, by adequate record or inscriptions, and gives priority of date to those of the most massive proportions; and it is probable, that such proportion would have been adopted in imitation of those already prevailing in the massive pillars of Egyptian and other eastern edifices and excavations, although they were applied to works in which the principles of a lighter character were taken as the model for the arrangements and their parts.

With a view to exhibit the relative proportions of several examples of antiquity, the following Table is introduced, the scale being a division into sixty parts of the lower diameter of each column.

PROPORTIONS OF THE DORIC ORDER.

	Height of Column.		Top Diameter.	Architrave.	Frieze.	Cornice.	Intercolumniation.	
	D.	M.	M.	M.	M.	M.	D.	M.
Temple of Corinth,	4	4	44 2–3	48 2–3			1	14
Hypæthral Temple at Pæstum,	4	8	41 1–4	42 1–3	40 1–2	21 1–2	1	4 3–4
Temple of Selinus,	4	21 3–4	46	46 1–3	44 2–3		1	2 1–3
Temple of Minerva at Syracuse,	4	24 1–2	46	44 1–2	40		1	5 2–3
Pseudo-dipteral Temple at Pæstum,	4	27	40 1–3	50			59 1–2 & 67 2–3	
Temple of Jupiter at Selinus,	4	34 1–3	35 1–2	52	44 2–3	26		
Temple of Juno Lucina,	4	42	45 1–3	55	45		1	15
Temple of Concord,	4	45 1–4	46	46 4–5	46 1–3	25	1	10 2–3
Hexastyle Temple at Pæstum,	4	47 3–4	43	45 3–4	44 3–4	24 3–4	1	1 1–3
Temple of Jupiter Panhellineus,	5	24	44 1–2	51 1–3	51 1–2		1	41
Temple of Minerva at Athens,	5	33 1–2	47	43	43	32	1	17 2–3
Temple of Theseus,	5	42 1–3	46 2–3	50	49 1–2		1	37 1–2
Temple of Minerva at Sunium,	5	54	45 3–4	48 1–2	48 1–2		1	28
Portico of the Agora at Athens,	6	2 1–2	47	40	42	21		
Temple of Apollo,	6	3 3–4	42 1–2	49 2–3	42 1–2			
Temple of Jupiter Nemæus,	6	31	49	38 2–3	43 1–2			
Portico of Philip,	6	32 1–2	49 1–2	38 1–2	43 3–4	25 1–2	2	43 2–3
Theatre of Marcellus,	7	51 2–3	48	30	45 5–9	37 2–9		

Certainly the severe features of the Doric Order afford better means than the Ionic or Corinthian examples, wherein to trace the principles of design, particularly in Grecian architecture, and to teach the mode by which those architects proceeded and eventually arrived at excellence; but it is necessary for the student carefully to investigate the peculiarities of the latter orders, and to trace the principles upon which they were founded. The peculiarities referred to are chiefly those which give to the Orders new characteristics; and although the Grecian artists did not so cherish them as to afford many examples, and these have been greatly diminished, yet such of the Ionic Order as we have received from them, being designed with the same judgment, and being doubtless the result of similar study, exhibit what was perhaps esteemed by them as perfecting their intentions, a knowledge of which is a proper object of a student's inquiry.

If, as it is recorded by Vitruvius, the ancient architects sought in the beauties of the female form and proportions the types of imitation for the beauty and gracefulness of these latter Orders, they have well expressed a semblance characteristic of those excellencies, in spite of principles necessary to construction, and amidst the elements of strength and durability. The qualities

that seem naturally to appertain to building, are those that we identify as masculine; such as strength, firmness, and capability of resistance, and therefore congenial with the Doric Order, for which reason of conformity and suitableness, the Greeks probably preferred it; but that the difficulty should be so overcome, as we find it is by the examples of the Ionic and Corinthian Orders of the Greeks, is a matter of admiration; for we there find identified with the qualities of strength, the feminine delicacy which we admire in the lovely forms whence those characteristics originated.

The union of those properties of strength and beauty being satisfactorily accomplished in the Ionic Order, it became much more practicable in the hands of the architect than the Doric could be, controlled as it is by the ordinances of its triglyphs and the severity of its intercolumniations; and it is very probable that it was used for general purposes, as well for temples dedicated to public worship, as subsequently in buildings similar to the aqueduct of Hadrian, as it is called by Wheler and Spon. The descriptions of the Ionic temples of Greece, as given to us by early writers, and by several praiseworthy travelers and authors who have investigated their remains, induce us to infer that the Order was not usually applied to buildings of considerable magnitude. The temples on the Illyssus and of Erectheus and Minerva Polias, afford the most esteemed examples of the Order, but suffer the disadvantage of not exhibiting those sculptures or metalic embellishmeuts which are essential to the perfection of the buildings, which it is believed they formerly possessed, and which they would require to an extent at least as ample as was afforded to the Doric temples.

Although it is to be regretted that Vitruvius and other early writers have not transmitted much information on the principles which in Greece governed the erection of its Ionic edifices, still we may rejoice that so much remains yet of Athens, both abroad and at home, for our investigation, that we are in part enabled to supply the deficiency; for those venerable fragments of ancient art, however mutilated by time, ignorance, and barbarism, beautiful even in their decay, show that this style of art had there arrived at an excellence which probably was never surpassed in any other portions of Greece, notwithstanding its high reputation in Ionia, and the extolled and magnificent works at Ephesus and Miletus.

From these remains it may be inferred that the Ionic style was applied by the architects of Athens to small buildings chiefly, and probably to such as were dedicated to youthful or female divinities, and to the inferior order of mysteries; and whatever may have been the type on which its character was founded, it is evident, that its proportions and arrangements are rather suited to please, on account of its grace and richness, than to awaken the sublimer sentiments to which the Doric Order is so admirably and peculiarly addressed. Thus as the mind readily associates ideas of beauty with that which is comparatively little, and the idea of smallness with that which is lovely; a principle in addition to that of suitableness to its appropriation, may be deduced, by which those artists designed the sacred edifices in question; for it is probable, that when from want of magnitude in the edifice itself, they could not produce the èffect of grandeur, they judiciously aimed to substitute a character of elegance and beauty.

Some erroneous notions appear to have arisen respecting the embellishment of the Ionic Order, on account of the plainness which appears in some of the fragments themselves, and in the representations of the several edifices, as given by Stuart and others; but although the little temple on the Illyssus, since destroyed, did not exhibit sculpture in the moulded detail, there is no doubt, from the then existing appearances, that the frieze was decorated by sculpture, and that the tympanum of the pediment was equally adorned. The almost profuse embellishments of the capitals and mouldings of the Temple of Erectheum, and portico of Minerva Polias, proves that richness was a part of the character of the Order, and as it was sometimes the practice of the Greeks to adorn unsculptured surfaces by fresco and other painting, and as the architrave in the pronaos or porch of the Temple on the Illyssus retained to the last a considerable portion of painted ornament on its surface, it is improbable that its more visible parts were deficient in due proportion of embellishment.

With regard to the Order as exhibited in the remains of the Erectheum and portico of Minerva Polias, the lesser parts being exquisitely enriched, it is not reasonable to suppose that the whole would have been so incongruous as it must have been, had the cymatium, frieze, and tympanum been destitute of ornament; and the marks on the frieze, which show the places that in all probability formerly received the means of fixing sculpture or metal relievi, demonstrate that part, at least, to have corresponded with the perfect remains of the edifices.

The absence of such marks, if they are not to be found in the frieze of the portico of Minerva Polias, is no proof that the perfection of the Order did not require sculpture in that part, because it might have been there omitted, on account of its being situated to the northward, consequently in an aspect unsuited to display, and so near to the declivity of the Acrŏpolis, that it could not have been adequately viewed.

In both examples the subdivision of the epistylium increases the demand for sculpture in the frieze, and its upper moulding is made to project by it so far as to form an ample recess for the sculpture that seems to have been applied to the surfaces above it.

But for the free use of embellishment to this Order, the secondary parts themselves are too few and simple, and perhaps too ample for the necessary expression of delicacy, whence the Order has been termed imperfect: but, being sculptured, they are certainly the better suited to the object; for it should be considered that when delicate ornaments are sculptured in such quantities, the very circumstances of their breadth and simplicity augments the richness applied to them.

In all cases, the best examples show that the mouldings intended to be plain, should be more subdivided and irregular in their outlines than those that are to be sculptured. Painters are aware that youthful and feminine beauty depends on proportions illustrative of the principles producing interesting contrast; the distinguishing features of a child lose their infantine loveliness as they become large, in proportion to the other parts of the countenance, and the reverse of this Order is acknowledged to be incompatible with female beauty. These facts, in addition to the claims which the Ionic Order has in common with the Doric, lead us to infer that the entablature and pediment of the Ionic Order of Athens, was amply embellished, and besides, it is evident, that without the presence of the sculpture alluded to, the capitals of the columns and the embellishments of the inferior parts of the edifice would be out of harmony, nor is it reasonable to suppose that the finishings of those parts that are lost to us from the devastations of time and other causes of destruction, were less corresponding with the subject than we find

those to be, which so happily for architecture, are subject to our examination.

The capital of the Ionic column may be considered as the scale of embellishment to the whole Order; for, so far as can be deduced from existing documents, the other parts of the work are in complete harmony with it, whether they be compared in the simply elegant temple on the Illyssus, or in the abundantly decorated Erectheum, or Temple of Minerva Polias.

Although the revival of genuine architecture in Europe and America is so eminently indebted to Greece for instruction in the Doric and Ionic, it must remain a source of deep regret, with respect to the Athenian practice in large works of the Corinthian Order, that time has forever thrown over it an impenetrable veil; for although the Temple of Jupiter Olympius, if properly so called and correctly copied, is in this style of art, yet its remains are mutilated and few, but in magnitude and proportion such as to assert a claim to the reputation, as recorded by Vitruvius, of being "universally esteemed and accounted one of the rarest specimens of magnificence," and paralleled with the celebrated Doric Temple of Ceres and Proserpine, at Eleusis, and also with the Ionic Temples of Diana at Ephesus, and of Apollo at Militus.

The Corinthian Order in the little monument of Lysicrates, cannot but be admired for its elegance and beauty, but it is evidently not suited in its proportions and detail to works of magnificence: it may be esteemed a variety of the genuine order, as practised by the Greeks, applied with exquisite taste and feeling to an edifice in which the quality of grandeur was not attempted. But this interesting little work of art, by its own arrangement, teaches that the same dispositions occurred in this order as in the Doric,—solid support in the basement, in the middle a bold relief of light and shade, and in the upper part a display of enrichment and a decorative lightness; altogether not unaptly compared with the progressive variety in a tree, from its firm base at the earth in which it grows, to the lightness of its terminating leafage.

Whether those artists sought in nature the principles of art or not, it is certainly true that in architecture, the works most esteemed are those that are also most conformable to the laws of nature; and they are judged of through their operation, although the perception may not be aware of it; but it is, perhaps, one of the excellencies of the art, that in its imagery the type of its origin is not easily recognized.

On an examination of the architectural works of Rome, it will be found, that those examples of the Roman Orders are most esteemed, which are composed upon principles of design that approximate nearest to those employed by the Greek architects, particularly as relates to proportion, contour, and expression, and whether it be in the Doric, or in the Ionic Orders. The Greek entablature is so proportioned, that the epistylium and frieze exceeds considerably in length the diagonal of the cornice, by which it acquires the appearance of ample strength, and affords space for the broad shadow, projected from the facia; whereas the Roman entablature is sometimes so composed, that the frieze and architrave are of much less extent than the diagonal alluded to; from which circumstance the cornice appears too large, and the whole entablature acquires the effect of being heavy without the benefit of appearing strong; and from the want of force in the shadow projected from the facia, conspicuous in the Greek examples an equally effective relief is not obtained.

Even in the most approved remains of the Corinthian order—an Order decidedly preferred by the Roman architects—there is a defect of contour in the cornice itself, attended with that disturbed imperfect shadow of its members, which has been before referred to; for the projection of this cornice being nearly the same as its height, and the parts of the profile made to correspond with the inclination of a diagonal line, drawn from its projection to its base, the whole cornice becomes inharmonious and broken in its light and shade, so soon as the sun approaches to an elevation by which his rays descend parallel with its direction, even on the fronts of edifices directly opposed to his influence; soon after, all its members are veiled in shadow, and depend on reflected lights for relief, unless indeed, the facia is so arranged as to intercept his rays. In some examples this feature is nearly or wholly absent; in others it is very limited; and even in the most approved examples—those of the Temples of Jupiter Tonans, of Jupiter Stator, and the Parthenon—the magnitude and situation of the facia are not adequate to produce the contrast desired, and so admirably effected in Grecian architecture. The Forum of Minerva, the arch of Trajan and Temple of Mars Ultor, are examples wherein these defects appear conspicuously. In the Doric and Ionic Orders at least, it is known that the Greeks omitted the carona, except where the pediment required its aid, and studiously gave to the facia an importance which is essential to perfection in architectural design.

The capitals of the Roman and Grecian Ionic Order differ in the same respect of distinctness or expression; the parts of the former being mixed together and confused, whereas the latter have them separated and entire; the abacus and its volutes being placed on the echinus as independently as the abacus of the Doric. In the Corinthian capitals and foliage of Grecian examples known to us, this distinctness is always found, and particularly in the best documents, as though it were a principle on which their artist designed them, that they should have the power of interesting by their variety, without embarrassing the mind with encroachments on simplicity; and it will probably be found in all the Athenian examples, that the bases of the Orders, where they are used, are composed with similar attention to perspicuity.

In the edifices of Greece there exists a peculiarity, a degree of which is sometimes observed in the Roman practice, but which has been much neglected by the Italian masters, with respect to the bearing of the epistylium on the capital of the column, and its consequent width, and which, perhaps, has not been sufficiently noticed by writers on architecture. The soffit of the epistylium, in each, is nearly as wide as the inferior diameter of the column, whilst in the Roman and Italian practice it is chiefly governed by the dimensions of the superior diameter, being made of the same width. From this circumstance, it is probable that their capitals are less bold than those of the Athenian works; and to obtain a proper arrangement of triglyphs and modillions, they have been compelled to resort to the projection of the facia of the epistylium to extend the frieze; thus to assist the lower portion of the cornice, without which those embellishments could not be separated so as to suit the intercolumniation desired; for these must, in all cases, be relatively adapted to each other.

The practice of the Greeks and Romans, in these particulars, differed so much, that they may perhaps afford the means by which, in a critical examination where doubt exists, the works of one country may be satisfactorily distinguished from the other. With regard to the connection of the transverse soffits of the epistylium with the walls of the Grecian Temples opposite to the antæ, as the latter are of a similar width, and do not profess to be imitations of the columns, no difficulty occurs; but in the

Roman practice, square pillars, in proportions and decorations corresponding with the columns, are made to receive the soffits, it has been difficult to follow the proceedings of the Greeks in the desirable increase of the soffits in question, because it would require that the pilasters behind the columns should not diminish, as was usual in the order, or that the soffit should overhang the pilasters, which is always offensive to the eye.

The principles alluded to in all the foregoing observations are applicable throughout the vast field of regular architecture, to which the want or desires of mankind may require its aid; unless they were so, the student would uselessly seek amongst the ruins of Greece and Rome for the practices followed in their designs; but if they are rightly understood, and judiciously applied, to the church, the palace, the theatre, or the humbler private dwelling, according to their respective demands, whatever they may be, of sublimity, splendor, magnitude, beauty, or accommodation, or whatever changes the local circumstances of climate or habit may require, they will adorn his exertions. Still the architect must for himself supply by study and reflection—the same means by which the ancients acquired their well earned reputation—the demands made on his ingenuity and judgment; for it would be absurd to suppose that these can be supplied by a mere copying of works, prepared for, and suitable to, partial purposes only, amidst other habits, and in other climates.

As examples of the powers of the mind, and the readiness with which the principles of ancient art have been made to apply to other and widely differing purposes, the works of the Italian architects are eminently encouraging; in the noble and splendid buildings of Italy they may be unquestionably traced, and each is amenable to the test of similar criticism, as all afford proper subjects for the contemplation of the student. In its unquestionable unity will be found the source of dignity in the palace, notwithstanding the variety of its form. Its continued terraces provide a secure and extended base, from which, like the columns of the Greek or Roman temple, arises its aspiring arcades, pillars and towers, bound together by its architraves and cornices, above which the sky is met in parts of less magnitude, but equally enriching, formed by its domes, turrets, balustrades, vases and figures, altogether combined in the greatness of relative proportions, of simplicity and ornament, rendered the more striking by its well disposed light and shadow, and the brilliancy of the material of which the edifice is constructed.

Thus in works that could not have been accomplished by mere imitation, we recognize the principles of ancient art in their designs, even at distances in which form and contour only are discernible. These works are not copies; they merit the reputation of originality; for although in fact they are the offspring of the same art, and governed by similar laws of design, the Grecian temple and the Italian palace are as unlike to each other, as are the Greek and Roman temples themselves to their Egyptian and Assyrian precursors, and from which their origin may probably be traced, and even beyond them, to the rude excavations of yet earlier times.

*Chambers** seems to have preferred the profiles of Vignole,† Palladio,‡ and Scammozzi,‖ whose works, more than any other of the Italian masters, approach the peculiarities of Athenian or Greek architecture, as they also correspond with the best remaining examples of Roman art in the Theatre of Marcellus, in the Temple of Fortuna Virilis, and in the Corinthian order of the Pantheon, and the Temples of Jupiter Stator, and of Jupiter Tondans, as also of the beautiful Temple at Tivoli, probably erected in the time of the emperor Augustus. In the Tuscan order he avowedly employs the proportions as given by Vignole, and the cornice of Scammozzi. Compared with masculine firmness of the Grecian Doric, this order is feeble and ineffective, perhaps arising from the insufficient height of the entablature; but this would be less obvious if the order were affixed to the walls of an edifice, as was usual in Italian buildings. The diagonal of the cornice being a line drawn from the top of the cornice to its connection with the frieze, is four minutes less than the collective heights of the frieze and architrave.

Chambers has also followed Vignole in the Doric order, in preference to other Italian masters, avoiding however, the errors into which some speculations on relative harmony had seduced him. The diagonal of the cornice is two minutes and a half less than the height of the architrave and frieze. The example given from the Theatre of Marcellus is two minutes less; and that by Palladio, from the Basilica at Vicenza, is six minutes less. In the Grecian Doric of the Temple of Minerva, the diagonal of the cornice, as given by Stuart, is less one module thirteen minutes, and that of the Temple of Theseus something more; in both cases the member that terminates the cornice of the pediments being omitted.

In the Ionic order of Chambers, the diagonal of the cornice is five minuies less than the heights of the entablature beneath it; whereas that of the Ilyssus, as given by Stuart, is little more than half the height of the frieze and architrave. In the Temple of Minerva Polias, the diagonal is but one third of both, and the portico of the Erectheum is the same; but this is without the upper member of the pediments, and which probably formed no part of the lateral cornice. The examples from the Villa Capra and Basilica at Vicenza, have diagonals precisely corresponding with the heights of their architraves and friezes, although in both cases the projections of the cornice is very nearly the same as its height.

The Composite and Corinthian order of Chambers present diagonals six minutes less than their architraves and friezes, whilst the examples of the Composite of Palladio, have both of the same dimensions.

The diagonal of the Jupiter Stator at Rome exceeds the architrave and frieze thirteen minutes and a half, and that of the Pantheon is less eight minutes. From these circumstances it will appear that the Roman and Italian orders presented a much larger appearance of cornice, as proportioned to the architrave and frieze beneath them, than was practised by the earlier architects; and as, except in the Doric order, it was usual with the former to make the height and projections of the cornice alike, so that the diagonal would be inclined at an angle of forty-five degrees; it is obvious, that in consequence of the parts deviating little from the course of this line, they would be greatly immersed in shadow except at an early and late period of the day; the cornices of the Grecian examples, on the contrary, are evidently designed to avoid this as a defect, and to prevent, at any time, a division of the shadow projected from the facia, as, perhaps, desired by the Roman and Italian architects, in the Ionic, Composite, and Corinthian orders.

* Sir William Chambers, an eminent architect, by birth a Swede, but was brought over to England at two years of age. His name will be transmitted to late posterity, as the builder of that great national ornament, Somerset Place. He died March 8th, 1796.

† James Baroggio Vignole, an eminent Italian architect. Died in 1573.

‡ Andrew Palladio, a celebrated Italian architect, of the sixteenth century. He immortalized his name by four books on architecture. He was born in 1508, and died in 1580.

‖ Vincent Scammozzi, a native of Vincenza, the most celebrated architect of his time. He wrote in Italian, "Ideas on Universal Architecture," in ten books, and died in 1616.

PLATE I.

VILLA IN THE COTTAGE STYLE.

This building is built of wood, in the City of New Haven, Con. The sides are covered with pine boards one inch thick, planed and matched; the boards do not exceed four and a half inches in width—the edges thoroughly painted before put on. The roof is shingled. The principal story is ten feet in the clear; the chambers eight feet six inches in the clear, with the exception of one corner in each of the wings, which is cut off by the roof. The Parlor and Dining Room are twelve by eighteen feet each. Bedroom twelve by fifteen feet—Library eight by twelve feet—Kitchen twelve by fourteen feet. There are no fireplaces in the Parlor or Dining Room; the building is warmed by a hot air furnace placed in the cellar. If a furnace is not used, it may be warmed by stoves. The building is well and neatly finished, and cost twenty-four hundred dollars.

PLATE II.

This Plate is designed for a building thirty-four by twenty-six feet, with an addition in the rear for sink room and pantry, nine by seventeen feet. The sides covered with inch boards planed and matched, each board not to exceed four and a half inches in width, the edges well painted before being put on. The roof covered with tin or duck cloth, put on matched boards and thoroughly painted. The principal story ten feet in the clear,—the Chambers eight feet in the clear,—Parlor thirteen by seventeen feet,—Bedroom thirteen by twelve feet,—Kitchen thirteen by nineteen feet.

PLATE III.

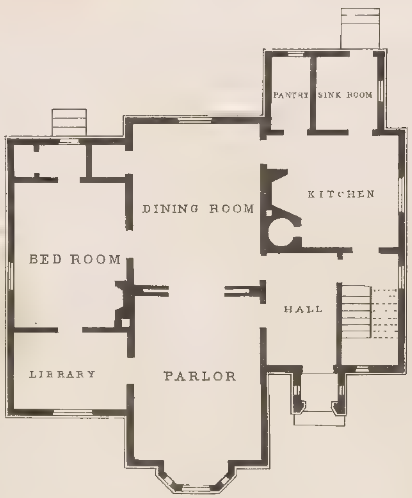
This building is built of wood, in the City of New Haven, Con. The sides are covered with inch boards planed and matched. The boards do not exceed four and a half inches in width, the edges thoroughly painted before being put on. The roofs are covered with tin. The front is twenty-three feet. The wings project eight feet. The rear addition twenty-three by ten feet. The principal story is ten feet in the clear—Chambers eight feet in the clear.

PLATE IV.

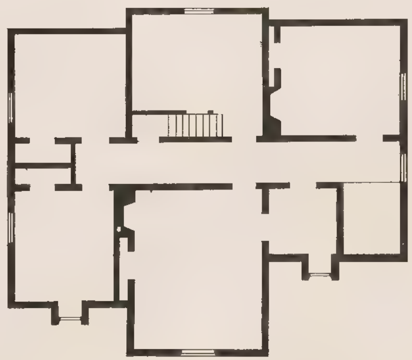
This Villa is built in the City of New Haven, Con. There is no building in the city which has been so much admired for chasteness and beauty as this. It is built of wood, the roof covered with tin. The principal story is ten feet in the clear,—the Chambers eight feet in the clear. The main building is nineteen feet front—wings nine feet each.

FRONT ELEVATION.

END ELEVATION.



GROUND PLAN.



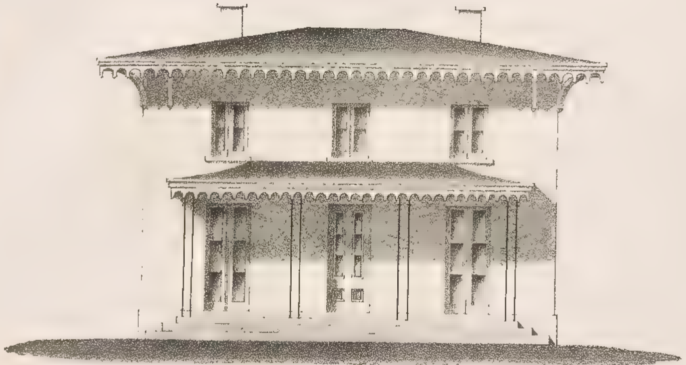
CHAMBER PLAN.

H. Austin, Arch't

VILLA IN THE COTTAE STYLE.

Lith of E.B.&E.C. Kellogg

Pl 2



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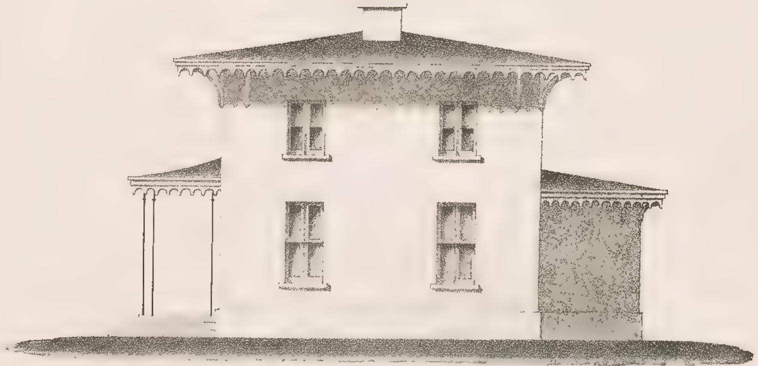

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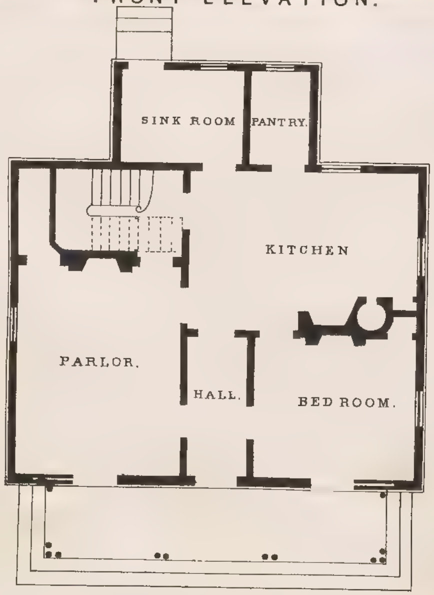

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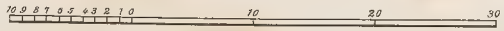
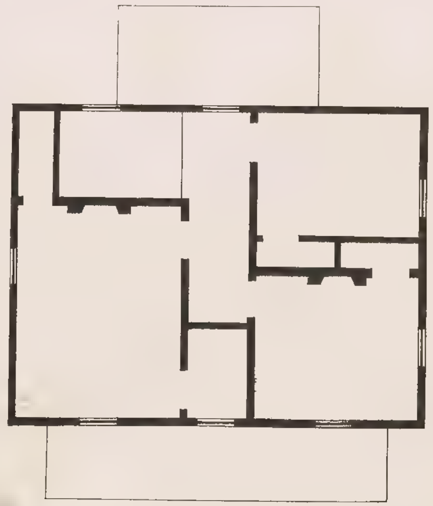


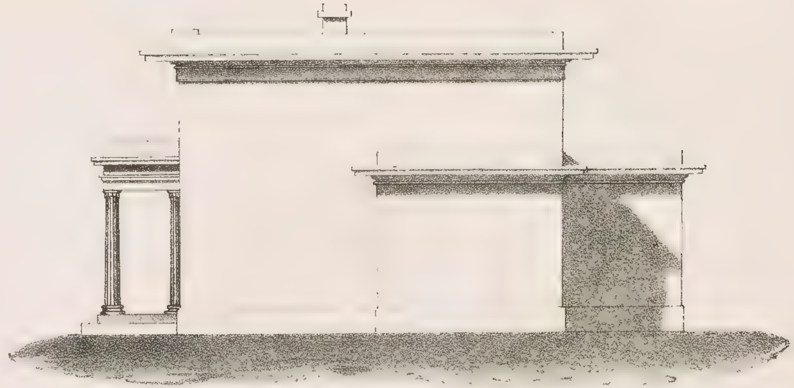


CHAMBER PLAN.

H. Austin Archt

Lith of E B & E C Kellogg

FRONT ELEVATION.



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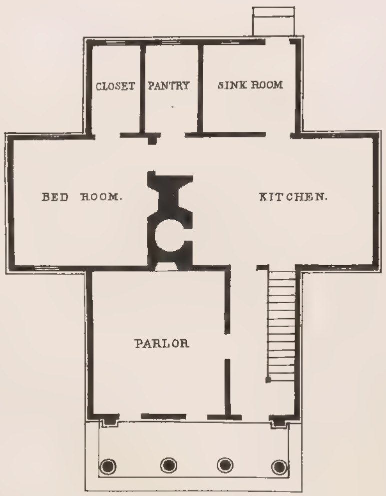

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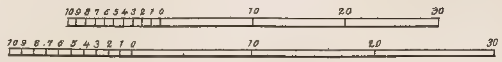
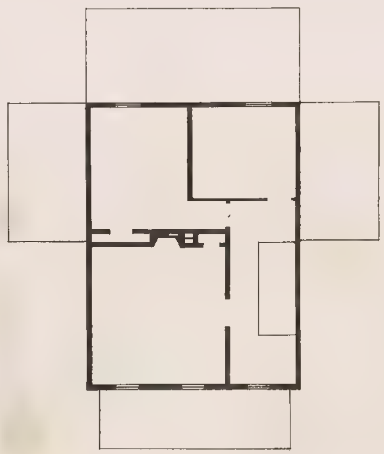
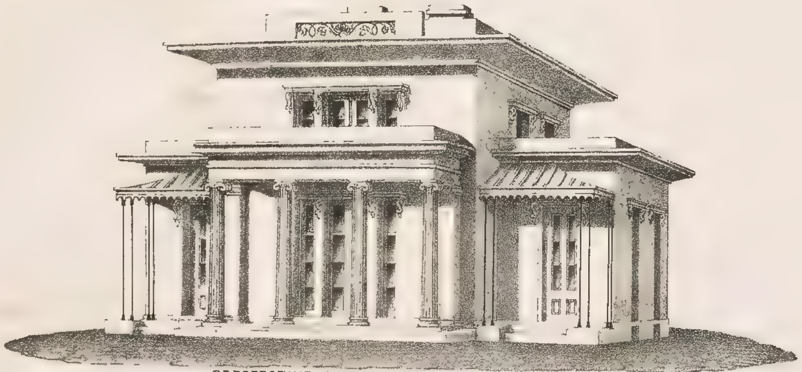




CHAMBER PLAN.

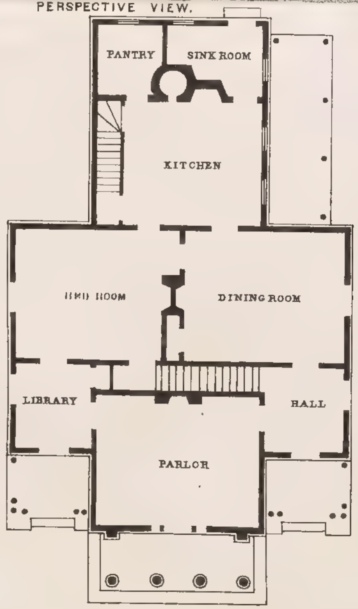
H. Austin Archt.

Lith. of E.B & E.C. Kell

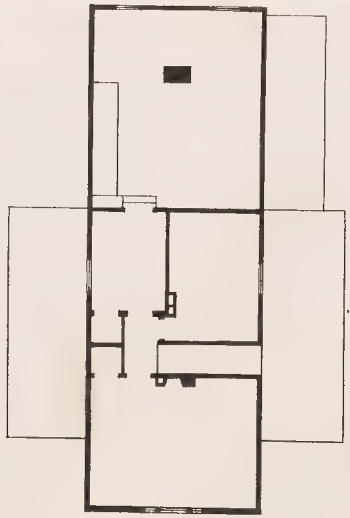


PERSPECTIVE VIEW.

FRONT ELEVATION.



GROUND PLAN.



CHAMBER PLAN.

H. Austin, Archt.

Lith of E.B.&E.C. Kellogg

MATHEMATICAL INSTRUMENTS, SCALES, &C.

PLATE 5.

The Mathematical Instruments, commonly included in a case for the purposes of drawing, are *compasses* with their appendages, viz. *steel drawing pen, pencil holder,* with black lead pencil, *protractor,* or graduated semicircle, *plane scales,* and parallel ruler; to which, are often added other scales and implements, adapted to particular purposes, as land surveying, &c.

The use of the compasses is too well known to require particular explanation. There are in the case, generally, two kinds, one with fixed steel points, and the other with one point fixed. When the unfixed point is taken off, there may be put in its place, a steel drawing pen point. The steel point is put in on the compass, when it is intended therewith to describe circles, or arcs, with ink, which are intended to remain. Occult arcs or such as are to be rubbed out again are most conveniently described with a pencil holder. The other steel pen is used for drawing right lines from any given points in any direction. An explanation of the use of the compasses has been fully given under the head of Practical Geometry.

The Scale is so called from a Greek word, which signifies a wooden measure of length. It is a thin broad rule of wood, ivory, or brass, divided into different lines of various names and use. The best and most useful scales for architectural purposes, are represented in *figs.* 1, 2, *and* 3, *pl.* 5; and are of the exact size in which they are usually made. The graduations of these scales have been made with such care, that I believe they may be relied on, for practice, by them who have not the instruments at hand.

Fig. 1. The breadth is divided into seven parts, and is numbered, (on the left of the scale,) viz. 55, 45, 40, &c. to 20. These numbers are decimals of an inch, as may be seen in the first divisions at the right, that is on the lower part of each division; which is divided into ten parts, or the tenth part of a foot, and the upper part of which is divided into twelve parts, or the twelfth part of a foot. At the right, through these several divisions, they are numbered from 1, 2, 3, &c. to 10: again 1, 2, 3, &c. to 20, and so on. And in the upper division on the right, is a line of *chords,* which is numbered from C, viz. 10, 20, 30, &c. to 90. The construction and use of the line of chords is given on the *plane scale.*

In plotting and making architectural drawings, it is most convenient to work upon a scale of one quarter of an inch to a foot; or half an inch. The scale or division which is numbered 40 at the left, is just one quarter of an inch; and that which is numbered 20 in the lower division, is half an inch.

To take of feet and inches from the scale.—Suppose for example, it was required to take the distance of ten feet and six inches from the scale, (say the quarter inch,) set one foot of the compasses on 10, and open the other leg to the centre of the first division on the left, (which is divided into twelve parts.) The extent will be the distance required. Again, suppose it is required to take the distance of ten feet and five-tenths of a foot, (say from the same scale:) set one foot of the compasses on 10, and open the other leg to the centre of the first division on the lower part of the scale, (which is divided into ten parts.) The extent will be the distance required.

Fig. 2, exhibits the back of the same scale, with inches, eighths, and tenths of inches, &c. And it contains also a *Decimal Diagonal Scale,* for plotting or planning.

The *diagonal scale* is subdivided to hundredth parts of one half and one quarter of an inch; its principle and application will be obvious on inspection; as it may be seen that the perpendiculars are divided into ten equal parts, and through the divisions parallel lines are drawn, the whole length of the scale. Again, the length of the first division is divided both at top and bottom, into ten equal parts, and the points are connected by diagonal lines, so as to take off dimensions or numbers of two or more figures.

Examples.—If the largest divisions be taken as units, the *exterior* smaller divisions will be tenth parts, and the divisions in the heighth will be *hundredth* parts. If the larger divisions be taken as tens, the next smaller will be hundredths, and the smallest thousandths, &c. Each set of divisions being tenth parts of the former ones.

To take the distance representing one and four-tenths from the scale, (say the half inch,) set one foot of the compasses on the upper line, to the larger division 1, and open the other leg to 4, in the subdivision on the right. The extent will be the distance required.

To take a distance equal to 25, set, in like manner, one foot of the compasses on the larger division 2, and extend the other to the subdivision 5, which will be the distance.

For 346, the larger division being in this case taken as hundredths, set one leg in 3, upon the line marked 6 at the end, and extend the other to the diagonal 4, which will be the extent required.

Fig. 3, represents another set of plotting scales for half an inch, one quarter of an inch, three eighths, and one-eighth of an inch to the foot, &c. and subdivided diagonally for greater exactness. The uses of these are too clear to require farther explanation.

Fig. 4, represents another which is called the *Plane Scale.* The construction of this scale is as follows:

1. With the radius you intend for your scale, describe a semicircle A D B, as in *fig.* 6, and from the centre C draw C D perpendicular to A B, which will divide the semicircle into two quadrants, A D, B D: continue the line C D to S: and draw the tangent B T perpendicular to A B.

2. Divide the quadrant B D, into nine equal parts, then will each of these be ten degrees; subdivide each of these parts into single degrees, and if your radius will admit of it, into minutes, or some aliquot parts of a degree greater than minutes.

3. Set one foot of the compasses in B, and transfer each of the divisions of the quadrant B D to the right line B D, then will B D be a line of *Chords*.

4. From the points 10, 20, 30, &c. in the quadrant B D, draw right lines parallel to C D, to cut the radius C B, and they will divide that line into a line of *sines*, which must be numbered from C towards B.

5. If the same line of *sines* be numbered from B towards C, it will become a line of *versed sines*, which may be continued to 180°, if the same divisions be transferred on the same line on the other side of the centre C.

6. From the centre C, through the several divisions of the quadrant B D, draw right lines till they cut the tangent B T, then will the line B T, become a line of *Tangents*.

7. Setting one foot of the compasses in C, extend the other to the several divisions 10, 20, 30, &c. in the tangent line B T, and transfer these extents severally to the right line C S, then will that line be a line of *Secants*.

8. Right lines drawn from A to the several divisions 10, 20, 30, &c. in the quadrant B D, will divide the radius C D into a line of *Semi-tangents*.

9. Divide the quadrant A D into eight equal parts, and from A as a centre, transfer these divisions severally to the right line A D, then will A D be a line of *Rhumbs*, each division answering to 11° 15′ upon the line of chords. The use of this line is for protracting and measuring angles, according to the common division of the mariner's compass. There is another line also on the plane scale, that is not described in this figure, and is marked M L, which is joined to a line of chords; and shows how many miles of easting or westing correspond to a degree of longitude in every latitude. To describe this line let the quadrant A B C, *fig.* 7, be described with the same radius as in *fig.* 6; thence divide the line A B into sixty equal parts, and through each point draw lines parallel to A C, to intersect the arc B C: on B as a centre, transfer the several points of intersection to the right line B C, and then number it from C towards B, from 0 to 60, and it will be the line of longitude. And then take these several lines of chords, sines, tangents, secants, &c., and place them upon a ruler, as represented in *fig.* 4, they will form the instrument called the *Plane Scale*.

Fig. 5, represents an *Instrument*, or *Scale*, called the *Sector;* and consists of two rules or legs, moveable round an axis or joint as a centre, having several scales drawn on the faces, some single, others double; the single scales are like those upon a common *Gunter's Scale;* the double scales are those which proceed from the centre, each being laid twice on the same face of the instrument, viz. once on each leg. From these scales, dimensions or distances are to be taken when the legs of the instrument are placed in an angular position.

The single scales are used exactly like those upon a common *Gunter Scale*.

And those of the double scales, the number of which is seven, viz. the scale of lines marked Lin. or L., the scale of chords marked Cho. or C., the scale of sines marked Sin. or S., the scale of tangents to 45°, and another scale of tangents from 45° to about 76°, both of which are marked Tan. or T., the scale of secants marked Sec. or S., and a scale of Polygons marked Pol.

The scale of lines, chords, sines and tangents under 45°, are all of the same radius, beginning at the centre of the instrument, and terminating near the other extremity of each leg, viz. the lines at the division 10, the chords at 60°, the sines at 90°, and the tangents at 45°; the remainder of the tangents, or those above 45°, are on the scales beginning at a quarter of the length of the former, counted from the centre, where they are marked with 45°, and extend to 76°. The secants also begin at the same distance from the centre, where they are marked with 0, and are from thence continued to 75°. The scales of polygons are set near the inner edge of the legs, and where these scales begin, they are marked with four, and from thence are numbered backward, or towards the centre, to 12.

In describing the use of a sector the terms *lateral distance* and *transverse distance* often occur. By the former is meant the distance taken by the compasses on one of the scales only, beginning at the centre of the sector; and by the latter, the distance taken between any two corresponding divisions of the scales of the same name, the legs of the sector being in an angular position.

The use of the sector depends upon the proportions of the corresponding sides of similar triangles. Suppose for example the triangle A B C, *fig.* 8, be the form of the *sector* when set in an angular position; then if we take A B equal to A C, and A D equal to A E, and join B C and D E, it is evident that B C and D E will be parallel, therefore by the above mentioned proposition, A B : B C :: A D : D E; so whatever part A D is of A B, the same part D E will be of B C: hence if D E be the *chord, sine* or *tangent* of any arc to the radius A D, B C will be the same to the radius A B.

The most useful scales or lines on the *sector* for *Architectural* purposes is the *Line of Lines*, marked L, the *Line of Chords*, marked C, and the line of *Polygons*, marked Pol., as represented in *fig.* 5.

Uses of the Line of Lines.—The line of lines is useful to divide a given line into any number of equal parts, or in any proportion, or to find third and fourth proportionals, or to increase a given line in any proportion.

Example 1.—To divide a given line into any number of equal parts, (suppose 8,) make the length of the given line a transverse distance to 8 and 8, the number of parts proposed; then will the transverse distance of 1 and 1 be one of the eight parts, or the eighth part of the whole: and the transverse distance of 3 and 3 will be 3 of the equal parts, or $\frac{3}{8}$ of the whole line, &c.

Example 2.—If a ship sails 59 miles in 9 hours, how far would she sail in 3 hours at the same rate?

Take 59 in your compasses as a transverse distance and set it off from 9 to 9, then the transverse distance 3 and 3 being measured laterally, will be equal to $19\frac{2}{3}$ miles.

Example 3.—Having a plan or chart constructed upon a scale of 5 feet to an inch, or 5 miles to an inch, it is required to open the sector so that a corresponding scale may be taken from the line of lines.

Make the transverse distance 5 and 5 equal to 1 inch, and this position of the sector will produce the given scale.

Example 4.—It is required to reduce a scale of 6 inches to a degree, another of 3 inches to a degree.

Make the transverse distance 6 and 6, equal to the lateral distance 3 and 3; then set off any distance from the chart laterally, and the corresponding transverse distance will be the reduced distance required.

Example 5.—One side of any triangle being given of any length, to measure the other two sides on the same scale.

Suppose for example the triangle A B C, *fig.* 7, *pl.* 28, being constructed, and the side A B is 15 feet, to find the side A C and B C.

Take A B in your compasses, and apply it transversely to 15 and 15; to this opening of the sector apply the distance A C in your compasses to the same number on both sides of the rule transversely; and where the two points fall, will be the measure on the line of lines of the distance required; the distance A C will fall against 14, 14, and B C against 13, 13, on the line of lines.

Uses of the line of Chords on the Sector.—The use of the line of chords upon the sector, is convenient for protracting any angle, when the paper is so small that an arc cannot be drawn upon it with the radius of a common line of chords.

Suppose for example that the triangle A B D, *fig.* 9, it was required to set off 20° from the point B on the arc B C. Take the radius A B in your compasses, and set it off transversely from 60° to 60° on the chords, then take the transverse extent from 20° to 20° on the chords, and place one foot of the compasses in B, the other will reach to C, and B C will be the arc required. And by the converse operation any angle or arc may be measured, viz. with any radius describe an arc, and then set that radius transversely from 60° to 60°; thence take the distance of the arc intercepted between the two legs, and apply it transversely to the chords, which will show the degrees of the given angle.

Note.—When the angle to be protracted exceeds 60° you must lay off 60°, and the remaining part; or if it be above 120° lay off 60° twice, and then the remaining part. And in a similar manner any arc above 60° may be measured.

Use of the line of Polygons on the Sector.—This line is also very useful to inscribe a regular polygon in a circle, in plotting on Paper. For example, let it be required to inscribe a regular octagon in a circle. Open the sector till the transverse distance 6 and 6 be equal to the radius of a circle; then will the transverse distance 8 and 8 be the side of the inscribed polygon.

To inscribe any regular polygon in a circle.—Suppose, for example, it is required to inscribe a hexagon as represented in the circle A B C D A, *fig.* 10. First find the angle of centre E, by dividing 361° by the number of sides of which the proposed figure is to consist: then from the centre E draw the radiating lines E A, E F, E G, &c. in the angle given by the quotient, and the chord of the angle included between them, will be the side of the hexagon required. Thus, may any regular polygon be described, the radius being always 60°. In the like manner, any regular polygon may be described in a circle with the radius of a common line of chords as represented on the *plane scale.*

Use of the line of chords on the plane scale.—The definition of a chord is given among those under the head of Geometry, and the lines of this name are divided for the purpose of laying off and measuring angles, on the established principle, that the radius or semi-diameter of a circle is equal to the side of a hexagon inscribed in the same circle; or, in other words to the chord of 60°. Hence by taking the extent of the chord of 60° in the compasses, applying one foot to an angular point, and sweeping an arc with the other from leg to leg, (*produced if required,*) the exact measure of the angle may be found.

Example 1.—In the triangle A B D, *fig.* 9, it is required to construct an angle at the point A of the line A B of any number of degrees, suppose 23.

From the line of chords take in your compasses the extent of 60; and setting one foot in A, describe the arc B C; then take 23, the number proposed, from the same line of chords, in your compasses, and set it off from B to C. Join A C, and the angle B A C will contain 23 degrees as required.

Example 2.—Suppose it is required to find the pitch of a roof, the angle of which is 25 degrees.

Take, as above, in your compasses, the extent of 60, and set one foot in A, and describe the arc B C; then take 25, the number proposed, in your compasses and set it off from B to C, and join A C. Then will the angle B A C contain 25° as required.

This method of constructing a roof will be found the simplest and best. For if the span of the roof is given, we have only to take one half for the base. Then (by Problem 2, Trigonometry,) you may ascertain the length of the rafter, and also the perpendicular heighth. Though *Carpenters* generally divide the breadth of the building, or span, into some number of equal parts, and then give one of these parts for the heighth. Suppose it is required to make the height of the roof equal to ' of the span. The angle of $\frac{1}{6}$ will be found to contain 18°. If it be equal to $\frac{1}{5}$ the angle will be 21° 30′. If $\frac{1}{4}$, the angle is 26°. If $\frac{1}{3}$, 33° 30′, &c.

Fig. 11, exhibits an *Instrument* or *Hemistant,* suitably constructed, to take levels and angles of elevation or depression in measuring heights, distances, &c. This instrument should be made of hard wood. The diameter A B of the semicircle may be from 2 to 4 feet in length; the larger, the more accurate; and the straight part which extends out from A B, may be from 4 to 5 inches wide; this straight part, and the graduated semicircle, should be made from one piece of board; and $\frac{3}{4}$ of an inch will be sufficient for the thickness. To construct the graduated semicircle, let the line A B be drawn, and from D with the radius A D sweep the semicircle A C B; and from the centre D, draw D C at right angles with A B, which will divide the semicircle into two quadrants. To divide these quadrants into degrees; first divide them into 9 equal parts each; then will each of these be 10 degrees; thence subdivide each of these parts into single degrees, and if the radius will admit of it, into minutes, or some aliquot part of a degree greater than minutes, and number them from 0° to 90° each quadrant from C.

In taking angles of elevation, you can use a plummet as represented at H, or there may be a line drawn through the centre of the staff G H, and then let the degrees be drawn across the edge of the board, across the graduated semicircle, which will answer all purposes of a plummet, or even better. E represents the level, and is of the same with that of the straight part A B, and the thickness may be from one and a quarter, to one and a

half inches. F represents the spirit level as let into the upper edge.

Fig. 12, shows the plan and section of the same. A represents that part of the level; B the graduated semicircle; C the mortice which is harved out of that part of the level for the staff to slide through; E represents the screw to hold it fast when the level is placed in its right position.

A, at the upper part of this plate, shows a representation of a *Draught board,* commonly called the *Trustle board,* to which the paper used in drawing is to be fixed. This board is composed of a frame of mahogany or other hard wood: the edges of the frame should be made perfectly straight and square, with a panel about half the thickness of the frame, which is to be let in from the back, and to lie in a rabbet, in the frame, and there to be secured by small buttons; B shows a section of the board, and the buttons by which the panel is kept in its place. Eight in number will be sufficient for a common sized board. It would not be amiss before making the board, to ascertain the size of the paper to be used, and make the panel about two inches less than the sheet. In applying this board to use, lay the paper on a table, and moisten one side of it with a sponge; then place the board upside down near it; take out the panel and lay it on the paper, (one inch of which will extend beyond the panel all around,) take hold of the edges of the paper and lift them both into the frame; then fasten the buttons, and when it becomes dry it will be perfectly smooth.

C and D represents the T square for drawing right lines; the blade as represented at D, should not exceed $\frac{3}{16}$ of an inch in thickness.

E represents Pool's Geometrical Protractor. It is an instrument highly useful for the practical illustrations of elementary principles. I have used it for a number of years, and would recommend it to all Architects on account of its simplicity, and the ease with which it can be applied. Those wishing to purchase, will find them in all of our principal cities.

ELEMENTS OF GEOMETRY.

INTRODUCTORY REMARKS.

It is necessary to the understanding of this work, that the Practical Builder should have a knowledge of the properties, relations, and positions of lines, as explained in some common treatise for representing geometrical truths.

The course adopted by the Ancients is generally regarded as the most satisfactory; it not only accustoms the Architect to reason correctly, which is indispensable, but also directs him in a course distinct from analysis, which in important mathematical research renders propositions clear to his mind, without the assistance of rule and compass. It is only by being competent to demonstrate from principles previously established, and continue a chain of reasoning in such a manner as will render the conclusion from one truth, part of the data requisite for the proof of a succeeding proposition, that the Architect can become distinguished.

This science being fully explained in the writings of Archimedes and Euclid, and having served as a model for all successive works, I have thought proper to adopt in this treatise, so much of those "elements" as will be required by the Architect in building.

If the builder attempts to apply the rules of Geometry to his art, without the knowledge of theory, his efforts will prove abortive; or should he at all succeed, yet his work would be void of proportion and incomplete. It is only by a competent knowledge of this science, that the Architect can accomplish his work in a simple and elegant style; or the Artist so construct his lines as to be able to complete his design.

The completion of a design is usually left to the skill of the workman, who is supposed to be thoroughly acquainted with the execution of the task he undertakes. But if he is ignorant of the "geometrical construction" of the object to be executed, he is not only incompetent for the task he has commenced, but at every additional advance, displays to the world his inability and ignorance: such a person, so long as he remains uninformed, will be unprepared for any undertaking, and his labors in no manner useful, unless under the guidance and direction of another.

The definitions, theorems and problems which are here subjoined, are intended to instruct the uninformed, and prepare him to proceed with that part of this work, wherein their application will be found absolutely necessary. The terms are as clearly stated as the nature of the present work will admit; and the theorems and problems are placed in succession, so that nothing is introduced "as taken for granted," but every thing proved and explained.

DEFINITION OF GEOMETRICAL TERMS.

PLATE 6.

1. A *Point.*—Abstractly considered, a point is said to have position without magnitude, and is therefore represented to the eye by the smallest *visible mark*, or *dot*.

2. A *Line* is length without breadth or thickness; it is represented by the motion of a visible point.

3. A *right* or *strait line* is the shortest that can be drawn between two given points—as A B *fig.* 1.

Every line which is not strait, or composed of strait lines, is a *curve line*, thus A B *fig.* 2.

4. A *superficies* or *surface* is extension of length and breadth without thickness.

5. A *Plane Superficies* is a flat surface which coincides in every point, with a right line.

6. A *Plane Figure*, or *Diagram*, is the lineal representation of any object on a plane surface. If the lines forming the figure be straight, the figure is said to be *rectilineal.*

7. *Parallel Lines* are in every part equally distant from each other, and cannot meet how far soever, either way, they be produced, as A B and C D *fig.* 3 *and* 4.

8. An *Angle* is the space enclosed between two lines meeting in a point. See A B C *fig.* 5; the letter A denotes the vertex or angular point, this is a rectilineal angle, which is formed by two straight lines meeting in the point A.

9. *Converging Lines* are right lines so inclined to each other as to meet if produced to a certain point. Thus, A B and C D *fig.* 6, converge to each other, and if produced will meet in the angle O.

10. *Right and Oblique Angles.*—If one right line meets another, so as to make the angles on each side equal, each angle is called a *Right Angle*, and the line which meets the base, or lower line, is called a *perpendicular.* Thus in *fig.* 7, the line C D is drawn perpendicular to A B and makes the angles on both sides of C D equal; each of these angles is called a *right angle.* In *fig*, 8, the line C D does not make the angles on each side of it equal to each other; therefore C D is said to be drawn obliquely to the lines A B; while in *fig.* 7, C D is at right angles with the line A B.

11. An *Acute Angle* is less than a right angle.

12. An *Obtuse Angle* is greater than a right angle. In *fig.* 8, the line C D makes the angles on each side of it unequal; therefore, one must be greater than the other: the greater is an *obtuse*, and the lesser, an *acute* angle: for it is obvious that whatever be the position of the line C D, relative to A B, the excess of one angle more than a right angle the other must want in order to be equal to the same.

Examples.—*Fig.* 9 is an *Acute* Angle; *fig.* 10, a *Right* Angle, and *fig.* 11, an *Obtuse* Angle.

13. A *Plane Triangle* is a space inclosed by three right lines, as A B C, *fig.* 12.

14. A *Right-angled Triangle* is that which has one right angle; as A B C, *fig.* 12. The longest side is called the *Hypothenuse*, and the other two, *Legs*, or base and perpendicular. Thus, A C the hypothenuse, A B the base, and B C the perpendicular.

15. An *Acute-angled Triangle* is a triangle which has all its angles acute; as *fig.* 13 *and* 14.

16. An *Obtuse-angled Triangle* is a triangle having one obtuse angle; as *fig.* 15.

17. An *Equilateral Triangle* is a triangle having all its sides equal; as *fig.* 13.

18. An *Isosceles Triangle* is a triangle having two equal sides; as *fig.* 14.

19. A *Salene Triangle* is a triangle having no two of its sides equal; as *fig.* 15.

20. A *Parallelogram* is a figure of which the opposite sides are parallels. Thus the *fig.* 16, 17, 18 *and* 19 are parallelograms.

21. When the *parallelogram* has a right angle, it is called a *rectangle.* Thus the *figures* 16 *and* 17 are rectangles.

22. If the sides of the *rectangle* be equal, it is called a square. See *fig.* 16.

23. If the two adjacent sides be unequal, the rectangle is termed an *oblong;* as *fig.* 17.

24. If only two opposite angles of a parallelogram be equal, the figure is called a *rhombus;* as *figures* 18 *and* 19.

25. If two adjacent sides of a rhombus be equal, the figure is called a *rhomboid;* as *fig.* 18.

26. Every figure, inclosed by four right lines, is called a *quadrangle*, or *quadrilateral.* Thus *figs.* 16, 17, 18, 19, 20 *and* 21, are quadrangles or quadrilaterals.

27. When all the sides of a quadrilateral are unequal, it is called a *trapezium;* as *fig.* 20.

28. If two sides of a trapezium be parallel, it is called a *tra-pe-zo-id:* as *fig.* 21.

29. Figures having equal sides and equal angles, or equilateral and equiangular figures, formed by more than four right lines, are called regular *polygons.*

30. A regular polygon of five sides is called a *pentagon;* as *fig.* 22.

31. A regular polygon of six sides is called a *hexagon;* as *fig.* 23.

32. A regular polygon of seven sides is called a *heptagon:* as *fig.* 24.

33. A regular polygon of eight sides is called an *octagon;* as *fig.* 25. Of nine sides, an *eneagon* or *nonagon.* Of ten sides, a *decagon.* Of eleven sides, an *undecagon.* Of twelve sides, a *duodecagon.* Of fifteen sides, a *quindecagon;* but polygons having more than twelve sides are commonly expressed as such, with the number of sides given.

34. A circle is a plane figure formed by one uniform curved line which is called its circumference; as *fig.* 26.

35. The centre of a circle is the point in the middle of it, as A, in *fig.* 26, and the line A B drawn from the centre to the circumference, is the radius of the circle; all lines thus drawn are equal.

36. The diameter of a circle is a right line drawn through the centre and terminated by the circumference; as A B *fig.* 27.

37. A chord of a circle is a right line drawn from one point of a circle to another, and dividing it into unequal or equal parts or segments. In the latter case, the chord is also the diameter. Thus C D *fig.* 28, is a chord as well as A B *fig.* 27.

38. A *Semi-circle* is one half of a circle, as divided into two equal parts by the diameter.

39. A *Segment* of a circle is that portion which is cut off by a chord. Thus, in *figs.* 28 *and* 29, C D and F H are segments; and *fig.* 30, though a semi-circle, is still a segment, terminated by the diameter.

40. A *Sector* is the portion of a circle formed by two radii and the intercepted part of the circumference; as A B C, *fig.* 31.

41. A *Quadrant* is the fourth part, or quarter of a circle; or in other words, a sector contained by two radii, forming a right angle at the centre, and the intercepted part of the circumference; as A B C *fig.* 32.

42. An *Arc* is any portion of the circumference of a circle.

43. The *Sine* of an arc is a line drawn from one end of the arc perpendicular to a diameter drawn through the other end of the same arc. Thus K B *fig.* 33, is the sine of the arc A B, K B being a line drawn from one end B of that arc, perpendicular to D A which is the diameter passing through the other end A of the arc.

44. The *Co-sine* of an arc is the sine of the complement of that arc, or of what that arc wants to make it a quadrant; thus A H being a quadrant, the arc B H is the complement of the arc A B; B E is the sine of the arc B H, or the co-sine of the arc A B.

45. The *Versed* sine of an arc, is that part of the diameter contained between the sine and the arc; thus K A is the versed sine of the arc A B, and D C K is the versed sine of the arc D H B.

46. The *Tangent* of an arc is a right line drawn perpendicular to the diameter passing through one end of the arc, and terminated by a line drawn from the centre through the other end of the arc; thus A G is the tangent of the arc A B.

47. The *Co-tangent* of an arc is the tangent of the complement of that arc to a quadrant; thus H F is the tangent of the arc H B or the co-tangent of the arc A B.

48. The *Secant* of an arc is a right line drawn from the centre through one end of the arc to meet the tangent drawn from the other end; thus C G is the secant of the arc A B.

49. The *Co-secant* of an arc is the secant of the complement of that arc to a quadrant; thus C F is the secant of the arc B H, or co-secant of the arc A B.

50. What an arc wants of being a semicircle is called the supplement of the arc; thus, the arc D H B is the supplement of the arc A B. The sine, tangent or secant of an arc is the same as the sine, tangent or secant of its supplement; thus the sine of eighty degrees is equal to the sine of one hundred, and the sine of seventy is equal to the sine of one hundred and ten, &c.

51. *Equivalent figures* are such as have equal surfaces, without regard to their form.

52. *Identical figures* are such as would entirely coincide, if the one be applied to the other.

53. In *Equiangular figures*, the sides which contain the equal angles, and which adjoin equal angles, are homologous.

54. Two figures are similar, when the angles of the one are equal to the angles of the other each to each, and the homologous sides are proportionals.

55. In two circles, similar sectors, similar arcs, or segments, are those which have equal angles at the centre. Thus if the sector A B C, *fig.* 34, be similar to the sector D E F in *fig.* 35, then the angle A B C will be equal to the angle D E F; or if the arc A C be similar to the arc D F, then the angle at B will be equal to the angle at E. Also if the segment G M H *fig.* 36, be similar to the segment K N L in *fig.* 37, then the angle I, will be equal to the angle R.

56. The altitude of a figure is a right line drawn from the top, or vertical angle perpendicular to the base or opposite side, or to the base produced or continued. Thus, C D is the altitude of the triangle A B C in *fig.* 38, or C D is the altitude of A B C *fig.* 39.

The altitude of a parallelogram is the perpendicular which measures the distance of two opposite sides, taken as bases. Thus, E F *fig.* 40, is the altitude of the parallelogram D B.

57. The area of a figure is the quantity of surface containing a certain number of units of any given scale; as of inches, feet, yards, &c.

DEFINITIONS OF THE ELLIPSE.

58. That portion of the primary line terminated at each extremity by the vertices of the curve, is called the major, or transverse axis.

59. A straight line, drawn perpendicularly to the axis major, from any point in it to meet the curve, is called an ordinate.

60. The middle of the axis major is called the centre of the figure. Thus, A A *fig.* 41, is the axis major, P M an ordinate to it, and the point C in the middle of A A is the centre of the ellipse A M A.

61. A straight line drawn through the centre perpendicularly to the axis major, and terminated by the curve, is called the axis minor, or conjugate axis.

62. A third proportional to the axis major and minor, is called the parameter, or the latus rectum of the axis.

63. That point in the axis, cut by an ordinate, which is equal to half the parameter, is called the focus. In *fig.* 42, B B, drawn through C, in the semi-axis-minor; and if L L be a third proportional to A A, B B, then L L is the parameter, and the point F, where it cuts A A, is the focus. Or thus; if two pins are fixed at the points A and B, as in *fig.* 43, a string being put about them, and the ends fastened together at C; the point C being moved round, keeping the string stretched, it will describe a curve called an ellipsis, and the two points A and B, about which the string is made to revolve, is the foci.

64. Any line drawn through the centre, and terminated at each extremity by the curve, is called a diameter.

65. A diameter which is parallel to a tangent at one extremity of another diameter, is called a conjugate diameter to that other diameter.

66. A straight line parallel to a tangent, at the extremity of any diameter, terminated at one extremity by that diameter, and the curve at the other, is called an ordinate to that diameter.

67. The portion of a diameter between the centre and an ordinate, is called the abscissa of that ordinate, or of that diameter. In *fig.* 44, the straight line A A, drawn through the centre, C, is a diameter, and if S T be a tangent at A, and the diameter B B be drawn parallel to S T, the diameter is called the conjugate diameter of A A and P M, parallel to S T or B B, is an ordinate to the diameter A A; and the distance C P, on the diameter A A, is called the abscissa.

EXPLANATION OF TERMS AND SIGNS USED IN THIS WORK CONTINUED.

68. An Axiom is a self-evident proposition.

A Theorem is a truth, which becomes evident by means of a train of reasoning called a demonstration.

A Problem is a question proposed, which requires a solution.

A Lemma is a subsidiary truth, employed for the demonstration of a theorem, or the solution of a problem.

The common name proposition, is applied indifferently to theorems, problems, and lemmas.

A Corollary is an obvious consequence deduced from one or several propositions.

A Scholium is a remark on one or several preceding propositions, which tends to point out their connexion, their use, their restriction, or their extension.

An Hypothesis is a supposition, made either in the enunciation of a proposition, or in the course of a demonstration.

The sign + is pronounced, *plus:* it indicates addition, and denotes that whatever number or quantity follows the sign, must be added to those that go before it; thus 6 +5 signifies that 5 is to be added to 6, or A+B implies that the quantities represented by A and B, are to be added together.

Again, A+B+C+D implies that B is to be added to A; C to the sum of A, and B and D, to the sum of A, B, C.

The Sign — is pronounced minus; it indicates subtraction, and denotes that the number following it must be subtracted from those going before it; thus 7—3 signifies that 3 must be subtracted from 7 or m—n implies that the quantity represented by n, is to be subtracted from that represented by m. Suppose for instance that m is 7, and n 3; then 7—3 will be 4. Therefore m—n denotes the remainder arising by subtracting n from m.

The sign × indicates multiplication; and shows that the numbers placed before and after it are to be multiplied; thus 5×8 signifies that 5 is to be multiplied by 8, which makes 40; and 4×6×2, signifies the continued product of 4 by 6 and by 2 which makes 48; or A×B signifies that A is to be multiplied by B.

The sign ÷ indicates division, and signifies that the numbers that stand before it are to be divided by the number following it, as 75÷16 shows that 75 is to be divided by 16; division may also be denoted by placing two points between the numbers; thus 75:16 represents 75 divided by 16, or by placing the numbers thus, $\frac{75}{16}$ which signifies 75 divided by 16, ( ) or —. either of these marks are used for connecting numbers together: Thus $\overline{4+5\times8}$, or (4+5) +8, signifies that the sum of 4 and 5 are to be multiplied by 8.

The sign = indicates equality, and shows that the numbers or quantities placed before it are equal to those following: thus 8×10=80, or 8 multiplied by 10 is equal to 80, and 6+2×7=56, or A×B=C D.

The sign : :: : signifies proportion, and is marked thus 6:12::10:20, that is, as 6 is to 12, so is 10 to 20, or A:B::C:D, that is, as A is to B, so is C to D.

Roots are usually represented by the following characters or exponents:—

$\sqrt{3}$, or $3^{\frac{1}{2}}$, denotes the square root of the number 3.

$\sqrt[3]{5}$, or $5^{\frac{1}{3}}$, denotes the cube root of the number 5.

7^2, denotes that the number 7 is to be squared.

8^3, denotes that the number 8 is to be cubed.

The square root of the line A B is designated by A B 2; its cube by A B 3.

A number placed before a line, or a quantity, serves as a multiplier to that line or quantity; thus 3 A B, signifies that the line A B is taken three times; $\frac{1}{2}$ A signifies the half of the angle A.

° Signifies Degrees; thus 35° represents 35 degrees.

′ Signifies Minutes thus; thus 26′ or 26 minutes

″ Signifies Seconds; thus 54″ or 54 seconds.

‴ Signifies Thirds or sixtieth parts of seconds; thus 43‴ or 43 thirds.

Again, 35° 26′ 54″ 43‴ implies 35 degrees, 26 minutes, 54 seconds, and 43 thirds.

S signifies Sine.—Sec. signifies Secant.—Tan. signifies Tangent.

Co-sine, Co-tangent, or Co-secant of an arc signifies the sine, tangent or secant of the complement of that arc respectively.

∠ Signifies Angle, with an s at top angles $\angle^s$ or with a d angled $\angle^d$.

△ Signifies Triangle. □ Signifies a Square.

AXIOMS.

69. 1. Things that are equal to the same thing, or to equal things, are equal to one another. 2. If equal things be added to equals, the sums are equal. 3. If equal things be taken from equals, the remainders will be equal. 4. If equal things be added to unequals, the sums are unequal. 5. The halves of equal things are equal, and double the parts of equal things are equal. 6. Magnitudes that mutually agree, or fill equal spaces are equal to one another. 7. The whole is greater than a part, and equal to the sum of all its parts. 8. Only one right line can be drawn from one point to another. 9. Two right lines cannot be drawn through the same point parallel to another right line, without coinciding with each other. 10. All right angles are equal to each other. 11. Equal circles have equal semi-diameters.

POSTULATES OR DEMANDS.

70. A Postulate signifies something which may be assumed as granted: 1. That a right line may be drawn from any one point to any other point. 2. That a right line may be produced or continued at pleasure in a right line. 3. That a circle may be described from any centre with any radius.

THEOREMS.

THEOREM I.

71. *All right angles are equal to each other.* Let the straight line C D *figs.* 45 *and* 46, *pl.* 6, be perpendicular to A B, and G H to E F; the angles A C D and E G H will be equal to each other.

Demonstration.—Take the four distances C A, C B, G E, G F, all equal; the distance A B will be equal to the distance E F, and the line E F being placed on A B, so that the point E falls on A, the point F will fall on B. Those two lines will thus coincide entirely; for otherwise there would be two straight lines extending from A to B, which (Def. 69. 8.) is impossible: and hence G the middle point of E F, will fall on C, the middle point of A B. The side G E being thus applied to C A, the side G H must fall on C D. For suppose, if possible, that it falls on a line C K, different from C D: then since by hypothesis (10) the angle E G H=HGF, A C K would in that case be equal to K C B. But the angle A C K is greater than A C D; and K C B is smaller than B C D, but by hypothesis A C D=

BCD; hence ACK is greater than KCB. Therefore the line GH cannot fall on a line CK different from CD; therefore it falls on CD, and the angle EGH on ACD; therefore all right angles are equal to each other (69. 6.)

THEOREM II.

72. Every straight line, which meets another, makes with it two adjacent angles, the sum of which is equal to two right angles. Let AB and CD *fig.* 47, be the straight lines meeting each other at C, then will the angle ACD + the angle DCB be equal to two right angles.

At the point C, erect CE perpendicular to AB. The angle ACD, is the sum of the angles ACE, ECD; therefore ACD + BCD is the sum of the three angles ACE, ECD, BCD; but the first of those three angles, is a right angle, and the other two together make up the right angle BCE hence the sum of the two angles ACD and BCD is equal to two right angles.

73. Cor. 1. If one of the angles ACD, BCD in *fig.* 48 is right, the other must be right also.

74. Cor. 2. If the line DE is perpendicular to AB, reciprocally AB will be perpendicular to DE.

For since DE is perpendicular to AB, the angle ACD must be equal to its adjacent one DCB and both of them must be right. But since ACD is a right angle, its adjacent one ACE must also be right; hence the angle ACE=ACD; therefore AB is perpendicular to DE.

75. Cor. 3. *fg.* 49. The sum of all the successive angles BAC, CAD, DAE, EAF, formed on the same side of a straight line BF, is equal to two right angles; because their sum is equal to that of the two adjacent angles, BAC, CAF.

THEOREM III.

76. Whenever two straight lines intersect each other, the opposite or vertical angles which they form are equal.

Let AB and DE *fig.* 50, be the given straight lines intersecting each other at C; then is the angle ECB=ACD, and the angle ACE=DCB.

For since DE is a straight line, the sum of the angles ACD, ACE, is equal to two right angles; and since AB is a straight line, the sum of the angles ACE, BCE, is also equal to two right angles; hence the sum ACD+ACE is equal to the sum ACE+BCE. Take away from both, the same angle ACE; there remains the angle ACD, is equal to its opposite or vertical angle BCE. It may be shown in the same manner, that the angle ACE is equal to its opposite angle BCD.

77. Scholium. The four angles formed about a point by two straight lines which intersect each other, are together equal to four right angles; for the sum of the two angles ACE, BCE, is equal to two right angles; and the other two ACD, BCD, have the same value; therefore the sum of the four, is four right angles.

In general, if any number of straight lines, CA, CB, CD, &c. as in *fig.* 51, meet in a point C, the sum of all the successive angles ACB, BCD, DCE, ECF, FCA, will be equal to four right angles; for if four right angles were formed about the point C by means of two lines perpendicular to each other, the same space would be occupied either by the four right angles, or by the successive angles ACB, BCD, DCE, ECF, FCA.

THEOREM IV.

78. Two triangles are equal when an angle and the two sides which contain it, in the one are respectively equal to an angle and the two sides which contain it, in the other.

Let the angle A *figs.* 52 *and* 53 be equal to D, the side AC equal to the side DF, the side AB equal to DE; then will the triangle ABC be equal to DEF. For these triangles may be applied to each other, so that they shall perfectly coincide. If the side DE be placed on its equal AB the point D will fall on A, and the point E on B; and since the angle D is equal to the angle A, when the side DE is placed on AB, the side DF will take the direction AC. Besides, DF is equal to AC; therefore the point F will fall on C, and the third side EF will exactly cover the third side BC; therefore (69. 6.) the triangle DEF is equal to the triangle ABC.

79. Cor. When in two triangles, these three things are equal, namely, the angle A=D, the side AB=DE, and the side AC=DF, the other three are equal also, namely, the angle B=E the angle C=F and the side BC=EF.

THEOREM V.

80. In an isosceles triangle, the angles opposite to the equal sides are equal.

Let the side AB *fig.* 54, be equal to AC, the angle C will be equal to B.

Join A the vertex, and D the middle point of the base BC. The triangles ADB, ADC, have all the sides of the one respectively equal to those of the other, AD being common, AB=AC (hyp.) and BD=DC by construction; therefore by the last proposition, the angle B is equal to the angle C.

81. Cor. An equilateral triangle is likewise equiangular, that is to say has all its angles equal.

82. Scholium. The equality of the triangles ABD, ACD, proves also that the angle BAD is equal to DAC, and BDA to ADC; hence the latter two are right angles; hence the line drawn from the vertex of an isosceles triangle, to the middle point of its base, is perpendicular to that base, and divides the angle at the vertex into two equal parts.

In a triangle which is not isosceles, any side may be assumed indifferently as the base; and the vertex is, in that case, the vertex of the opposite angle. In an isosceles triangle, however, that side is specially assumed as the base which is not equal to either of the other two.

THEOREM VI.

83. From a given point without a straight line, only one perpendicular can be drawn to that line.

Let A *fig.* 55, be the point, and DE the given line.

Let us suppose we can draw two, AB and AC. Produce one of them AB, till BF is equal to AB and joins FC.

The triangle CBF is equal to ABC; for the angles CBF and CBA are right, the side CB is common and the side BF=AB; therefore (78) those triangles are equal, and the angle BCB=BCA. The angle BCA is right by hypothesis; therefore BCF must be right also. But if the adjacent angles BCA, BCF, are together equal to two right angles, the line ACF must be straight; from whence it follows, that between the same two points A and F, two straight lines can be drawn: which is impossible; hence it is equally impossible that two perpendiculars can be drawn from the same point to the same straight line.

84. Scholium. At a given point C in the line AB, it is eqally impossible to erect two perpendiculars to that line, for (see the diagram of Art. 72) if CD and CE were those two perpendiculars, the angle BCD would be right as well as BCE, and the part would thus be equal to the whole.

THEOREM VII.

85. Two right-angled triangles are equal when the hypothenuse and a side of the one are respectively equal to the hypothenuse and a side of the other.

Suppose the hypothenuse AC=DF, *figs.* 56 *and* 57, and the side AB=DE; the right-angled triangle ABC will be equal to the right-angled triangle DEF.

Their equality would be manifest, if the third sides BC and EF were equal. If possible suppose that those sides are not equal, and that BC is greater. Take BG=EF; and join AG. The triangle ABG is equal to DEF; for the right angles B and E are equal, the side AB=DE, and BG=EF; hence these triangles are equal (78) and consequently AG=DF. Now (Hyp.) we have DF=AC; and therefore AG=AC. But the oblique line AC cannot be equal to AG, which lies nearer the perpendicular AB; therefore it is impossible that BC can differ from EF; therefore the triangles ABC and DEF are equal.

THEOREM VIII.

86. If a straight line falling upon two other straight lines makes the alternate angles equal to one another, these two straight lines are parallel.

Let the straight line E F, *fig.* 58, which falls upon the two straight lines AB, CD, make the alternate angles A E F, E F D equal to one another; A B is parallel to C D.

For if it be not parallel, A B and C D being produced shall meet either towards B, D, or towards A, C; let them be produced and meet towards B, D, in the point G; therefore G E F is a triangle, and its exterior angle A E F is greater than the interior and opposite angle E F G; but it is also equal to it which is impossible: Therefore, A B and C D being produced do not meet towards B, D. In like manner it may be demonstrated that they do not meet towards A, C; but those straight lines which meet neither way, though produced ever so far are parallel (7) to one another, A B therefore is parallel to C D.

THEOREM IX.

87. If a straight line falling upon two other straight lines makes the exterior angle equal to the interior and opposite upon the same side of the line; or makes the interior angles upon the same side together equal to two right angles; the two straight lines are parallel to one another. Let the straight line E F *fig.* 59, which falls upon the two straight lines A B, C D, make the exterior angle E G B equal to G H D, the interior and opposite angle upon the same side; or let it make the interior angles on the same side B G H, G H D together equal to two right angles; A B is parallel to C D.

Because the angle E G B is equal to the angle G H D, and also to the angle A G H, the angle A G H is equal to the angle G H D; and they are the alternate angles; therefore A B is parallel (86) to C D. Again, because the angles B G H, G H D are equal (by. hyp.) to right angles, and A G H, B G H are also equal (71. 1.) to two right angles, the angles A G H, B G H are equal to the angles B G H, G H D: Take away the common angle B G H; therefore the remaining angle A G H is equal to the remaining angle G H D; and they are alternate angles; therefore A B is parallel to C D.

THEOREM X.

88. To draw a straight line through a given point parallel to a given straight line.

Let A *fig.* 60, be the given point, and B C the given straight line, it is required to draw a straight line through the point A parallel to the straight line B C.

In B C take any point D, and join A D; and at the point A in the straight line A D make the angle D A E equal to the angle A D C: and produce the straight line E A to F.

Because the straight line A D which meets the two straight lines B C, E F, makes the alternate angles E A D, A D C equal to one another, E F is parallel (87) to B C, therefore the straight line E A F is drawn through the given point A parallel to the given straight line B C, which was to be done.

THEOREM XI.

89. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

Let A B C *fig.* 61, be a triangle, and let one of its sides B C be produced to D; the exterior angle A C D is equal to the two interior and opposite angles C A B, A B C; and the three interior angles of the triangle, viz. A B C, B C A, C A B, are together equal to two right angles.

Through the point C draw C E parallel (87) to the straight line A B; and because A B is parallel to C E and A C meets them, the alternate angles B A C, A C E are equal. Again, because A B is parallel to C E, and B D falls upon them, the exterior angle E C D is equal to the interior and opposite angle A B C, but the angle A C E was shown to be equal to the angle B A C; therefore the whole exterior angle A C D is equal to the two interior and opposite angles C A B, A B C, to these angles add the angle A C B, and the angles A C D, A C B are equal to the three angles C B A, B A C, A C B, but the angles A C D, A C B, are equal (71) to two right angles; therefore also the angles C B A, B A C, A C B are equal to two right angles.

90. Cor. 1. All the interior angles of any rectilineal figure are equal to twice as many right angles as the figure has sides, wanting four right angles.

For any rectilineal figure A B C D E *fig.* 62, can be divided into as many triangles as the figure has sides, by drawing straight lines from a point F within the figure to each of its angles. And by the preceding proposition, all the angles of these triangles, are equal to twice as many right angles as there are triangles, that is, as there are sides of the figure; and the same angles are equal to the angles of the figure, together with the angles at the point F, which is the common vertex of the triangles; that is, together with four right angles. Therefore twice as many right angles as the figure has sides, are equal to all the angles of the figure, together with four right angles, that is, the angles of the figure are equal to twice as many right angles as the figure has sides, wanting four.

91. Cor. 2. All the exterior angles of any rectilineal figure are together equal to four right angles.

Because every interior angle A B C *fig.* 63, with its adjacent exterior A B D is equal (71) to two right angles; therefore all the interior, together with all the exterior angles of the figure, are equal to twice as many right angles as there are sides of the figure; that is by the foregoing corollery, they are equal to all the interior angles of the figure, together with four right angles; therefore all the exterior angles are equal to four right angles.

THEOREM XII.

92. The opposite sides and angles of a parallelogram are equal to one another and the diameter bisects it, that is, divides it into two equal parts.

Note.—A parallelogram is a four sided figure; (20) of which the opposite sides are parallel; and the diameter is the straight line joining two of its opposite angles.

Let A C D B *fig.* 64, be a parallelogram of which B C is a diameter; the opposite sides and angles of the figure, are equal to one another; and the diameter B C bisects it.

Because A B is parallel to C D, and B C meets them, the alternate angles A B C, B C D are equal to one another; and because A C is parallel to B D, and B C meets them, the ultimate angles A C B, C B D are equal to one another, wherefore the two triangles A B C, C B D have two angles A B C, B C A in one, equal to two angles B C D, C B D in the other, each to each and the side B C which is adjacent to these equal angles, common to the two triangles; therefore their other sides are equal, each to each, and the third angle of the one to the third angle of the other, viz: the side A B to the side C D, and A C to B D, and the angle B A C equal to the angle B D C. And because the angle A B C is equal to the angle B C D, and the angle C B D to the angle A C B, the whole angle A B D is equal to the whole angle A C D: and the angle B A C has been shown to be equal to the angle B D C; therefore the opposite sides and angles of a parallelogram are equal to one another, also its diameter bisects it; for A B being equal to C D, and B C common, the two A B, B C are equal to the two D C, C B, each to each; now the angle A B C is equal to the angle B C D; therefore the triangle A B C is equal (78) to the triangle B C D, and the diameter B C divides the parallelogram A C D B into two equal parts.

THEOREM XIII.

93. Parallelograms upon the same base and between the same parallels, are equal to one another.

Let the parallelograms A B C D, E B C F in *figs.* 65, 66, *and* 67, be upon the same base B C, and between the same parallels A F, B C; the parallelogram A B C D is equal to the parallelogram E B C F.

If the sides A D, D F of the parallelograms A B C D, D B C E opposite to the base B C be terminated in the same point D; it is plain that each of the parallelograms is double (92) of the triangle B D C; and they are therefore equal to one another.

But if the sides A D, E F, opposite to the base B C of the parallelograms A B C D, E B C F, be not terminated in the same point; then because A B C D is a parallelogram, A D is equal (92) to B C; for the same reason E F is equal to B C; wherefore A D is equal (1 Ax) to E F; and D E is common; therefore the whole, or the remainder, A E is equal (2 or 3 Ax) to the whole, or the remainder D F; now A B is also equal to D C, therefore the two E A, A B are equal to the two F D, D C, each to each; but the exterior angle F D C is equal to the interior E A B, wherefore the base E B is equal to the base F C, and the triangle E A B (78) to the triangle F D C. Take the triangle F D C from the trapezium

A B C F, and from the same trapezium take the triangle E A B; the remainder will then be equal (3 Ax,) that is, the parallelogram A B C D is equal to the parallelogram E B C F.

THEOREM XIV.

94. Equal triangles upon the same base, and upon the same side of it, are between the same parallels.

Let the equal triangles A B C, D B C *fig.* 68, be upon the same base B C, and upon the same side of it; they are between the same parallels.

Join A D: A D is parallel to B C; for, if it is not, through the point A draw (88) A E parallel to B C, and join E C. The triangle A B C is equal to the triangle E B C, because it is upon the same base B C and between the same parallels B C, A E. But the triangle A B C is equal to the triangle B D C; therefore also the triangle B D C is equal to the triangle E B C, the greater to the less, which is impossible: Therefore A E is not parallel to B C. In the same manner it may be demonstrated that no other line but A D is parallel to B C: A D is therefore parallel to it.

THEOREM XV.

95. If a parallelogram and a triangle be upon the same base, and between the same parallel: the parallelogram is double of the triangle.

Let the parallelogram A B C D *fig.* 69, and the triangle E B C be upon the same base B C and between the same parallels B C, A E; the parallelogram A B C D is double of the triangle E B C.

Join A C; then the triangle A B C is equal to the triangle E B C, because they are upon the same base B C and between the same parallels B C, A E.

But the parallelogram A B C D is double (92) of the triangle A B C, because the diameter A C divides it into two equal parts; wherefore A B C D is also double of the triangle E B C.

THEOREM XVI.

96. The complements of the parallelograms which are about the diameter of any parallelogram, are equal to one another.

Let A B C D *fig.* 70, be a parallelogram of which the diameter is A C; let E H, F G be the parallelograms about A C, that is through which A C passes, and let B K, K D be the other parallelograms, which make up the whole figure A B C D, and are therefore called the complements. The complement B K is equal to the complement K D.

Because A B C D is a parallelogram, and A C its diameter, the triangle A B C is equal (90) to the triangle A D C: And because E K H A is a parallelogram, and A K its diameter, the triangle A E K is equal to the triangle A H K: For the same reason, the triangle K G C is equal to the triangle K F C. Then because the triangle A E K is equal to the triangle A H K, and the triangle K G C to the triangle K F C; the triangle A E K together with the triangle K G C is equal to the triangle A H K together with the triangle K F C. But the whole triangle A B C is equal to the whole A D C; therefore the remaining complement B K is equal to the remaining complement K D.

THEOREM XVII.

97. In any right-angled triangle, the square which is described upon the side subtending the right angle, is equal to the squares described upon the sides which contain the right angle.

Let A B C *fig.* 71, be a right-angled triangle, having the right angle B A C; the square described upon the side B C is equal to the squares described upon B A, A C.

On B C describe the square B D E C, and on B A, A C the squares G B, H C; and through A draw (86) A L parallel to B D or C E and join A D, F C; then because each of the angles B A C, B A G is a right angle (22) the two straight lines A C, A G upon the opposite sides of A B, make with it at the point A the adjacent angles equal to two right angles; therefore C A is the same straight line with A G; for the same reason A B, and A H are in the same straight line. Now because the angle D B C is equal to the angle F B A, each of them being a right angle, adding to each the angle A B C, the whole angle D B A will be equal to the whole F B C; and because the two sides A B, B D, are equal to the two F B, B C, each to each, and the angle D B A equal to the angle F B C, therefore the base A D is equal (78) to the base F C, and the triangle A B D to the triangle F B C; but the parallelogram B L is double (95) of the triangle A B D, because they are upon the same base B D, and between the same parallels B D, A L, and the square G B is double of the triangle B F C, because these also are upon the same base F B and between the same parallels F B, G C. Now the doubles of equals are equal to one another, therefore the parallelogram B L is equal to the square G B: And in the same manner, by joining A E, B K, it is demonstrated that the parallelogram C L is equal to the square H C. Therefore the whole square B D E C is equal to the two squares G B, H C, and the square B D E C is described upon the straight line B C, and the squares G B, H C, upon B A, A C: Wherefore the square upon the side B C, is equal to the squares upon the sides B A, A C.

98. Cor. Hence in any right-angled triangle, if we have the hypothenuse and one of the legs, we may easily find the other leg, by taking the square of the given leg, from the square of the hypothenuse the square root of the remainder will be the leg sought. Thus if the hypothenuse was 50, and one leg was 40, the other leg would be 30, for the square of 40 is 1600, and the square of 50 is 2500, by subtracting 1600 from 2500, it leaves 900, and the square root of 900 is 30. If both legs be given, the hypethenuse may be found by extracting the square root of the sum of the squares of the legs, for we know that 6, 8 and 10 will form a right-angled triangle, therefore the square of 6 is 36, and the square of 8 is 64, thus adding 36 and 64 together gives 100 whose square root is 10, which is the sought hypothenuse.

THEOREM XVIII.

99. The segments of two chords, which intersect each other, in a circle are reciprocally proportional.

Let the chords A B, and C D, *fig.* 72, intersect at O, then will A O : D O :: O C : O B. Join A C and B D. In the triangles A C O, B O D the angles at O are equal, being vertical; the angle A is equal to the angle D, because both are inscribed in the same segment; for the same reason the angle C=B; the triangles are therefore similar, and the homologous sides give the proportion A O : D O :: C O : O B.

100. Cor. Therefore A O.O B=D O.C O; hence the rectangle under the two segments of the one chord is equal to the rectangle under the two segments of the other.

THEOREM XIX.

101. If from the same point without a circle, two secants be drawn terminating in the concave arc, the whole secants will be reciprocally proportional to their external segments.

Let the secants O B, O C *fig.* 73, be drawn from the point O: then will O B : O C :: O D : O A. For joining A C, B D, the triangles O A C, O B D have the angle O common; likewise the angle B=C; these triangles are therefore similar; and their homologous side give the proportion O B : O C :: O D : O A.

102. Cor. The rectangle O A.O B is hence equal to the rectangle O C.O D.

103. Scholium. This proposition, it may be observed, bears a great analogy to the preceding, and differs from it only as the two chords A B, C D, instead of intersecting each other within, cut each other without, the circle. The following proposition may also be regarded as a particular case of the proposition just demonstrated.

THEOREM XX.

104. If from the same point without a circle, a tangent and a secant be drawn, the tangent will be a mean proportional between the secant and its external segment.

From the point O *fig.* 74, let the tangent O A, and the secant O C be drawn; then will, O C : O A :: O A : O D, or OA^2=O C.O D.

For joining A D and A C, the triangles O A D, O A C have the angle O common; also the angle O A D, formed by a tangent and a chord, has for its measure half of the arc A D; and the angle C has the same measure; hence the angle O A D=C, therefore the two triangles are similar, and we have the proportion, O C : O A :: A O : O D, which gives OA^2=O C.O D. (100.)

THEOREM XXI.

105. If either angle of a triangle is bisected by a line terminated on the opposite side, the rectangle of the sides including the bisected angles is equal to the square of the bisecting line together with the rectangle contained by the segment of the third side.

Let A D *fig.* 75, bisect the angle A; then will $A{:}B\,A\,C = A\,D^2 + B\,D.D\,C$.

Describe a circle through the three points A, B, C; produce A D till it meets the circumference, and joins C E.

The triangle B A D is similar to the triangle E A C; for by hypothesis the angle B A D=E A C: also the angle B=E since they both are measured by half of the arc A C; hence these triangles are similar, and the homologous sides give the proportion, B A : A E :: A D : A C; hence B A. A C=A E.A D; but A E=A D+D E and multiplying each of these equals by A D, we have $A\,E.A\,D = A\,D^2 + A\,D.D\,E$; now A D.D E= B D.D C hence finally B A.A C=A D+B D.D C.

THEOREM XXII.

106. In every triangle the rectangle contained by two sides, is equal to the rectangle contained by the diameter of the circumscribed circle, and the perpendicular let fall upon the third side.

In the triangle A B C *fig.* 76, let A D be drawn perpendicular to B C; and let E C be the diameter of the circumscribed circle; then will A B A C=A D C E.

For joining A E. the triangles A B D, A E C are right-angled, the one at D, the other at A; also the angle B=E; these triangles are therefore similar, and they give the proportion, A B : C E :: A D : A C and hence A B.A C=C E.A D.

107. Cor. If these equal quantities be multiplied by the same quantity B C there will result A B, A C, B C=C E, A D, B C; now A D, B C is double of the surface of the triangle; therefore the product of the three sides of a triangle is equal to its surface multiplied by twice the diameter of the circumscribed circle.

The product of three lines is sometimes called a *solled*, for a reason that shall be seen afterwards. Its value is easily conceived, by imagining that the lines are reduced into numbers, and multiplying these numbers together.

108. Scholium. It may also be demonstrated, that the surface of a triangle is equal to its perimeter multiplied by half the radius of the inscribed circle.

For the triangles A O B, B O C, A O C as in *fig.* 77; which have a common vertex at O, have for their common altitude the radius of the inscribed circle; hence the sum of these triangles will be equal to the sum of the bases A B, B C, A C, multiplied by half the radius O D; hence the surface of the triangle A B C is equal to the perimeter multiplied by half the radius of the inscribed circle.

THEOREM XXIII.

109. In every quadrilateral inscribed in a circle, the rectangle of the two diagonals is equal to the sum of the rectangles of the opposite sides.

In the quadrilateral A B C D *fig.* 78; we shall have A C.B D=A B. C D+A D.B C.

Take the arc C O=A D, and draw B O meeting the diagonal A C in I.

The angle A B D=C B I, since the one has for its measure half of the arc A D, and the other half of C O, equal to A D; the angle A D B=B C I, because they are both inscribed in the same segment A O B; hence the triangle A B D is similar to the triangle I B C, and we have the proportion A D : C I :: B D : B C. hence A D.B C=C I.B D. Again the triangle A B I is similar to the triangle B D C; for the arc A D being equal to C O, if O D be added to each of them, we shall have the arc A O=D C; hence the angle A B I is equal to D B C; also the angle B A I to B D C, because they are inscribed in the same segment; hence the triangles A B I, D B C, are similar, and the homologous sides give the proportion A B : B D :: A I : C D; hence A B.C D=A I.B D.

Adding the two results obtained, and observing that A I.B D+C I.B D=(A I+C I.) B D=A C.B D, we shall have A D.B C+A B.C D=A C.B D.

110. Scholium. Another theorem concerning the inscribed quadrilateral may be demonstrated in the same manner.

The similarity of the triangles A B D and B I C give the proportion B D : B C :: A B : B I; hence B I.B D=B C.A B. If C O be joined the triangle I C O, similar to A B I, will be similar to B D C, and will give the proportion B D : C O :: D C : O I; hence O I B D=C O.D C, or because C O=A D, O I.B D=A D. D C.

Adding the two results, and observing that B I.B D+O I.B D is the same as B O, B D, we shall have B O.B D=A B.B C+A D.D C.

If B P had been taken equal to A D and C K P been drawn, a similar train of reasoning would have given us, C P.C A=A B.A D+B C.C D.

But the arc B P being equal to C O, if B C be added to each of them it will follow that C B P=B C O; the chord C P is therefore equal to the chord B O, and consequently B O.B D and C P.C A are to each other as B D is to C A; hence, B D : C A :: A B.B C+A D.D C : A D.A B+B C. C D.

Therefore the two diagonals of an inscribed quadrilateral are to each other, as the sums of the rectangles under the sides which meet at their extremities.

These two theorems may serve to find the diagonals when the sides are given.

THEOREM XXIV.

111. If a point be taken on the radius of a circle, and this radius be then produced and a second point be taken on it without the circumference of the circle, these points being so situated that the radius of the circle shall be a mean proportional between their distances from the centre, then if lines be drawn from these points to any point of the circumference, the ratio (of these lines) will be constant.

Let P, *fig.* 79, be the point within the circumference, and Q the point without; then if C P : C A :: C A : C Q, the ratio of Q M and M P will be the same, for all positions of the point M.

For by hypothesis, C P : C A :: C A : C Q: or substituting C M for C A, C P : C M :: C M : C Q; hence the triangles C P M, C Q M, have each an equal angle C contained by proportional sides; hence they are similar (102), and hence the third side M P is to the third side M Q as C P is to C M or C A.

But by division, the proportion C P : C A :: C A : C Q gives C P : C A :: C A—C P : C Q—C A or C P : C A :: A P : A Q; therefore M P : M Q :: A P : A Q.

PRACTICAL GEOMETRY.

PLATE 6.

PROBLEM I.

112. At a given distance, parallel to a given straight line A B, *fig.* 80. pl. 6. to draw a straight line, C D.

In the given straight line A B, take any two points as A D or B C in your compasses, with one foot in A describe the arc at D, also in B describe another at C draw a line through C and D and it is done; for the line C D will be parallel to the line A B.

PROBLEM II.

113. To bisect or divide a given straight line A B, *fig.* 81, by a perpendicular.

Take any distance in your compasses greater than half the line A B, then with one foot in B describe the arc C F D; with the same distance, and one foot in A, describe the arc C G D; cutting the former arc in C and D; draw the line C D, and it will bisect A B in the point E perpendicular.

PROBLEM III.

114. To make an angle that shall contain any proposed number of degrees, from a given point, in a given line.

Case 1. When the given angle is right or contains 90° let C A *fig.* 82 be the given line, and C the given point.

On C erect a perpendicular C D and it is done, for the angle D C A is an angle of 90°, or thus on the point C, as a centre, with the chord* of

For the description of the line of chords see *fig.* 6, *pl.* 5, which will be explained hereafter.

60° in your compasses, describe an arc G H and set off thereon from G to H the distance of the chord of 90° and from C through H draw C H D, which will form the required angle D C A of 90°.

115. Case 2. When the angle is acute as *fig.* 83; as for example 36° 30′, let C B be the given line and C the point at which the angle is to be made.

With the chord of 60° in your compasses, and one foot in C as a centre, describe the arc F B, on which set off from B to F, the given angle 36° 30′ taken from the line of chords; through F and the centre C draw the right line A C, and it is done, for the angle A C B will be an angle of 36° 30′ as was required.

116. Case 3. When the given angle is obtuse, as for example 127° 20′ let C B, *fig.* 84, be the given line, and C the angular point.

Take the chord of 60° in your compasses, and with one foot in C as a centre, describe an arc B G H E, upon which set off the chord of 60° (which you already have in your compasses) from B to G, and from G to H; then set off from G to E, the excess of the given angle above 60° which is 67° 20′ taken from the line of chords; or you can set off from H to E the excess of the given angle above 120° which is 67° 20′ draw the line C E and it is done, for the angle E C B will be an angle of 127° 20′ as required.

PROBLEM IV.

117. To bisect a given arc of a circle as A B, *fig.* 85, whose centre is C.

Take in your compasses any extent greater than the half of A B, and with one foot in A describe an arc; with the same extent and one foot in B, describe another arc, cutting the former in D; join C D and it is done, for this line will bisect the arc A B in the point E, and divides it into two equal parts.

PLATE 7.

PROBLEM V.

118. From a given point B, *fig.* 1. at the extremity of a given straight line, A B, to draw a perpendicular.

Take any point, E above the line A B, and, with the radius B E, describe the arc *d* B C cutting A B in *d*: draw the straight line *d* E C, and join B C, which will be the perpendicular required.

PROBLEM VI.

119. From a given point C *fig.* 2, to let fall a perpendicular to a given straight line A B.

From the point C, with any radius greater than the distance of A B, describe an arc cutting A B at *e* and *f*; from the points *e* and *f*, as centres with any equal radius greater than the half of A, describe arcs cutting each other in D, and draw C D, which will be the perpendicular required.

PROBLEM VII.

120. To describe the segment of a circle, which shall have a given length or chord, A B, *fig.* 3, and a given breadth or versed sine, C D.

Bisect the straight line A B by a perpendicular C E; from the point D, where the perpendicular cuts the chord A B, make D C equal to the breadth, or versed sine; join A C; and make the angle C A E equal to the angle A C E; from E as a centre, with the radius E A or E C, describe the arc A C B which will be the segment required.

PROBLEM VIII.

121. Through three given points A, B, C, *fig.* 4, to describe the circumference of a circle.

Join A B, B C and bisect each of the lines A B and B C, by a perpendicular, and let the perpendiculars meet each other in I: from the centre I, with the distance I A, I B, or I C, describe the circle A B C, which is that required.

PROBLEM IX.

122. Upon a given straight line A B, *fig.* 5, to describe an equilateral triangle.

From the centres A and B, with the radius A B, describe arcs cutting each other at C, join A C and B C, then A B C will be the equilateral triangle required.

PROBLEM X.

123. Upon a given straight line A B, *fig.* 6, to describe a square.

From the point B draw B C perpendicular to A B, make B C equal to A B: from the points A and C as centres, with a radius equal to A B or B describe arcs cutting each other in D and join A D and D C, then A B C D is the square required.

PROBLEM XI.

124. Upon a given straight line A B, *figs.* 7 *and* 8, to describe a regular polygon of any number of sides.

Produce the side A B to P, and on A P, from the centre B, describe a semicircle A C P; divide the semi-circumference A C P into as many equal parts as the number of sides intended: through the second division, from P draw the line B C; bisect A B and B C by perpendiculars cutting each other in S; from S. with the radius A S, B S or C S describe a circle A B C D E, then carry the side A B or B C round the remaining part of the arc, which will be found to contain the remaining sides of the number required.

Fig. 7 is an example of a pentagon; *fig.* 8 is an example of a hexagon; but in this figure, we need not proceed by the general method; we have only to make a radius of the given side A B and take the points A and B as centres; and from the arcs A G and B G and strike a circle with the radius G A or G B, which will contain the side A B six times.

PROBLEM XII.

125. In a given square, A B C D, *fig.* 9, to inscribe a regular octagon, so that four alternate sides of the octagon, may coincide with four sides of the square.

Draw the diagonals A C and B D, cutting each other in S: on the sides of the square make A L, A F, B E, B H: C G, C K; and D I, D M, each equal to half the diagonal; join M E, F G, H I, K L; then will F G H I K L M E F be the octagon required.

PROBLEM XIII.

126. In a given triangle A B C, *fig.* 10, to enscribe a circle.

Bisect any two angles, A and B by the straight lines A E and B E and the point E, the intersection of these two lines will be the centre of the inscribed circle; draw E D perpendicular to A B cutting A B in D; from E with the radius E D, describe the circle D F G, which will be enscribed in the triangle A B C, as required.

PROBLEM XIV.

127. A circle, D E F, *fig.* 11, and a line A B, touching it, being given, to find the point of contact.

From the centre C draw the perpendicular C D cutting A B in D, which is the point of contact required.

PROBLEM XV.

128. Two straight lines, A B, B C, *fig.* 12, forming any angle, being given to describe a circle to touch each of these lines at a given point A, in one of them.

Make B C equal to B A, and draw A D perpendicular to A B, and C D perpendicular to B C: from the point of intersection D, with the radius D A or D C describe the circle A C E which is that required.

PROBLEM XVI.

129. In a given circle A B C D *fig.* 13, to enscribe a square.

Draw the diameters A C and B D at right angles, and join A B, B C, C D, D A, then A B C D will be the square required.

PROBLEM XVII.

130. To describe a segment, A A C of a circle by means of an angle.

Let A B *fig.* 14, be the length or chord, and D C the versed sine, join C A and C B, produce C A to F and C B to E making C E and C F of any length, not less than the chord A B, prepare two straight edges, C E and C F, and fasten them together at the angle C, so that their outer edges may form the angle A C B, and to keep them to the extent, fix another

slip across them at D to keep them tight: put in pins at A and B, and move the angle thus formed round these pins, hold a pencil to the angular point at C, it will describe the segment required.

Another Method.

131. Let A B, *fig.* 15, be the length of the chord, and C D the versed sine, (or the perpendicular height in the middle) join A D, and draw D E parallel to A B, making D E of any length not less than A D, form a triangular piece of wood, A D E: bring the angular point D of the triangle, to the point A, (put in pins at the points A D B) and move the triangle, so that the side D A may slide upon A, and the side D E upon D; then if during the motion a pencil be held at the angular point D, with its point tracing over the plane, the arc A D will be described by the point of the pencil, the arc A D being described, the arc D B will be described, in a similar manner: and consequently, the whole segment of the circle, as required to be done.

PROBLEM XVIII.

132. To describe an ellipsis, or any segment of an ellipsis, having a diameter and a double ordinate, by means of points being found in the curve, without finding the parameter.

Let A B, *fig.* 16, be a diameter (or double ordinate) let C D be its conjugate, and let E D be the height of the segment. Through D draw F G parallel to A B; also through the points A and B draw A F, and B C parallel to D E, cutting F G in F and G. Divide A E and E B into a like number of equal parts, as four; likewise B G, and A F, into the same number of equal parts. From the point D, through the points 1, 2, 3, in A F, and B G, draw 1 D, 2 D, 3 D. From the point C, through the points 1, 2, 3, in A B, draw C*a*, C*b*, C*c*, cutting the lines 1 D 2 D 3 D, in *a*, *b*, *c*, they will be in the periphery of the ellipsis; a curve being traced through these points, will form the ellipsis required.

But if the curve is very large, as in practical works, the best way is to put in nails or pins at the points, *a*, *b*, *c*, &c., bend a slip round them, and draw a curve by it, it will appear quite regular.

Figs. 17, 18, 19, 20 *and* 21 are drawn in a similar manner.

PROBLEM XIX.

133. To describe a hyperbola by finding points in the curve, having the diameter or axis A B in *figs.* 22 *and* 23, its abscissa B C and double ordinate D E. Through B draw G F parallel to D E; from D and E draw D G and E F parallel to B C cutting G F in F and G. Divide C D and C E each into any number of equal parts, as four through the points of division 1, 2, 3, draw lines to A.

Likewise divide D G and E F into the same number of equal parts, viz. four from the divisions on D G and E F draw lines to B and a curve being drawn, through the intersections at B *a b c* E, will be the hyperbola required.

Figs. 24, 25, 26, 27, 28, 29 *and* 30. Need no explanation, they being drawn in a similar manner.

PROBLEM XX.

134. To describe a parabola upon a given ordinate, A E, and a given abscissa E D *fig.* 31. Make E C equal to E A and complete the rectangle A F G C, so that the opposite side may pass through D. Proceed as in problem 15, excepting that instead of drawing the lines 1, 2, 3, &c. to A as in *figs.* 22 *and* 23, draw them perpendicular to A C.

PROBLEM XXI.

135. To describe the figure of the sines, *fig.* 32, describe the quadrant F H G equal to the height of the figure, and divide the arc H G into any number of equal parts, the more of these, the more perfect will be the operation; and extend the chords to double the number of parts upon the line A C, which is a continuation of F H, and make the points of division.

Draw the lines 1 *k*, 2 *l*, 3 *m*, &c. perpendicular to A C; and from the points 1, 2, 3, &c. of division in the quadrant, draw lines 1 *k*, 2 *l*, 3 *m*, &c. parallel to A C, and through the points A *k l m*, &c. draw a curve, which will be the figure of the sines as required.

PROBLEM XXII.

136. To describe a conic section, to touch two right lines A B *fig.* 33, and B C, in the points A and C, and to pass through a given point H.

Join the points A and C; through D, draw D E and D F, parallel to B A and B C, through D, draw G A parallel to A C, cutting B A and B C in G and H, and divide D G, and D H, D E, and D F each into the same number of equal parts. From C through the points 1, 2, 3, in D F draw the lines C *a*, C *b*, C *c*, &c. from A through the points 1, 2, 3, in D G, draw 1 A, 2 A, 3 A, cutting the former in *a*, *b*, *c*, which are in the curve.

In the same manner may points be found between D and C.

PLATE 8.

PROBLEM XXIII.

137. To describe a triangle, whose three sides shall be equal to three given lines provided that any two of them are greater than the third.

Let A, B, C, *fig.* 1, be the three given straight lines.

Take one of the given lines as A, and make the base of the triangle D E. Upon E, with the length of B in your compasses, describe an arc at F. Upon D with the length of C, describe another arc intersecting the former at F, and join D F, and F E, then D F E, is the triangle required.

In this manner a triangle may be made equal to another given triangle; for this is only making the sides of the triangle equal to those of the given triangle.

PROBLEM XXIV.

138. To form a trapezium from a given triangle, let G, I, H, *fig.* 2, be the given triangle; and described in the same manner as *fig.* 1. The two sides G K, and I K being given, take the length of G K in your compasses, and upon G describe an arc at K; upon I with the length of I K describe another arc intersecting the former at K, and join G K and I K: then is G K I H the trapezium required.

The line drawn G I, divides the trapezium into two obtuse-angled triangles, and is called the base line, and to let fall the perpendiculars K, H to M and N on the base line G I; then it becomes four right-angled triangles.

PROBLEM XXV.

139. To make a rectangle equal to a given triangle A D C, *fig.* 3. It is required to make a rectangle equal to the given triangle. Draw D C perpendicular to A B, and divide D C into two equal parts at *g*, through *g* draw E F, parallel to A B from B draw B F, perpendicular to A B, through A draw A E parallel to B F. Then the rectangle A B E F will be equal to the triangle A B C, as required to be done.

PROBLEM XXVI.

140. To make a square equal to a given rectangle. Let A B C D, *fig.* 4, be the given rectangle, produce the side A B of the rectangle to *h*, and make B *h*, equal to B C draw B G perpendicular A B; and on *i h* as a diameter, describe the semicircle A G *h*, and on the straight line B G describe the square B G F E; which is the thing required to be done.

We now see that a triangle may be reduced to a rectangle, and a rectangle may be reduced to a square; therefore a triangle may be reduced to a square.

Figures 5 *and* 6, these two figures are described similar to the figures 1 and 2.

PROBLEM XXVI.

141. Given two circles to find a third equal to them both.

Let A B and A C, *fig.* 7, be the diameters of the given circles, perpendicular to each other at the point A; join C B on which, describe a circle and the thing is done.

PROBLEM XXVIII.

142. Given any two similar figures, to find another equal and similar to them both.

Let E and F, *fig.* 8, have their sides A B and A C placed perpendicular to each other join B C, on which describe a figure similar to E or F, and the thing is done.

PROBLEM XXIX.

143. Given three straight lines to find a fourth proportional.

Let A C, *fig.* 9, be one of the given lines, make any angle with the line C E and from B, one extremity of another given line, draw B D the third any how to meet C E, and through A draw A E parallel to B D, and A E will be the fourth proportional required.

PROBLEM XXX.

144. Having given two lines, to find the third proportional.

Let C B and C D, *fig.* 10, be the given lines, and let them have any inclination at the point C; join B D, and produce C D and C B to A and E making C A equal to C D; through A draw A E parallel to B D, and C E will be the third proportional required.

PROBLEM XXXI.

145. To find a line equal to a given arc of a circle.

Let A B C, *fig.* 11, be the given arc, join A C which bisect in D, by the perpendicular B D; join A B, and produce A C till A E be equal twice A B; divide C E into three equal parts, and make E F equal to one of these parts; then will A F be nearly equal to the arc A B C.

PROBLEM XXXII.

146. To divide a given line in the same proportion as another line is divided.

Let *d e fig.* 13, be the line proposed for division. On *d e* describe an equilateral triangle, *d* C *e*; produce the sides C *d* and C *e* till each of them be equal to A B; join A B and from C to the points of division, *f*, *g*, *h*, draw the lines C *f*, C *g*, and C *h*, which will cut *d e* in the points *i*, *h*, *l*, in the same proportion as A B is cut. Or thus—

147. Let B D *fig.* 13, be the given line whose divisions are required.

Upon B D raise B A perpendicular at the point B, which make equal to B D; produce A B till A C be equal to the given divided line C E, draw C E perpendicular to A C, and produce A D to E, to the points of division *f*, *g*, *h*, *i*; from A draw the lines A *f*, A *g*, A *h*, and A *i*, which will divide the line B D, in the given proportion.

PROBLEM XXXIII.

148. To divide a quadrant of a circle into any number of equal parts.

Bisect the diameter A B *fig.* 14, perpendicularly in C, produce C E till E F be equal to three-fourths of A C or B C, join F A which produce to meet D G, drawn through D parallel to A B: divide D G into the proposed number of equal parts in the points *h*, *i*, *k*, *l*, and join F *h*, F *i*, F *k*, and F *l*, which will divide the quadrant A D, into the proposed number of equal parts nearly.

PROBLEM XXXIV.

149. Having the abscissa E D, *figs.* 15 *and* 16, and a double ordinate A B to describe a parabola.

Produce E D to B, making D B equal to D E and join A B and C B. Divide A B into any number of equal parts, numbering them from A to B, and divide B C into the same number of equal parts, numbering them from B to C, join 1 1, 2 2, 3 3, &c. and the parts of the straight lines, comprehended between the intersections, will form the parabola, being all tangents at different points.

PROBLEM XXXV.

150. To describe a parabola by a continued motion.

Let G H, *fig.* 17, be the straight edge of a ruler, and K L Q the internal right-angle of a square, and let the edge which is parallel to K L coincide with the straight edge G. H. Suppose now one end of a string fastened at F, and the other end to the end G of the square. Let F I be perpendicular to G H, meeting G H in I. Suppose the ruler to remain fixed, and the square to be moved, keeping the upper edge against the straight edge G H: then if both parts, F M, M Q, be kept straight by a pencil at M, the point M will describe the half of the parabolic curve.

PROBLEM XXXVI.

151. Given the axis major and two foci of a conic section, to describe the curve.

Let A B, *figs.* 21 and 27, be the axis major, and the points F *f*, the foci. Produce B A, if necessary, to F, and make A I equal to A F; from the remote focus *f*, describe an arc Q I. Through Q draw *f* M: then find the point M, by sloping with a compass or dividers, so that F M may be equal to M Q; then M is a point in the curve.

By employing F the same manner as has now been done, in respect to *f*, we shall obtain the curve *r* B *s*.

N. B. In the ellipse, *fig.* 27, it will be most convenient to describe one-half by means of the focus *f*, and then the other by the other focus F.

In the parabola, *fig.* 20, the arc Q I will become a straight line, this being understood the point M, and every other point will be found as in *figs.* 21 *and* 27.

PROBLEM XXXVII.

152. To describe an ellipse having the two axes given.

On A B, *fig.* 22, describe the rectangle G H I K, whose sides G H and G K are equal to the given axes. Divide C D into any number of parts, *q*, *r*, *s*, *t*, and D G into the same number *a*, *b*, *c*, *d*; from B through the points of division *q*, *r*, *s*, *t*, draw lines B M, &c. and from A to the points of division, *a*, *b*, *c*, *d*, draw A *a*, A *b*, &c. meeting the former in M, then will M be a point in the curve: and thus may all the points be found, through which draw the curve itself. If the curve is large, it would be well to put in pins or nails at the points, and bend a slip of wood around to draw the curve by.

PROBLEM XXXVIII.

153. The transverse and conjugate axes A B and D D in *fig.* 23, of an ellipsis being given to find the two foci, from thence to describe an ellipsis.

Let F and *f* be the foci, make F M + *f* M equal to A B, supposing F M + *f* M to be a cord or string. Then move the point M, round, taking care to keep the string always tight, and the point M will, in its motion, trace out the curve A D B D, which is the ellipse required.

PROBLEM XXXIX.

154. To describe an ellipse round a given rectangle.

In *fig.* 24, let I T, be half the longest side of the rectangle, and make I G equal to I T: join T G, cutting A B in H, then G I is the semi-transverse and G H the difference of the semi-axes. Therefore, the curve may be described. Or thus—

In *fig.* 25, *g*, *i*, is the semi-transverse, and *g*, *h*, the difference of the semi-axes. The point *g*, is supposed to move in the groove exhibited in one arm of the trammel, while *h*, moves in the other and the point *i*, traces out the curve.

PROBLEM XL.

155. To describe an ellipse by means of circles.

From the centre C, *fig.* 26, with a distance equal to the semi-conjugate, describe the quadrant D *h*, *g*, and from the same centre, with the semi-transverse as a distance, describe the arc *i* B, bisect D *h g* in *h*; join C *h* which produce to *i*, through *h* draw *h*, *k* parallel to C B and from *i* draw *i k* perpendicular to *h k* then will *k* be a point in the curve, produce D C to L, and bisect the distance D *k*, perpendicularly, which bisecting line produce to meet D C in L, then will L be the center of the circle, describing one part of the curve.

Again; through *l* draw *l m* parallel to *h k* meeting the arc D *n m*, in *m*; join *m* B, which produce to meet the curve in *n*, join *n l* cutting A B in *g*; *g* is the centre of the circle whence the vertical part of the curve is described.

HYPERBOLA PROBLEMS XLI.

156. Given the transverse axis of an hyperbola and an ordinate, to find the conjugate axis and assymtotes, (which are two straight lines such as, if produced indefinitely with the curve will never meet each other) and thence to describe the curve itself.

Let A *a*, *fig.* 28, be the transverse axis, and let P M be an ordinate. Make P D equal to A P. Then on *a* D describe the semicircle *a* N D, produce P M to N. Draw A R perpendicular to C D, and make A R equal to C A. Join N R and produce N R and D A, if necessary, to meet each other in S; and draw M S, cutting A R in Q, produce Q A to T and make A T equal to A Q. Then Q T will be the conjugate axis or A Q, A T, will each be the semi-conjugate axis. Through the points C, T, draw J H; and through the points C Q, draw I K: then J H and I K are the assymtotes by which the curve may be described.

157. The three curves of the conic sections,—Problem.

The vertical section of a right cone being given and the position of the axes of a conic section, to describe that section.

Let A V B, *fig.* 29, be the section of a cone through its axes; let *i g* be

the line of the axes, and let it cut the sections A V B at h, and the opposite side B V, produced, at g. On $g\,h$ describe the semicircle $h\,q\,s\,g$. Draw Vp parallel to A B, cutting the axis in p. Bisect $h\,g$ in r and draw $p\,q$, $r\,s$, perpendicularly to $h\,g$. Make $p\,w$ equal p V; then with the transverse axis $h\,g$, at the ordinate $p\,w$, describe the ellipse $h\,w\,t\,g$, cutting $r\,s$ at t; then $r\,t$ is the semi-conjugate axes.

GEOMETRY OF SOLIDS.

DEFINITIONS OF SOLIDS.

158. A Right Cylinder is that which is formed by the revolutions of a rectangle about one of its sides; the line round which the rectangle revolves is called the axis (plural axes); and the circles generated by the two opposite sides of the rectangles perpendicular to the axes, are termed the ends or bases. The surface of the cylinder, generated by the line parallel to the axis, is termed the curved surface, which is either straight or convex, according as a strait edge is applied, parallel to the axis, or in any other direction.

159. A Right Cone is that which is formed by supposing a right-angle triangle to revolve about one of its legs or perpendicular sides; the fixed leg, or line, is called the axis; the surface generated by the other leg is called the base; and the surface formed by the hypothenuse, or sides opposite the right angle is denominated the curved surface, which is either straight or convex, according as a straight edge is applied upon the surface from the vertex, or in any other direction.

160. A Sphere or Globe is that which is formed by supposing a semicircle revolves upon its diameter; the diameter upon which the semicircle revolves is called the axis, and the surface formed by the arc of the semicircle is called the curved surface which is convex, in whatever way it may be tried by a straight edge.

161. An Ellipsoid is formed or generated by supposing a semi-ellipse to revolve upon one of its axis, the axis thus fixed is called the axis of the ellipsoid, and the surface generated by the curve is termed the curved surface.

PLATE 9.

PROBLEM XLII.

162. To describe a conic section, from the cone, through a line given in position in the section passing through the axis.

Let A B C, *figs.* 1, 2, *and* 3, *plate* 9, be the section of a right cone, and let D E be the line of section. Through the apex or top of the cone, C, draw C F parallel to the base A B of the section, and produce E D to meet A B in D, as in *figures* 2 and 3 or A B produced in G, as in *fig.* 1, as also to meet C F in F. On A B describe a semicircle which will be equal to half the base of the cone; in the semicircle take any number of points, a, b, c, &c. Draw Dd, in *figures* 2 and 3 and Gd in *fig.* 1, perpendicular to G F; as also Dd, *figs.* 2 *and* 3, perpendicular to D F: From the points a, b, c, &c. draw lines $a\,e$, $b\,f$, $c\,g$, &c., cutting Gd *fig.* 1, and Dd, *figs.* 2 *and* 3, in the points e, f, g, &c. in *fig.* 1, make in Ge, Gf, Gg: &c. equal to Ge, Gf, Gg, &c. and in Dd, *figs.* 2 *and* 3, make De, Df, Dg, &c. equal to De, Df, Dg, &c. Through the points e, f, g, &c. draw lines to F. Through the points a, b, c, &c., draw lines perpendicular to A B. From the points of section in A B, draw lines to the vertex C of the cone cutting the sectional line, D E, in l, m, n, &c. Through the points of section l, m, n, &c. draw lh, mi, nk, &c. perpendicular to D E. Through the points D, h, i, k, in *fig.* 1, or d, h, i, k, &c. in *figs.* 2 and 3, draw a curve which will be the conic section required.

Observations.

163. In the first of these figures, the line of section cuts both sides of the section of the cone, in this case the curve D $h\,i\,k$ and e E is an Ellipse. In *fig.* 2 the line of section D E is parallel to the side A C of the section of the cone; in this case the curve $d\,h\,i$ &c. E is a Parabola. In *fig.* 3, the line of section, D E is not parallel to any side of the cone; it must therefore, when produced with the sides of the section through the axis, meet each of these two sides in different points, in this case, the section d, h, i, &c. E is either an Ellipse or *Hyperbola*, but the case is determined to be an *hyperbola* by the line of section meeting the opposite side B C at A C, where it cuts above the vertex, at the point B.

Here we may observe, that the line of section, D E is the same as that which has before been called the abscissa, the part E B produced, contained between the two sides of the section is called the axis major: and the line Dd, perpendicular to D E, an ordinate.

Hence the same section may be found by the method already shown in the problem; viz. by drawing any straight line $d\,e\,b$, *fig.* 4; make $d\,e$ equal to D E, *fig.* 3, and $e\,b$ equal to E B, *fig.* 3. Through d draw the straight line D D at right angles to $d\,b$: make d D equal to D d, *fig.* 3, then with the axis major $b\,e$, the abscissa $e\,d$, and the ordinate d D, on each side of the abscissa describe the curve of the hyperbola, which will be of the same species as that shown in *fig.* 3.

PROBLEM XLIII.

164. To describe a cylindric section through a line given in position upon the section passing through its axis, *fig.* 4.

This is no more than a particular case of the last problem.

For a cylindric may be considered as a cone, having its apex at an infinite distance from its base; or practically, at a vast distance from its base, in this case all the lines for a short distance, would differ insensibly from parallel lines; and this is the construction shown at *fig.* 5, which is therefore evident. But as the section of a cylinder so frequently occurs I shall here give a more practical description of it. Thus:—

Let A B H I, *fig.* 5, be a section of a right cylinder, passing through its axis, A B being the side which passes through the base, and let D E be the line of section. On A B describe a semicircle; and in the arc take any number of points a, b, c, &c. from which draw lines perpendicular to the diameter A B, cutting it in Q, R, S, &c.: perpendicular to A B or parallel to A I or B H, draw the lines Qq, Rr, S s, &c. cutting the line of sections D E, in the points q, r, s, &c. from the points of section q, r, s, &c. draw the lines qi, rk, sl, &c. perpendicular to the line of section, D E, make the ordinates qi, rk, sl, &c. each respectively equal to the ordinates Qa, Rb, Sc, &c.; and through the points D, i, k, l, &c. to E draw a curve, which will evidently be the section of the cylinder, as required.

The same may be done in this manner; viz:—Bisect the line of section D E in the point t. Draw $t\,m$ perpendicular to D E.

Make $t\,m$ equal to the radius of the circle which forms the end of the cylinder; then with the axis major D E, and the semi-axis minor tm describe a semi-ellipse, which will be the section of the cylinder required.

165. A Definition. A cuneoid is a solid ending in a straight line, in which, if any point be taken, a perpendicular from that point may be made to coincide with the surface, the end of the cuneoid may be of any form whatever.

The cuneoid which occurs in architecture, has a semi-circular or a semi-elliptical end, parallel to the straight line to which the perpendicular is applied.

PROBLEM XLIV.

166. To find the section of a cuneoid with a semi-circular base, the given data being a section through the axis, perpendicular to the vertex or sharp end, and the line of section upon that end.

Let A B C, *fig.* 6, be the section through the axis perpendicular to the sharp edge, and let D E be the line of section.

This construction is similar to that of finding the section of a cylinder excepting that instead of drawing parallel lines from the base A B, they are in this figure drawn from the points of section in A B to the point C which is the vertex of the cuneoid: the ordinates Qa, Rb, Sc, &c. being transferred respectively to qi, rk, sl, &c.; and the curve D, i, k, l, and to E being drawn through the points D, i, k, l, &c. by hand.

CARPENTRY AND JOINERY.

Carpentry is the art of applying timber in the construction of buildings. The cutting of the timbers, and adapting them to their various situations so that one of the sides of every timber may be arranged according to some given surface as indicated in the designs of the architect, requires profound skill in geometrical construction.

For this purpose it is necessary, not only to be expert in the common problems generally given in a course of practical geometry, but to have a thorough knowledge of the sections of solids and their coverings. Of these subjects, the first has already been explained in the series of problems given in the geometrical part of this work, and I am now about to treat on the other; that is the method of covering them.

As no line can be formed on the edge of a single piece of timber so as to arrange with a given surface, nor on the intersection of two surfaces, (by workmen called a groin) without a complete understanding of both, the reader is required not to pass them, until the operations are perfectly familiar to his mind.

For the more effectually riveting the principles upon the mind of the student, it is requested that he should model them as he proceeds, and apply the sections and coverings found on the paper to the real sections and surfaces by bending them around the solids.

The surfaces which timbers are required to form are those of cylinders, cylindroids, cones, cuneoids, spheres, ellipsoids, &c. either entire or as terminated by cylinders, cylindroids, cones, and cuneoids.

The formation of Arches, Groins, Niches, Angle brackets, Lunettes, Roofs, &c. depend entirely upon their sections, or upon their covering or upon both.

This branch of carpentry from its being subjected to Geometrical rules and described in schemes or diagrams upon a floor, sufficiently large for all the parts of the operation, has been called *descriptive carpentry.*

In order to prepare the reader's mind for this subject, it will be necessary to point out the figures of the sections, as taken in certain positions.

All the sections of a cylinder, parallel to its base are circles.

All the sections of a cylinder parallel to its axis are parallelograms. And if the axis of the cylinder be perpendicular to its base, all these parallelograms will be rectangles. If a cylinder be entirely cut through the curved surface and if the section is not a circle it is an ellipse.

All the sections of a cone parallel to its base are circles; all the sections of a cone passing through its vertex are triangles; all the sections of a cone which pass entirely through the curved surface and which are not circles are ellipses: all the sections of a cone which are parallel to one of its sides, are denominated parabolas, and all the sections of a cone which are parallel to any line within the solid passing through the vertex are denominated hyperbolas.

All the sections of a sphere or globe made plane are circles.

The solid formed by a semi-ellipse revolving upon one of its axis is termed an ellipsoid.

All the sections of an ellipsoid are similar figures: those sections perpendicular to the fixed axis are circles, and those parallel thereto are similar to the generating figure.

OF THE COVERINGS OF SOLIDS.

PROBLEM I.

Let A B C D *fig.* 7, *pl.* 9, be the generating section of the frustum.

On B C describe the semi-circle B E C, and produce the sides B A and C D, of the generating section A B C D to meet each other in F.

From the centre F with the radius F A describe the arc A H: and from the same centre F with the radius F B, describe the arc B G; divide the arc B E C of the semicircle into any number of equal parts; the greater the number the more correct will be the result of the operation; repeat the chords of one of those equal arcs, upon the arc B G, as often as the arc B E C contains equal parts; then through G, the extremity of the last part, draw G F, cutting the arc A H at H then will A B G H be the covering required, of the frustum of a right cone.

To find the covering of the frustum of a right cone when cut by two concentric cylindric surfaces, perpendicular to the generating section.

Let A B C D *fig.* 7, *pl.* 9, be the given section, and A D, B C, the line on which the cylindric surface stands, (find the arc B G as before described) and mark the points 1, 2, 3, &c. of division both in the arc B G and in the semi-circumference: from the points 1, 2, 3, &c. draw lines to F; also from the points 1, 2, 3, &c. in the semi-circumference draw lines perpendicular to B C so that each line thus drawn may meet or cut it. From the points of division in B C draw more lines to F, cutting the arc B C in *a*, *b*, *c*, &c. from the points *a*, *b*, *c*, &c. draw lines parallel to B C to cut the side B A from the centre F through each point of section in B A, describe an arc, cutting the lines drawn from each of the points 1, 2, 3, &c. in B G at *a*, *b*, *c*, &c. then will B *e* G be the curve which will cover the line B C on the plan, or B C will be the seat of the line B *e* G.

In the same manner A H the original of the line A D, will be found; and consequently, B *e* G H A will form the covering over the given seat A B C D as required to be done.

PROBLEM II.

To find the covering of a right cylinder.

Let A B C D *fig.* 8, *pl.* 9, be the seat or generating section.

Produce the sides D A and C B to H and G, and on B C describe a semicircle and make the straight line B G equal to the semi-circumference; draw G H parallel to A B, and A H parallel to B G, then will A B G H be the covering required.

PROBLEM III.

To find the covering of a right cylinder contained between two parallel planes perpendicular to the generating section, *fig.* 9, *pl.* 9. Through the point B draw I K, perpendicular to A B; and produce D C to K, on B K describe a semicircle and make B I equal to the length of the arc of the semicircle by dividing it into equal parts and extending them on the line B I. Through the points of section 1, 2, 3, &c. in the line B I, draw lines 1*a*, 2*b*, 3*c*, &c. parallel to B A, and through the points 1, 2, 3, &c. in the arc of the semicircle, draw the other lines 1*a*, 2*b*, 3*c*, &c. parallel to B A cutting A D in *a*, *b*, *c*, &c. draw *aa*, *bb*, *cc*, &c. parallel to B K: then through the points *a*, *b*, *c*, &c. draw the curve A H and A H, will be the edge of the covering over A D.

In the same manner the other opposite edge B G will be found, and the whole covering will therefore be A B G H.

PROBLEM IV.

A B C D, *fig.* 10, *pl.* 9, being the seat of the covering of a semi-cylindric surface, contained between the surface of two other concentric cylinders of which the axis is perpendicular to the given seat; it is required to find the covering.

Through B draw I K perpendicular to A B; and produce D C to K. On B K describe a semicircle and divide its circumference into equal parts, at the points 1, 2, 3, &c; the more of these the truer will be the operation; and repeat the chord on the straight line B I, as often as the arc contains equal parts, and mark the points 1, 2, 3, &c. of division. Through the points 1, 2, 3, &c. in the arc of the semicircle, draw the lines 1*a*, 2*b*, 3*c*, &c. parallel to B A; and, through the points 1, 2, 3, &c. in B I, draw lines 1*a*, 2*b*, 3*c*, &c. parallel to B A,—Draw *aa*, *bb*, *cc*, &c. parallel to K I, and through all the points *a*, *b*, *c*, &c. draw the curve line A H, which is one of the edges of the covering.

In the same manner the other edge B G will be found; and consequently, the whole covering A B G H.

PROBLEM V.

To find the covering of that portion of a semi-cylinder contained between two concentric surfaces of two other cylinders, the axis of these cylinders being perpendicular to A B C D, *fig.* 11, *pl.* 9. Join B C and in this case B C will be perpendicular to A B. Produce C B to G; and on B C describe a semicircle. Divide the arc of the semicircle into any number of equal parts, and extend the chords upon the straight line B G marking the points of section both in the semicircle and in the straight line B G. Through the points 1, 2, 3, &c. in the arc of the semi-circle, draw lines 1*a*, 2*b*, 3*c*, &c. parallel to A B; and through the points 1, 2, 3, &c. in B G, draw the lines 1*a*, 2*b*, 3*c*, &c. parallel to A B; also draw *aa*, *bb*, *cc*, &c. parallel to B G, and, through the points *a*, *b*, *c*, &c. draw a curve, which will form one of the edges of the soffit: the opposite edge is formed in the same manner.

HIP ROOFS.

GEOMETRICAL CONSTRUCTION.

To find the bevels for cutting the various timbers in a hipid roof, and the length and backing of the hip rafters.

Let A B C D, *fig.* 12, *pl.* 9, be the plan of the building, or the outlines of the wall-plates, A E, B E, and C E, D E, the seats of the hips, draw E A, E B, and E C, E D; the base lines over which the hip rafters are to stand, let P *n*, *m*, be the pitch of the roof, and *o n* the perpendicular height, draw E F at right angles with E A and equal to the height of the perpendicular *o n*, then draw A F which is the length of the hip rafter, draw *h*, *k*, *h*, at any distance from the angle A and at right angles with A E make *k i* equal to *k r*, or from *k* to the nearest point of the top line of the hip rafters draw *h i* and *i h*, which is the backing of the hip rafter required. This method will give the backing of the hip rafter, whether the building be square or beveling.

To find the bevels of a purlin upon a hip rafter giving the seat of a common rafter, and the seat of the hip rafter, and the angle which the common rafter makes with its seat.

Let A B C D, *figs.* 13 and 14, *pl.* 9, be the outlines of the wall-plates A F, D F, and B E, C E, the seats of the hips and E F the seat of the ridge-piece. Place the section-purlin in its real position with respect to the common rafter. Produce that side of the section of the purlin, of which the bevel is required, upon the hip toward the seat; from one extremity of the line thus produced, and, with the length of the said line as a radius, describe a circle. Draw three lines, parallel to the wall-plate, to meet the hiped line; viz. one from the centre of the circle, one from the point where the line meets the circle and the third a tangent to the circle. From the point in the seat of the hip-rafter where the middle line meets the said seat, draw a line perpendicular to that middle line to meet the tangent; join the point where this perpendicular meets the tangent to the point where the line drawn from the centre meets the seat of the hip-rafter, and the angle formed by the line thus joining, and the line drawn from the centre of the circle will be the bevel of the purlin.

Example, *fig.* 13, *pl.* 9, A F being the seat of a hip-rafter, I F that of a common rafter and F I L the angle which the common rafter makes with its seat, and *a b c d* the section of the purlin. Now suppose it were required to find the bevel of that side of the purlin represented by *a d*. Produce *a d* to any point, *f*; and from *a*, with the radius *a f*, describe a circle *f g h*, parallel to the adjacent wall-plate, A B draw three lines to cut the seat A F, of the hip; viz. from the centre *a* draw *a i* and from the point *f*, where *a f* meets the circle draw *f k*, the former cutting the seat in *i*, and the latter in *k*. Draw *k l*, perpendicular to *f k*, and draw the tangent *e l*, cutting *k l* in *l*; and join *i l* then *l i a* is the angle required.

In the same manner by producing *a b*, we shall find the angle formed upon the end of the side of which its section is *a b*.

In order that the different inclined planes, which form the sides of a roof, should have an equal inclination to the horizon, the seats of the hip-rafters ought to bisect the angles of the wall-plates.

When a roof is wider at one end than at the other as in order as in *fig.* 15, *pl.* 9, to prevent its winding, let I K and O P be the seats of the two common rafters, passing through each extremity

of the ridge-piece and let the rafters I L and K L be found as before; divide O P into two equal parts, in E; draw E R perpendicular to O P. Make the angle E P R equal to the angle F K L; then E R will be the height of the roof upon the seat O P.

If this should be objected to, because it makes the ridge higher at one end than at the other, let E, *fig.* 17, *pl.* 9, be the end of the seat of the ridge next the narrow end of the roof.

Bisect all the four angles of the roof by the straight lines A F, B E, C E, D F; and through E, draw E G, parallel to A B cutting A F in G; and draw E H parallel to C D, cutting D F in H; and join G H; then G H will be parallel to A D. This is true, because, since all the angles are bisected, if we imagine perpendiculars drawn from E to the three sides, the three straight lines thus drawn will be equal; and because E G is parallel to A B, the perpendiculars drawn from the points E and G, to the straight line A B, are equal; from the same reasons, because E H is parallel to C D, the perpendiculars drawn from the points E and H to the straight line C D, are equal, therefore the perpendicular drawn from the point G, to the straight line A B, is equal to the perpendicular drawn from H to the straight line C D.

And, since the angles B A D and C D A are bisected by the straight lines A G and D H, the two perpendiculars, drawn from G to the sides A B and A D, are equal, as also the two perpendiculars from the point H to the sides D A and D C; but the perpendicular drawn from G, to the side A B, is equal to the perpendicular drawn from H to the side C D; therefore the perpendiculars, drawn from points G and H, to the straight line A D, are equal to each other; but when the perpendiculars drawn between two straight lines are equal, these two straight lines are parallel: therefore the straight line G H is parallel to A D.

Whence if all the angles of a roof be bisected, and if any point be taken in any one of the bisecting lines, and if a line be drawn through the point thus assumed, parallel to one of the adjacent sides, to meet the next bisecting line, and so on from one to another, till only one line remains to be drawn, then if the point assumed be joined to the point where the parallel meets the last bisecting line, the line thus joining will be parallel.

Fig. 16 is one half of the plan of the roof and the stretch-out of the rafters of *fig.* 15.

DOMES.

PLATE 10.

Fig. 1, *pl.* 10, is a design for an hemispherical dome, the ribs are constructed of thin boards and small pieces of plank. The principle of this form of roof consists in placing a number of hoops one above the other, and of such sizes as when properly placed, will form the contour of the dome. These hoops are here formed by pieces of plank as represented on the plan of the dome in *fig.* 1, *No.* 2. Near each one of these is a long mortice, the position of these is shown in the section *d d d* &c. in *fig.* 1, *No.* 1, one of the ribs or rafters is shown in *fig.* 1, *No.* 3, with a mortice in the middle of it long enough to receive the thickness of two hoops, as also may be seen in *fig.* 1, *No.* 4. At each end of these ribs is a sliding mortice of half the length, as represented in the section *c c c* &c. in *fig.* 1, *No.* 1, when these are to be put together, the wall-plate (which should be of two thicknesses of boards, and made to break joint,) should be first laid, and then a piece of the rafter, as *fig.* 1, *No.* 3, should be fixed upright in its proper place and secured by a tenon at the lower end, which must go through the plate. It should be observed, that the rafters are of two lengths which should break joint; of course one of the first pieces should be but half the length of *No.* 4, when one set of the rafters are fixed all round, the pieces which form the hoops, or which I shall call the purlins, are fixed in them and secured by wooden keys which are driven, on each side of the rafter, through the mortice. By driving these keys, more or less, the hoop may be lengthened or contracted, so as to bring it to the exact form or contour of the dome. After the first set of purlins are fixed and properly keyed, another set of rafters are placed, and then another set of purlins, until the dome is complete. The figure in the plate, for the sake of making its parts more clear has been drawn considerably out of proportion, the materials being much too large, and a much greater number of purlins would be proper. This principle of covering may be extended to a great span, and when the rafters come too close together, at the top every other one may be left out as may be seen in the preceding figures.

Figures 2 and 3, are so similarly constructed to each other, a particular explanation is not required of both. Therefore I shall proceed with *fig.* 3.

Fig. 3, *No.* 1, *pl.* 10, is a design for an ellipsoidal dome, the plan being elliptic, and one of the vertical sections circular. The ribs are constructed without trusses. In order to divide them as equally as possible, a purlin is introduced to support the upper ends of the jack-ribs. As this dome is supposed to arise from an elliptic well hole, the timbers are carried below the base, from *a, b, c, d, e, f.* *No.* 1 is the elevation, *No.* 2 the plan, showing the upper face of the wall-plates, purlin and curb, *fig.* 3, *No.* 3, *fig.* 3, *No.* 5, *fig.* 3, *No.* 7, are the entire ribs, to be placed upon A, C, E, in the plan, and *fig.* 3, *No.* 4, *fig.* 3, *No.* 6, *fig.* 3, *No.* 8, are the jack-ribs to be placed upon B, D, F, on the plan the upper ends of all the ribs terminate upon the curb, or upon the purlin, with a sally, or birds mouth, which is the usual method of fitting them. *Fig.* 2, *No.* 1, is a design for an hemispherical dome, constructed in the same manner as the elliptic dome, *fig.* 3.

In large roofs constructed of a domic form, without trussing, the ribs may be made in two or more thicknesses, in such a manner that the common abutment of every two pieces, in the same ring may fall as distant as possible from the abutment of any other two pieces, in a different ring. The number of purlins must depend upon the diameter of the dome.

To find the form of the boards for an ellipsoidal dome, the plan being an ellipse, and the vertical section upon the axis-minor a semi-circle; so that the joints of the boards may be in planes passing through the axis-major of the plan.

Let A B C D, *fig.* 4, *No.* 1, *pl.* 10, be the plan of the dome, A C the axis-major and D B the axis-minor; E the centre. From E, with the distance E D, or E B describe the semicircle B F D. Divide the arc into such a number of equal parts, that one of them may be equal to the breadth of a board, and let the points of division be at 1, 2, 3, 4, &c. draw the lines 1*a*, 2*b*, 3*c*, 4*d*, perpendicular to B D, cutting B D at the points *a, b, c, d*. Then upon A C, as an axis-major, and upon E*a*, E*b*, E*c*, E*d*, as so many axes-minor, describe the semi-ellipse A*a*C, A*b*C, A*c*C, A*d*C, which will represent the joints of the boards upon one side of the dome. Now since all the sections of this dome,

through the line A C are identical figures, the vertical section upon the line A C, will be identical to the half plan A B C or A D C. Divide, therefore, B A into any number of equal parts, by the points of division, *e, f, g, h, i, k, l;* the more correct will be the operation. Draw the straight lines *em, fn, go, hp, iq, kr, ls,* perpendicular to A C, cutting A C at the points *m, n, o, p, q, r, s,* and the semi-ellipse A*d*C, in the points *t, u, v, w, x, y, z,* on the straight lines G H, *fig.* 4, *No.* 2, set off the equal parts E*m, mn, no,* &c. from each side of the centre E each equal to one of the equal parts B *e, e f, f g,* &c. in the semi-elliptic curve, A B C, in the plan No. 1.

Through the points *m, n, o, p,* &c., No. 2, draw *t t, uu, v v,* &c. perpendicular to G H. Make *mt, mt,* each equal to *mt,* in the plan No. 1, and *nu, nu,* No. 2, each equal to *nu,* in the plan No. 1; then through all the points *t, u, v,* &c. draw a curve on each side of the line A C, to reach from A to C, and each curve will be the edge of a board. If the work be large it would be well to tack in nails at the several points and bend a slip around to describe the curve.

Fig. 4, *No.* 3, shows the longitudinal elevation: viz. on the line A C of the plan.

Fig. 4, *No.* 4, exhibits the transverse elevation, the contour being identical to that of the section on the line A B.

To find the covering for a sphere or globe which is supposed to be divided into four parts or quarters.

Let A C B D, *fig.* 5, *pl.* 10, be the circumference, and A B and C D the diameters which divide the circle, into four quadrants; divide the arc A C into a like number of equal parts, (six in this example,) draw E G through the arc at L, and take the six divisions from the arc A C and set them off from E to F on the line E G, produce the same number of parts from F to G, draw F M and F N at right angles with E G; take three of the divisions from the arc A G, and set them off from F to I, and from F to H, on the line M N, on M as a centre describe the arc E H G, cutting the point in H, on N as a centre describe the arc E I G, cutting the point in I.

Then will E I G H be one quarter of the covering required.

CIRCULAR ROOFS.

METHODS OF BOARDING.

With regard to boarding of roofs for slates there are two methods.

In the first place, if a round solid be cut by two planes, each parallel to the base, the portion of the surface of the solid, between these planes, will nearly coincide with a conic surface, contained between sections perpendicular to the axis of the cone of the same diameter each as those made by cutting the round solid; therefore the whole of the round solid may be looked upon as so many conic frustums, laying one upon another; therefore to cover all the conic frustums is to cover the round solid.

The other method of covering a round solid is to suppose the base divided into equal parts and the solid to be cut by planes passing through the points of division, and through the fixed axis; then the surface of the body will be divided into as many equal and similar parts: so that if any one of these portions of the solid be covered, the cover will of course fit any other portion thus divided; and as all the horizontal sections of each portion of the solid is the sector of a circle, the chords of all the sectors will be parallel to each other; therefore the curved surface will be nearly prismatic. This therefore affords another method of forming the boarding.

The first of these methods is called the horizontal method and the second the vertical method of covering a dome.

Let A B C, *fig.* 6, *pl.* 10, be a vertical section of a circular dome, through its axis and let it be required to cover the dome horizontally; bisect the base A C, in the point H, and draw H I perpendicular to A C cutting the semi-circumference in B, divide the arc B C into such a number of equal parts that each part may be less than the breadth of a board; that is to say, allowing the boards to be of a certain length, each part may be of the proper width, allowing for waste. Then if, between the points of division, we suppose the small arcs to be straight lines, as they will differ very little from them, and if horizontal lines be drawn through the points of division to meet the opposite side of the circumference, the trapezoids will be the sections of so many frustums of a cone and the straight line H I will be the common axis for every one of these frustums.

Now, therefore, to describe any board, which shall correspond to the surface of which one of these parts, *ab,* is the section, produce *ab* to meet H I in *c;* then will the radii *c b, c a,* describe two arcs; then radiating the end to the centre the lines thus drawn will form the board required.

In the same manner any other board may be found; as is evident from the principle described.

This kind of work should be described out on a floor or some other extensive plane; by so doing you can draw all your moulds and cut the joints, both for the frame and the covering before it is erected. In case the dome should be so large that it cannot be described on a floor, take a thin board of suitable length for the first course of boarding, bend it around on the plane of the rafters and scribe it down to the base line of the dome, when it is well fitted, guage it to a width and this will make you a pattern to mark out the remainder of the first course, and for the second course take the upper edge of the pattern of the lower course to mark out the lower edge of the second course, when this is done guage it to a width; and for the third course, proceed in the like manner and so on until the roof is completed.

To find the forms of the boards for covering an annular vault.

Let A D, *fig.* 1, *pl.* 11, be the outer diameter of the annulus, C G the inner, E the centre, and A C the thickness of the ring.

On A C describe the semicircle A B C: then if A B C be supposed to be set perpendicular to the plane of the paper, it will represent half the section of the ring. From E with the radius E A, describe the semicircle A F D; and from the same centre E, with the radius E C, describe the semicircle C H G, then A F D is the outer circumference and C H G is the inner circumference; and consequently A F D G H C, A is the section of the ring, perpendicular to the fixed axis; and the section A B C of the solid itself is perpendicular to the section A F D G H C.

To find the form of any board, divide the circumference A B C of the semicircle into such a number of equal parts as the boards or planks out of which they are to be cut will admit.

Let *a b* be the distance between two adjacent points; through the centre E draw H I perpendicular to A D: and through the points *a* and *b,* draw the straight line *a c,* meeting H I in the point *c;* from *c,* with the radius *c a,* describe an arc; and from

the same centre, c, with the radius $c\,b$, describe another arc, and inclose the space by a radiating line at each end; and the figure bounded by the two arcs, and the radiating line will be the form of the board required.

In the same manner the form of every remaining board may be found. It is obvious that, as common boards are not more than from ten to twelve inches in breadth, the boards formed for the covering cannot be long, if so they will produce much waste.

To cover an ellipsoidal dome the major-axis of the generating ellipse being the fixed axis.

Let A B C, *fig.* 2, *pl.* 11, be the section through the fixed axis or generating ellipse which will also be the vertical section of such a solid. Produce the fixed axis A C to I, and divide the curve A B C into such a number of equal parts that each may be equal to the proper width of a board. Then as before, draw a straight line through two adjacent points a and b, to meet the line A I in c; then with the radii ca and cb, describe arcs and terminate the board at its proper length.

Fig. 2, *No.* 2, is a horizontal section of the dome, exhibiting the plan of the boarding.

Fig. 3, *pl.* 11, is a section of a circular roof. The principle of covering it with boards bent horizontally, is exactly the same as in the preceding examples.

It is now necessary only to explain one general principle which extends to the whole of these round solids. The planes which contain the conic frustums are all perpendicular to the fixed axis, which is represented by H C, in all the figures. Produce $a\,b$ to meet the fixed axis H I in c; then with the radius $c\,a$, describe an arc, which two arcs will form the edges of the board, the ends are formed by radiating lines.

Either of these figures which we inspect, we shall find this rule to apply as the boards approach nearer to the revolving axis; they may be made either wider or longer, but as the boards approach near to the fixed axis the waste will be greater, and consequently, the boards must be shorter, when the boards come very near to the bottom of the dome, the centre, for describing the edges of the boards will be too remote for the length of a rod to be used as a radius.

In this case we may have recourse to the following method.

Let A B C, *fig.* 7, *pl.* 11, be the section of the dome as before, and let e be the point in the middle of the breadth of a board: draw $e\,d$ parallel to A C, the base of the section, cutting the axis of the dome in g, and join A e, cutting the axis in f. (Then by *Problem* 7 *Geometry*,) describe the segment of a circle through the three points d, f, e, and this will give the curve of the edge of the board as required.

Fig. 7, *No.* 2, exhibits the manner of using the instrument, (this instrument is also described in *Problem* 17, *Geometry*.) Thus suppose we make D E equal to $d\,e$, *fig.* 7, *No.* 1; bisect D E in G and draw G F perpendicular to D E and make G F equal to g, f, in *fig.* 7, *No.* 1, draw F H parallel to D E and make F H equal to F E, and join E H: then cut a piece of board into the form of the triangle H F E; then let H F E be that triangle; then move the vertex F from F to E keeping the leg F E upon the point E; and the leg F, and the angular point F of the piece so cut, will describe the curve, or perhaps as much of it as may be wanted.

It must be here observed that the line described is the middle of the board; but if the breadth of the board is properly marked off at each end, on each side of the middle, we shall be able to describe the arc with the same triangle, or if the concave edge of the board is hollowed out, the convex edge will be found by gauging the board off to its breadth.

As all the conic sections approach nearer and nearer to circles, as they are taken nearer to the vertex, so a parabola whose abscissa is small, compared to its double ordinate, will have its curvature nearly uniform, and will, consequently coincide very nearly with the segment of a circle, and as this curve is easily described, I shall employ it here instead of a circular arc as in Nos. 3 and 4.

Draw the chord D E as before, and bisect it in G. Draw G F perpendicular to D E and make G F equal to $g\,f$, in No. 1; so far the construction of the diagrams, Nos. 3 and 4 are the same, but in what follows they are different, therefore I shall speak of each separately—No. 3.

Divide each half, D G, G E, into the same number of equal parts; and, through the points of division draw lines perpendicular to D E; also from the points D and E at the extremities, draw perpendiculars; and make each of these perpendiculars equal to G F, then divide each into as many equal parts as D G or G E is divided into, and, through the points of division draw lines to F, intersecting the perpendiculars, and through the points of intersection, draw a curve on each side of the middle point F, and this will be the form of the edge of the board nearly.

In No. 4 make F H equal to $g\,f$, No. 1, and join D H and H E. Then divide D H and H E each into the same number of equal parts; then, through the corresponding points of division draw straight lines and the intersection of all the lines will form the curve sufficiently near for the purpose. The lines thus drawn being tangents to the parabolic curve.

The preceding method of covering round solids requires all the boards to be of different curvatures, and continually quicker as they approach nearer to the crown; but by the following method of covering a dome, with the joints in vertical planes, when the form of one of the moulders is obtained this form will serve for moulding the whole solid. The waste of stuff in this case is not less than in the other.

The method which I am about to explain is not only useful in the formation of the boards of a dome but in the covering of niches.

In *figs.* 4, 5 and 6, *pl.* 11, No. 1 is the plan, No. 2 the elevation; the counter of the latter being a vertical section passing through the axis. *Fig.* 4 is a dome which represents a round body of which the vertical section is an ogee or a curve of contrary flexure, to *figs.* 5 and 6. *Fig.* 5 represents a dome whose counter is a semicircle. *Fig.* 6 represents a segmental dome.

I shall proceed first with *figs.* 5 and 6. Through the centre of the plan G, draw the diameter A C; and the diameter B D, at right angles to A C; and produce B D to E. Let B D, *figs.* 5 and 6, be the base of the semi-section of the dome on B D, apply the semi-section C F D; and as the dome represented by *fig.* 5, is semicircular, the point F will coincide with the point A in the circumference of the plan. In *figs.* 5 and 6 divide the curve F D, of the rib into any number of equal parts, and extend the curve D F upon the straight line D E, from D to E; that is, make the straight line D E, equal in length to the curve D F. Through the points of division, in the curve D A, draw lines perpendicular to D G, cutting it at the points a, b, c, then extending the parts of the arc between the points of division upon the line D E, from D to 1, from 1 to 2, from 2 to 3, &c. make D d equal to half the breadth of a board, and join d G; produce

the lines $1a, 2b, 3c$, &c. draw through the curve D F to meet the line d G in the points d, c, f, &c. Through the points 1, 2, 3, &c. in D E, draw perpendiculars $1g, 2h, 3i$, &c. make $1g, 2h, 3i$, &c. respectively equal to $ad, be\ cf$, &c. and through the points d, g, h, i, &c. E draw a curve, which will form one edge of the board, the other edge being similar, we have only to describe a curve equal and similar, so as to have all its ordinates respectively equal from the same straight line D E.

In *fig.* 4, the form of the mould for the boards is found in a similar manner except that the curve D E is one side of the elevation, No. 2. Lines are drawn from the points of division in D E perpendicular to the diameter A C, which is parallel to the base of No. 2, and the points of division are transferred from the radius G C to the radius G D, which is the base of the section. The remaining part of the process is the same as in *figs.* 5 and 6.

In *fig.* 5, the curved edge of the board is a symmetrical figure of the sines; the curves of the mould, *fig.* 6, is a smaller portion of the figure of the same curve, and in *fig.* 4, the mould is a curve of contrary flexure; and if the curve D E be composed of two arcs of circles, the curve of the edges of the mould for the boards will still be compounded of the figure of the sines set on contrary sides; and if the curve D E be compounded of two elliptic segments, the edge of the mould for the formation of the boards will still be of the same species of curve; viz. the figure of the sines.

This figure occurs very frequently in the geometry of building.

COVERINGS OF POLYGONAL ROOFS.

The plans of these roofs are here supposed to be regular polygons, and all the sections parallel to the base, similar to the base and consequently similar to one another.

They are made of prismatic solids meeting each other in planes perpendicular to the plane of the base; and these mitre-planes meet each other in one common axis, which passes through the centre of each polygon.

In *fig.* 8, *pl.* 11, the plan is denoted by the letters A B C D E F A. Then the centre of the polygon being the point I, draw the lines A I, B I, C I, &c. Bisect any of the sides as A B, in the point L, and draw L I, then L I is perpendicular to A B.

Produce the line I L to M and let I L N be the section applied upon I L. In the curve L N, take any number of points 1, 2, 3, at equal distances, and transfer these distances to the line L M, so that L M may be equal to the arc L N. Through the points 1, 2, 3, &c. in L M, draw lines $1g, 2h, 3i$, &c. parallel to A B; and through the points 1, 2, 3, &c. in the arc L N, draw lines $1d, 2e, 3f$, &c. also parallel to A B, cutting L I at the points a, b, c, &c. and B I at the points d, e, f, &c. Make $1g$, equal to $a\,d$, $2h$, equal to $b\,c$, $3i$, equal to $c\,f$, &c. Through the points g, h, i, &c. draw a curve, which will be the edge of the joint over the mitre.

To find the angle-rib through the points d, e, f, &c. draw dk, el, fm, &c. perpendicular to B I. Make dk, el, fm, &c. respectively equal to $a1, b2, c3$, &c. through the points k, l, m, &c. draw a curve which will be the edge of the angle-rib as required.

Fig. 9, exhibits the method of framing the ribs for such kinds of roofs.

Fig. 10, shows the manner of describing the covering and ribs of a domical roof.

Fig. 11, shows the manner of describing the covering and ribs of a roof whose vertical section is a figure of contrary curvature.

Fig. 12, exhibits the manner of forming one of the ribs for an ogee roof, or that of a contrary curvature.

GROINS AND ARCHES.

Groins are the intersections of the surfaces of two arches crossing each other.

Construction of Groined Arches.

Groined Arches may be either formed of wood and lathed for plastering, or be constructed of brick or stone.

When constructed of brick or stone, they require to be supported upon wooden frames, boarded over so as to form the convex surface, which each vault is required to have, in order to sustain the cross arches during the time of turning them. This construction is called a centre, and is removed when the work is finished. The framing consists of equi-distant ribs fixed in parallel planes perpendicular to the axis of each body; so that when the undersides of the boards are laid on the upper edges of the ribs, and fixed, the upper sides of the boards will form the surface required to build upon.

In the construction of the centering for groins, one portion of the centre must be completely formed to the surface of its corresponding vault, without any regard to the cross arches, so that the upper sides of the boards will form a complete cylindric or cylindroidic surface. The ribs of cross vaults are then set at the same equal distances as that now described: and parts of the ribs are fixed on the top of the boarding at the same distances and boarded in, so as to intersect the other, and form the entire surface of the groin required.

Groins constructed of wood in place of brick or stone, and lathed under the ribs, and the lath covered with plaster, are called plaster groins.

Plaster groins are always constructed with diagonal ribs intersecting each other, then other ribs are fixed perpendicular to each axis, in vertical planes at equal distances, with short portions of ribs upon the diagonal ribs; so that when lathed over, the lathed angle may be equally solid to sustain the plaster.

When the axis and the surface of a semi-cylinder cuts those of another of greater diameter, the hollow surface of the lesser cylinder, as terminated by the greater, is called a cylindro-cylindric arch, and vulgarly a Welsh groin.

Cylindro-cylindric arches or Welsh groins, are constructed either of brick, stone or wood. If constructed of brick or stone, they require to have centres which are required to be formed in the same manner as those for groins; and if constructed of wood, lath and plaster, the ribs must be formed to the surfaces.

In the construction of groins, and of cylindro-cylindric arches, the ribs that are shorter than the whole width are termed jack ribs.

Cellars are frequently groined with brick or stone, and sometimes all the rooms of the basement stories of buildings, in order to render their superstructure proof against fire; the surfaces of brick or stone on which the first arch stones or course of bricks

are placed, are called the springing of the arches. It is evident that the more weight put on the side walls which sustain arches, the more will they be able to sustain the pressure of the arches; therefore the higher a wall is, the greater the weight will be on each of the side walls; and for this reason groins are often constructed with wood in upper stories, instead of brick or stone, as not being liable to thrust out the walls, or bulge them by the lateral pressure of the arches. The upper stories of buildings are never groined with stone or brick, unless when the walls are sufficiently thick to sustain the lateral pressure of the arches.

The ceilings of Gothic buildings were frequently constructed with groined arches of stone, which are obliged to be supported with buttresses, at the springing points of the arches.*

PLATE 12.

GROINS AND ARCHES.

Given the plan of a rectangular groined arch or vault, of which the openings are of different widths, but of the same heighth, and a section of one of the arches, as also the seats of the groins—to find the covering of both arches so as to meet their intersection.

Let A A A &c. *fig.* 1, *No.* 1, *pl.* 12, be the plan of the piers, and *ab*, *cd*, the seats of the groins.†

Let the section of the arch standing upon the lesser opening, B C, be a semicircle; it is required to find the section upon the greater opening and the ends of boards, so as to meet the groin, or line of intersection, of the two surfaces.

On the diameter B C describe a semicircle, and divide the quadrant into any number of equal parts *ef*, *fg*, *gh*, &c. and from the points *e*, *f*, *g*, &c. draw the line parallel to the axis F*k*, to meet the seat *ab* of the groined line, or line of intersection of the two surfaces. From the points *k*, *l*, *m*, &c. of intersection, draw the lines *k* Q, *l* R, *m* S, &c. parallel to the axis of the other vault, to meet the line V Q, perpendicular to that other axis in the points Q, R, S, &c. Then upon any line D E transfer the points Q, R, S, &c. to *q*, *r*, *s*, &c. and draw *qv*, *rw*, *sx*, &c. perpendicular to D E and transfer the ordinates F*e*, G*f*, H*g*, &c. of the semicircle to *qv*, *rw*, *sx*, &c. and through the points *v*, *w*, *x* &c. draw a curve; then *q*, *v* E will be half the section required.

To find the covering of the semi-cylinder. Upon any straight line Y Z, No. 2, set off the distances *lm*, *mn*, *no*, &c. each equal to the chord *ef* or *fg*, &c. in No. 1; and draw *l* K, *m* L, *n* M, &c. in No. 2, perpendicular to Y Z. Make *l* K, *m* L, *n* M, &c. in No. 2, equal to L *k* M *l* N *m*, &c. of No. 1, and through the points, K, L, M, &c. No. 2, draw a curve. Then will the figure K *l* Z be half the covering of the cylinder.

To construct the covering, No. 3, for the great opening.

In the straight line *vq*, No. 3, make *vu*, *ut*, *ts*, &c. equal to the parts, E*z*, *zy*, *yx*, &c. of the elliptic curve No. 1. In No. 3 draw *v* B, *u* O, *t* N, *s* M, &c. and make *v* B, *u* O, *t* N, *s* M, &c. No. 3, equal V *b*, U *o*, T *n*, S *m*, &c. No. 1: and in No. 3 draw a curve through the points B, O, N, M, &c.; then *qv* B K *q* will be the covering required.

To find the groin of a cylindro-cylindric arch.

Let A A A A, *fig.* 2, *pl.* 12, be the plans of four piers, which form the opening of different widths. On the lesser opening P M as a diameter, describe a semicircle. Divide the quadrant next to P into any number of equal parts, and through the points of section draw the lines 1 G, 2 H, 3 I, &c. perpendicular to P M, cutting P M in B, C, D, &c. and through the same points 1, 2, 3, &c. draw the lines 1 *a*, 2 *b*, 3 *c*, &c. parallel to P M, cutting a line *qe* perpendicular to P M, in the points *a*, *b*, *c*: produce the line which contains the points *a*, *b*, *c*, through the greater opening; and upon the part of the line thus produced, which is intercepted between the piers A, A, describe a semicircle.

Produce the line M P to *k*, and from *q* describe arcs *af*, *bg*, *ch*, &c. cutting B*k* in the points *f*, *g*, *h*, &c. draw *fk*, *gl*, *hm*, &c. parallel to the base of the greater semicircle to cut the arc of the same in the points *k*, *l*, *m*, &c. From the points *k*, *l*, *m*, &c. draw the lines *k* G, *l* H, *m* I, &c. parallel to P M; then through the points G, H, I, K, L, draw a curve, G, H, I, K, L, which will be the seat of the groin.

To find the diagonal or groin-ribs of a vault, of which the lesser openings are semicircles, and the groins in vertical planes passing through the diagonals of the piers.

On *ah*, *fig.* 3, *pl.* 12, the perpendicular distances between two adjacent piers of the lesser opening, describe a semicircle *abh*; and in the arc take 1, 2, 3, &c. any number of points, and draw the lines 1 *l*, 2 *m*, 3 *n*, &c. cutting the diagonal *ik* in *l*, *m*, *n*, &c. Draw as before *lq*, *mr*, *ns*, &c. perpendicular to *ik* and through the points *i*, *q*, *r*, *s*, &c. draw a curve; then *iuk* will be the edge of the rib to be placed in the groin.

The edge of the ribs for the other opening, will be found thus: From the points *l*, *m*, *n*, &c. draw the lines *l* I, *m* K, *n* L, &c. parallel to the axis of the opening of the larger body, cutting H B at the points C, D, E, &c. Make C I, D K, E L, &c. each equal to *c* 1, *d* 2, *e* 3, &c.; then through the points B I K L, &c. draw a curve: and the line thus drawn will be in the surface of the greater opening, so that B N H will be one of the ribs of the body-range.

The method of placing the ribs, is exhibited at the lower end of the diagram, *fig.* 3; the ribs of each opening being placed perpendicular to the axis of each groin.

To draw the arches of groins, whether right or rampant, so that their arches will intersect or mitre together as in *figs.* 4, 5, *and* 6. Fig. 4 is the plan of a rectangular groined arch or vault, of which the openings are of different widths, but of the same heighth.

This form is called the Gothic or pointed arch.

The section of the arch being given standing upon the lesser opening *abh*, and *ad* being divided off into a like number of equal parts as in *fig.* 3, you can proceed to find the ribs for the wider opening, as in the former examples.

A rampant arch, one of which, see *fig.* 5, the abutments or seats rise from an inclined plane.

This form of a groined arch is frequently met with in the ceilings of the under sides of galleries in Gothic churches.

It is but seldom the ceilings of galleries run horizontally, for they are generally raised up from the base line on the outer wall, for the sake of clearing the tops of the lower range of windows, or that more light may be produced.

To find the diagonal ribs of a vault as in *fig.* 5, suppose *jh* and *jk* to be the base or horizontal lines, and *a*, *h*, and *i*, *n*, the seats of the groins, draw *ja* and *ji* at right angles with *jh* and *jk*

* A specimen of this kind of structure may be seen in A. Pugin's eminent works on Gothic Architecture, taken from various ancient edifices in England, (vol. 2, pl. 47.)

† The difference between the plan of any body, and the seat of a point or line, is distinguished thus. The plan is a figure upon which a solid is carried up so that all sections, parallel to the plan, are equal and similar to that plan, and the surfaces are perpendicular; but the seat of a line is not in contact with the line itself, but a perpendicular erected from any point in the seat will pass through the corresponding point of the line itself.

and make ji equal in height to ja, draw the diagonals ah and ih, (the arch in the lesser opening being described as in *fig.* 27) divide the arc ab into a like number of equal parts (as above described) draw lines from the points 1, 2, 3, &c. through c, d, e, parallel to ai, until they intersect the diagonal line ik, then in the wider opening draw lines through l, m, n, &c. parallel to ij, and transfer the ordinates $1c$, $2d$, $3e$, &c. in the lesser opening, to lq, mr, ns, &c. in the wider opening, and through the points q, r, s, &c. draw the curve, then will i, p, u be half of the section required.

On the remaining part of the plate are a number of examples of the Gothic or pointed arch, as designed and described in the works of A. Pugin.

7. The semi-circular Arch was the principal one used in all buildings until about the middle of the 12th century, though a solitary instance of the pointed Arch may now and then be proved to be of earlier construction.

8. Arch described from one centre placed above the base line. This form has been denominated the horse shoe; it is common in some buildings of eastern countries.

9. Semicircular, but including a portion of the perpendicular jambs above the impost. This form is seen in a side-arch of the rood tower of Malmsbury Abby Church, where the transepts, being narrower than the nave and choir, two of the four arches were limited to a less breadth, though required to equal the others in height.

10, 11 and 12, Elliptical Arches described from three centres. Arches of this form are not only found in Norman buildings, mixed with the semicircular, but frequently over doors and windows in the early part of the fifteenth century, along with the pointed Arch and the other characteristics of the style of that period.

13. Semicircular Arches intersecting each other.—Some instances occur of intersecting pointed Arches, and others of Arches if they may be so called, described by strait lines forming a series of intersecting triangles raised on one base.

14. Semicircular and Lancet Arches combined.—Such mixture is commonly found in buildings of the twelfth century, when the pointed Arch began to prevail.

15. Elliptical, resembling a pointed Arch only rounded at the top.

16. Moorish.—This form may be classed with the horse-shoe, No. 8.

It is described from two centres placed above the impost Arches, somewhat of this form are occasionally met with in buildings of the early pointed or Gothic style; they are only found placed over narrow apertures.

17. Lancet Arch, described from two centres on the outside of the Arch.—Those termed lancet have been happily applied to the tall narrow windows which enlighten the structures of the thirteenth century.

Salisbury Cathedral is the most complete specimen of that style. These lights have each a pointed arch at top, and the arch is frequently raised on straight lines above the mouldings of the impost where such mouldings occur; this is indeed the lancet form, comparing the arch to the head of the lancet.

18. Equilateral where the point of the base and crown form an equilateral triangle.—This may be called the standard form of the pointed arch, and is perhaps the most beautiful.

19. Four-centred pointed.—Some beautiful varieties of decoration were struck out from this form, but it must still be regarded as less perfect than the simple arch struck from two centres, as in Nos. 17 and 18, 20, 21, and 22. The combination of circles, and portions of circles, being so infinitely diversified in specimens of florid tracery, especially in the larger windows of the fourteenth century, it would be in vain to attempt to analyze all their principles. We may observe, however, that most of them were divided at first into a few large forms, and these again subdivided into as many openings as the space would allow, so that the openings were never broader than those of the perpendicular lights of the window, and seldom less than one-half of the breadth of one of these. In proportioning the void and solid parts of windows, we seldom find the mullion exceed one-third of the light in the larger divisions, nor smaller than one-fifth.

23 and 24. Four-centred Arches, whose centres must be upon the same diagonal lines, which are found by dividing the base-line of the arch into more or less parts, according to the fixed height of the arch.—These are some of the various forms of what has been called the Tudor arch; being chiefly found in buildings erected under the reigns of the princes of the house of Tudor; we find, however, that this flattened arch was used more than fifty years before the accession of Henry VII, the first English sovereign of that name.

25 and 26. Methods of describing a pointed arch by the intersection of straight lines.—This arch may be classed with the four-centred, being of a less curve in the upper part than the lower. Many actual examples of arches appear to have been struck out, by the intersections of straight lines, in specimens of the latter period.

27. Rampant pointed, described also, by the intersection of straight lines.—(See what is said of fig. 5.)

28 and 29. Mode of describing a pointed arch by the crossing of straight lines.—This arch also may be classed with the four-centred.

30 and 31. Four-centred pointed of the same class as Nos. 23 and 24, but differently described.

ROOFING.

The roof is that part of a building raised upon the wall and extending over all the parts of the interior, in order to protect it from depredation, and from the severities and changes of the weather.

The Roof in Carpentry, consists of the timber-work, which is found necessary for the support of the external covering.

The several timbers of a roof are, *principal rafters, tie-beams, king-posts, queen-posts, struts, collar-beams, straining-sills, pole-plates, purlins, ridge-pieces, common rafters,* and *camber-beams.* The use of these will appear from the following description of them.

Principal rafters, are the two pieces of timber, in a framed roof, that form the two equal sides under the covering.

A *Tie-beam*, is a piece of timber, connecting the end of the principal rafters, in order to prevent them from spreading, by the weight of the covering. The *tie-beam* is therefore used as a string, and is in a state of tension.

A *king-post* or *principal-post*, is a vertical piece of timber, extending from the meeting of the two principal rafters to the

tie-beam, for the purpose of supporting the tie-beam in the middle.

Queen-posts, are two pieces of timber, equidistant from the middle of the truss, the one suspended from the head of one of the principal rafters, and the other with a level piece of timber between them.

Struts are those props which support the principal rafters in one or more points, so as to divide them into equidistant parts.

A *collar-beam* is the piece of timber framed between two queen-posts.

A *straining-sill* is a horizontal piece of timber, disposed between the end of the queen-posts, to counteract the efforts of the struts, in pushing the principals nearer to each other.

A *pole-plate* is a beam over each opposite wall, supported upon ends of the *tie-beam*, or upon the feet of the principal rafters.

Purlins, are horizontal pieces of timber, supported by the principal rafters.

A *ridge piece* is a beam at the apex of a roof, supported by the king-post, only the heads of the principals.

Common rafters are inclined pieces of timber, parallel to the principal rafters, supported by the pole-plates.

Camber-beams, are those timbers which are supported upon purlins over the collar-beams, and support the boarding for a leaden platform.

PLATE 13.

DESIGNS FOR ROOFS.

Fig. 1, is a design for a roof of a very narrow span, which ought not to be employed over a space exceeding fifteen or twenty feet.

Figs. 2, 3, 4, and 5, are examples which may be employed for a space of thirty or forty feet where the roof is shingled, or tinned.

Fig. 6, is a design for a roof for a narrow span, and calculated for an arched ceiling, having only one collar-beam, without a tie at the bottom.

Figs. 7 and 8 exhibit two designs of trusses, suitably constructed for the roofs of churches, and are calculated for a span of seventy or eighty feet.

Fig. 9, is a truss suitable for a small church of forty or fifty feet span.

Figs. 10, 11 and 12, show the elevations, and the construction of the timber work of a roof and cupola, for a small church of forty or fifty feet span.

Fig 10 represents the side of the truss and frame. Fig. 12, represents the frame as seen in the length of the roof. Fig. 11, shows the plan of the frame part of the deck of the belfry, which the upper part or story is framed into; these timbers should be well locked together and bolted; which I think will be sufficiently strong without the posts and timbers being extended down and framed into the collar beams, as they are generally, for it is a bad practice to load down a roof with unnecessary timbers; the least timber that can be put into a roof to make it sufficiently strong, the better.

The upper part of this cupola is framed out with planks or joist projecting out far enough to receive the entablature.

Fig. 13, is a design for a truss suitably constructed for a church or any other large building where it is designed to raise an arched ceiling, and is calculated for sixty or seventy feet span.

In this example it will be necessary to make the roof steeper than in the other examples heretofore described; and the timber also should be well seasoned, which will make the work better.

Fig. 14, shows the method of connecting the tie beam to that of the principal rafter.

Fig. 15, shows the longitudinal section of the tie-beams and king-post. The tie-beams are locked into each other, so that the outsides of each are in the same plane. The king-post is to be made in two parts, and a space cut away in each half, so as just to admit the tie-beams when locked together to pass through them.

Fig. 16, shows three different methods for scarfing timber.

CONSTRUCTION OF STONE OR WOOD BRIDGES.

PLATE 14.

Fig. 1, shows the centre of Westminster Bridge, which is partly supported by pieces strutting from the footings, and partly by piles driven into the bed of the river.*

Fig. 2, is the centre of Blackfriars Bridge; which is entirely supported by pieces strutting from the footings and piers. References.—A A A, timbers which support the centering.—B B. C C, upper and lower striking plates cased with copper.—D D, wedges between the striking plates for lowering the centre.—E E E, double trussing pieces to confine the braces.—F F F, apron pieces to strengthen the ribs of the centre.—G G G, bridgings laid on the back of the ribs.—H H H, blocks between bridgings to keep them at equal distances.—I I I, small braces to confine the ribs tight.—K K K, iron straps bolted to trussing pieces and apron pieces.—L L L, ends of the beams at the feet of the truss pieces.—M M M, principal braces.

Fig. 3, a longitudinal section of an arch of Waterloo bridge, showing the piles on which the piers are raised, the masses of

* The first bridge that was built in England of any note, was what is called the "London Bridge." This bridge was originally begun in the year 1176, by a priest, called Peter, curate of St. Mary Colechurch, a celebrated architect of those times. It was thirty-three years in building; but this period will not appear surprising, when it is considered that it was built over a river in which the tide rises twice every day, from thirteen to eighteen feet. The bridge at first consisted of twenty arches, but in 1758, the middle pier was taken down, and the two adjacent arches were converted into one, the span of which is seventy-two feet; its breadth is forty-five feet, and for many ages there were houses along each side of it, but these were removed when the middle pier was taken down in 1758. The remaining arches are narrow, and the piers inconveniently large, being from fifteen to twenty-five feet in thickness. The passage over the bridge is commodious, but in other respects there is nothing remarkable about it.

The Foundation Stone of Westminster Bridge was laid by the Earl of Pembroke, (a nobleman distinguished by his taste in Architecture) on the 24th of January, 1739. It consists of thirteen large and two small semicircular arches, of which the middle one is seventy-six feet span, and the parapets forty-four feet. The engineer was M. Labelye, a native of France; King James is believed to have aided in its design.

About ten years after this magnificent edifice was completed, another was begun about a mile lower down the river, known by the name of Blackfriars; the design was by Robert Mylne; it consists of nine arches of an elliptical form, of which the middle one is one hundred feet in span, and the breadth across the bridge is forty-three feet six inches. The arches being elliptical, and of wider span than those of Westminster, the bridge of course has a lighter appearance. The general style of it bespeaks a mind emboldened by the success of his predecessors, to advance with cautious step in the practice of bridge building. It is a work of great merit, and will stand a comparison with any other constructed in the same age. It was finished in ten and three-fourth years.

But the glory of England in bridge building, may be seen in the Strand or Waterloo Bridge, recently erected by Bennie, between Westminster and Blackfriars Bridges. Many other excellent Bridges have also been constructed in Great Britain, both of wood, iron and stone.

The Stone-Bridge over the Little River, Hartford, Con. was built in the year 1832. The design of its centre was given and erected by James Chamberlain, a practical architect and builder—and Elisha Rathbun being the master-mason. It consists of an arch of ninety feet span, and in breadth one hundred feet.

bricks composing the standards, and the centre supporting the arch. The dotted line shows the direction of a curve, in which the weight is so distributed, that the different pressures to which the edifice is exposed have no tending to change the form of the arch.

Fig. 4, shows a longitudinal section of the arch and centre of the Bridge that is built over the little river, crossing Main-street, in the city of Hartford, Con. The form of the centre of the bridge is different from those described: it is moreover well constructed. The bed of the river is a solid rock, and runs nearly on a horizontal plane between the two butments; during the summer season the river is generally shallow, which made it convenient for bedding the sills and timbers into which the struts are framed. References.—A, shows a part of the longitudinal and transversection of the timbers running horizontally in the bed of the river, into which the struts are framed.—B, shows the form of the wedges that are placed under the feet of each strut for lowering the centre.—C, shows a part of the transversection of the frame.—D, is a section of the arch ribs.—E, shows the manner in which the joints are connected.

The remaining part of the plate shows five designs which I have given for wooden bridges.

Fig. 5, is a side elevation of the frame, clearly showing all the timbers and the manner in which they are connected together. This design I think sufficiently strong for a bridge of ninety and even a hundred feet span. If there be any cause for giving away, it will be from the spreading of the two outer butments: if these be built sufficiently strong to keep the lower struts to their proper place, I think there will be no danger.

A bridge of this description should be filled in with puncheons between the principal timbers on each side of the truss or strings, standing vertically and not more than two feet apart from centre to centre, and covered over with boards on both sides, which should be matched or feather-edged, and strongly nailed with large nails or spikes, which not only keep the timber from being exposed to the storms, but makes the work more substantial, and it also prevents the timbers in a great measure from working or springing up and down, which is of great injury to a bridge. This is the reason that Town's patent bridge has so good effect.

It is not by common travel that our bridges are injured and sometimes broken down, but by driving over large droves of cattle and horses. Drovers should be particularly careful in crossing a bridge, not to let their whole drove enter the bridge at the same time, for if their immediate weight does not break the bridge, it will materially injure it.

Figs. 6 and 7, are designs for bridges, which may be from fifty to sixty feet span or even wider, if they are built of good timbers and strongly covered. Fig. 6 is supported by three iron bolts and two posts; the tenons may run through the sill, or tie-beam and plate, and may be keyed. Fig. 7 is supported by three posts which are locked to the sill and plate; the posts are in two parts, as represented in F. G shows the manner in which they are locked to the sill and plate. The posts should be made of white oak timber, as the ends which run below and above the sill and plate, will not be as liable to split. These being well locked and bolted together, will make a strong work, and will answer all the purposes of long iron bolts, and even better.

Figs. 8 and 9 are of the same principle as figure 7, only calculated for a less span.

PLATE 15.

This plate exhibits a number of useful machines, and their construction as made use of by builders and mechanics in general.

Fig. 1, No. 1, and fig. 1, No. 2, shows the elevation of a machine that is called a crane. This machine is used to raise stone and other heavy bodies on to buildings, and it is also placed on wharves for the purpose of unloading small vessels and boats. It can be constructed so as to be taken apart and put together again as occasion may require.

Fig. 1, No. 3, shows a plan of a frame drawn on a small scale.

Fig. 1, No. 4, shows a plan of the cap of the upper part of the frame; there must be a sufficient weight placed on the frame at A, to keep it in its place.

The cog wheel to which the wench or cylinder is attached, and about which the rope winds, is generally from one foot six inches to two feet in diameter, and the other wheel to which the crank is attached should be about one third of the size of the other.

Fig. 2, No. 1, fig. 2, No. 2, shows the elevation of a machine which is mostly used in hoisting piers for shop fronts, &c.

The frame part should be made of light timber; pine or spruce would be most proper, as it may be easily handled; the foot of the frame should be placed near the place where you intend to set your pier, and a little inclined, as represented at No. 2; it may be secured by guy ropes, or by two light poles being made fast at the upper part of the frame. The cast iron cog wheels may be of the size of those of the crane, heretofore described.

Fig. 3, No. 1, fig. 3, No. 2, and fig. 3, No. 3, show the plan and elevations of a wheel suitably constructed for raising goods, &c. The wheel and machinery is generally erected in the upper part of wholesale stores, for the purpose of hoisting and lowering goods from one loft to another.

The design that I have here given consists of three shafts; and to hoist heavy goods, the centre shaft should be used, which is attached to the great wheel, and to hoist light goods, such as boxes, bales, &c.; when they are to be raised many stories, the two outer shafts will answer all purposes, and the work may be performed with much greater rapidity; you can either hoist upon one, and lower on the other, or hoist and lower both at the same time. The great wheel is generally made from eight to fourteen feet in diameter, and the shaft from eight to twelve inches. A B C shows a plan of a check or lever to stop the force of the wheel, which should be strongly secured at A, and a rope attached at the end B, leading over a pully at C, then passing through to the lower story.

Fig. 4, shows a method of raising a stone column with *lewis* irons. D is the plan of a lewis, which should be let into the end of the column very tight, having no room to play. A A, views of the capstans. B and C, a view and plan of the frame of a capstan. The frame which the large falls are attached to, may be secured by guy ropes, or light poles as described in *fig.* 2, *No.* 2.

NOTE.—The weight of a stone column, or any other form of a stone may be accurately ascertained, by finding the number of cubic feet and inches it contains, by the rules given in section 2d of mensuration, and multiplying the number of feet thus found by 125, which is the number of pounds contained in a cubic foot of stone.

A column that is two feet six inches at the lower diameter, and two feet at the upper, and fifteen feet in length, contains

10

about sixty cubic feet, and 60×125=7500 pounds; a column of this size, and even larger may be raised.

Fig. 5, shows a method of raising a truss by a gin pole.

This should be of a suitable length to raise the truss to its destined height, and should be made either of pine or spruce, so as to be easily raised or lowered: a stick that is from ten to twelve inches in diameter at the bottom, and from six to eight at the top, will be sufficiently large to raise a truss from sixty to ninety feet span.

This when placed upright should be secured by small falls for the guys which will be more convenient, and safer than guy ropes; three in number will be all that is necessary, letting them run off at different angles, and properly secured at the bottom. A shows the manner in which they are secured at the top, and also the big block of the large fall; the fall part runs down and passes through a snatch block at B, then off on to the capstan at C.

In raising the trusses of a church, they should be put together on the main floor and well secured: commence raising them at one end of the building, and when you have got one raised and placed to its proper place and well braced, slip the gin along where the second one is to stand, and this being raised and properly secured to the other, proceed in this manner through the whole building; a good set of hands working under a master workman, will generally be able to complete the whole in one day.

Fig. 6, shows the manner of raising a purchase which is called by seamen a Spanish burden. A is a large single block which the runner is rove through, and one end of the runner is made fast to the double block at B, the other end hooks on to the weight at C, and also the single block of the fall, the fall part being rove through a snatch block at D, for the purpose of attaching a horse or a yoke of cattle, &c. This machine will be found useful for loading stone or large logs, on two set of wheels; this being placed near one end of a log and properly fixed for hoisting, fasten a horse to the fall and raise one end of the log up to a proper height, place one set of wheels under and lower the end; then the machine being placed at the other end, and raised in the same manner. This way of loading stone or logs is easily and quickly performed.

Figs. 7 and 8, show the elevations of a pile driver, a machine for driving piles into the ground, of which there are many kinds; some are worked by a number of men who raise a heavy weight to a small height, and then let it fall upon the pile, till by reiterated blows they drive it to the required depth. This machine is extremely simple. A long thick plank of wood is fixed up close to the pile, having a mortice through the upper end, in which a pully is fitted; a rope goes over this to suspend the rammer, which is a large block of hard wood, properly hooped to prevent it from splitting. In rising and falling, it slides against the base of the plank, and is guided by irons which are fixed to the ram and bent around the edges of the plank in the manner of hooks. The plank when placed upright, is secured by guy ropes in the manner of the mast of a ship. The end of the great rope which suspends the ram has ten or twelve small ropes spliced into it, for as many men to take hold of, to work it. They raise the ram up three or four feet by pulling the ropes all together, and then letting them go, the ram falls upon the pile head. When the pile becomes firm enough to cause the ram to rebound, they take care to pull the ropes instantly after the blow, that they may avail themselves of the leap it makes. This is the simplest form of the machine.

But for large works, such as bridges, &c. the piles are driven by a different kind of machine; this has a very heavy iron ram as represented at E and F, in *figs.* 7 *and* 8, with mechanical powers, by which it is raised to a very considerable height, and then let fall, instead of continually repeating small blows. This machine is constructed by two uprights erected on the frame, and supported by two braces, which are framed into the uprights and the cross feet or frame at the bottom; then let two pieces be framed horizontally, one on each side, running from the uprights into the braces at B; then let two maple planks be framed into these, and the bottom at A standing vertically, one on each side of the frame, for the crank and cylinder to run in, of which they should be from six to eight inches wide, and about two inches in thickness; the upper part of the frame may be constructed as represented in *figs.* 7 *and* 8, with a pully attached to the upper part of the frame, at I and J, for the rope to run over, of which one end of the rope is made fast to the cylinder at C, and the other end running over the pully at J, and so down to H, where it is made fast to the eye, or hoop of the tongs. H is a piece called a follower, consisting of a wooden block sliding between the uprights, and morticed to receive the iron tongs, which take hold of an eye on the top of a cast iron ram, or weight at E.

The rope is attached to the follower by an iron hoop, of which the form of it is seen in *fig.* 7, through which the centre pin of the tongs passes near H of the follower; the ram or weight E being held by the tongs, is drawn up by turning the crank at D till the tails of the tongs come to the inclined planes at the upper part of the frame, which opening the lower ends of the tongs, disengages them from the eye of the ram, and it falls upon the head of the pile. G is the plan, or form of the ram, showing the enter grooves made in the edges, by which it is guided as it rises and falls.

The fillets of iron are fixed withinside of the uprights, and they should be from 1 to $1\frac{1}{4}$ inches in thickness, and about 2 in width, and they should be let into the sides of the uprights, one inch, and properly fastened.

PLATE 16.

Figs. 1 and 2, exhibit two designs of antæ and entablatures which are more suitably constructed for frontispieces and interior finishing, than those of the orders.

Fig. 1, is imitated from the Choragic Monument of Trysallus, that stands at the foot of the Acropolis or citadel of Athens.

Fig. 2, is chiefly of my own design. The capital and mouldings are of the Grecian style. The heighths and projection of the mouldings and parts are expressed in minutes and eighths.

The remaining part of this plate is taken up with Grecian ornaments, and ornamented mouldings, &c. which seem to require no particular description.

Fig. 3, represents the enriched moulding of the antæ capital of the Temple of *Minerva Polias,* at Athens, as represented on *pl.* 36.

Fig. 4, *Egg and Tongue* with beads below, belonging to the same cap.

Fig. 5, represents the ornament between the bead and small projecting band of the same.

Fig. 6, is the ornament on the Cymatium of the door of the same temple, stretched out on a flat surface.

Figs. 9 and 10, represent a front and a side view of a console of the same door; most of the remaining part of these figures were taken from Grecian structures, which on account of their foliage are more graceful than those of the Romans.

STAIRS AND STAIR-CASING.

DEFINITIONS OF THE PARTS OF STAIRS.

A flight of steps, so called, means an assemblage of steps, so formed and united, that by walking on them, we ascend or descend from one height to another.

The surfaces on which we set our feet are called *treads;* and these for the convenience of walking, are set at equal distances and parallel to each other.

In order to give a solid appearance to the whole, every adjacent pair of treads are connected by a third and vertical piece, called a *riser*. Each riser and tread when fixed together, is called a step. The wall which supports the ends of the steps, is called the stair-case.

When the ends of the steps terminate upon a vertical prism or pillar, the prism or pillar is called a *newel.*

If the ends of the steps be cut through in the surface of the newel, and the pillar or prism be supposed to be removed, the space left open by the removal of the solid is called the *well-hole*.

Stairs that have a well-hole, or hollow in the centre, are called *Geometrical stairs*.

The meeting of the sides which form the external angles of the steps, is called the line of nosing: but sometimes the line of nosing is covered with a moulding, and then this moulding is called the *nosing*.

When the steps are of equal breadth, that is, when the distance from the line of nosing to the riser is every where equal, the steps are denominated *flyers.*

When the treads of the steps diminish in breadth toward the well-hole, the steps are called winders.

As the ends of the steps generally terminate upon a surface which is perpendicular both to the risers and treads, the surface on which they thus terminate is generally that of a cylinder.

A number of contiguous flyers are called a *flight*.

When the tread of the step is so broad as to be equal to two or more of the other steps, and situated between floors, it is called a *resting place*.

If the tread of the resting place form a right angle, that is, if the two risers be perpendicular to each other, the resting place is called a quarter-space, or quarter-pace.

When the breadth of the tread of a step is contained between the same vertical plane, or makes two right angles round the axis of the well-hole, the tread is called a half-space or half-pace.

Half-space and quarter-space are generally made on floors; and in this case are called *landing places.*

The carriage of a stair consists of several pieces joined, or framed together; when they are so constructed it is called the carriage of the stairs.

A flight of steps is generally supported by two pieces of timber, placed under the steps and parallel to the wall, being fastened at one or both ends, to pieces perpendicular thereto.

The pieces of timber or planks which are thus placed under the steps are called *rough strings.*

Dog-legged Stairs are those which have no well-hole, and consist of two flights without winders. The hand-rail, on both sides, is framed into vertical posts, in the same vertical plane, as well as a board which supports the ends of the steps. The boards are called *string-boards*, and the posts are called *newels.* The newels not only connect the strings, but they afford the principal support to the rail; and thus it may be affirmed that the newel posts and hand-rail, are all in one plane.

Open neweled Stairs are those which have a rectangular well-hole, and are divided into two or three flights.

Bracketed Stairs are those where the string-board is notched so as to permit risers and treads to lie upon the notches, and pass over beyond the thickness of the string-boards; the ends of the steps are concealed by means of ornamental pieces called brackets.

Geometrical Stairs are generally bracketed; but the dog-legged and open-neweled stairs, only those of the best kind are bracketed.

A *Pitching Piece* is a piece of timber wedged into the wall, in a direction perpendicular to the surface of that wall, for supporting the rough-strings at the top of the lower flight, where there is no trimmer, or where the trimmer is too distant to be used for the support of the rough-strings.

Bearers are pieces of timber or planks fixed into and perpendicular to the surface of the wall, for supporting the winders where they are introduced; the other end of the bearers is fastened to the string-board.

A *Notch Board* is a board into which the ends of the steps are let: it is fastened to the wall, or one of the walls of the stair-case.

Curtail Step is the lowermost step of the stairs, and has one of its ends next to the well-hole formed into an ornament representing a spiral line.

These are the principal parts which belong to a stair or stairs; other parts connected with it belong to the hand-rail.

PROPORTIONS OF STAIRS, &C.

The breadth of steps to common stairs is from nine to twelve inches. The breadth in elegant houses and public edifices, ought never to be less than ten, nor more than fifteen inches.

A step of greater breadth requires less height than that of a less breadth.

The general rule may therefore be as follows:—

Multiply the breadth and height of a given step together, and divide the product by the breadth of the required step, and the quotient will be the answer; or by reciprocal proportion, as the given breadth belonging to the height required, is to the breadth of the given step, so is the height of the given step to the height of the required step.

For example, taking as a standard a step of 11 inches in breadth, and 6 inches in height, we may easily find the height of another of a given breadth, which we shall suppose to be 9 inches. The operation is thus:

```
9 : 6 : : 11 :
          6
        ----
      9) 66 (7.33
         63
        ----
          30
          27
        ----
          30
          27
```

We find it to be $7\frac{1}{3}$ inches for the height, which agrees with what would be allowed in common practice.

Before we lay out the stairs in a building, we must consider the height of the story, and determine upon the height of the steps; which being done, we must bring the height of the story into inches and divide the number of inches in the height of the story, by the height of the step. Thus for example, suppose the height of the story to be ten feet three inches, and the height of the step to be seven inches, how many steps will be required in order to ascend to the given height. The operation is

```
  12              7) 123 (17
  10-3                7
  ----              ----
  120                53
    3                49
  ----              ----
  123 inches          4
```

Thus we find the number of steps to be seventeen and a trifle over, but as we must not have a fractional part, therefore the step must be over seven inches in height; we will proceed thus:

```
17) 123(7.23
    119
    ----
     40
     34
    ----
     60
     51
```

We find the height of the step to be $7\frac{23}{100}$ inches; being a trifle less than $7\frac{1}{4}$. This will answer very well; but, if we are still confined for room on the plan, we may throw the semi-circumference round the newel into winders.

The breadth of stair-cases may be from four to fifteen feet or more according to the destination of the building; but if the steps be less than two feet six inches in length, they become inconvenient for the passing of furniture, as is generally the case in small houses.

When the height of the story is very high, resting-places become necessary. In high stories, that admit of sufficient head-room and where the plan or area of the stairs is confined, the stairs may make two revolutions in the height of the story; that is, the ascendant or descendant may go twice round the newel or well-hole, and this becomes necessary; otherwise the steps would be enormously high, or extravagant floor-room must be allowed for the stairs.

In laying out the rise and run for a stair-case, there will be one more riser in number than treads, that is by not counting the upper one or landing, which forms that part of the floor. It may not be amiss to give an example here for a principal building, in order to show the number of steps and risers both in the grand and common stair-case.

For this purpose, suppose the story of a house to be twelve feet one inch high from floor to floor, and we may have sixteen feet ten inches for the run of the grand stair-case, the number of steps of the grand stair-case to be twenty in number, that is, there will be twenty risers and nineteen treads, what will be the breadth of the treads and risers?

Now the height of the story 12 feet 1 inch being reduced to inches is 145, and the run, which is 16 feet 10 inches, being reduced to inches gives 202, and first dividing by 19—for the run

```
19)202(10.63          divide by 20)145(7.25
   19                              140
   ----                            ----
   120                              50
   114                              40
   ----                            ----
    60                             100
    57                             100
   ----                            ----
     3
```

We find the breadth of the tread to be $10\frac{63}{100}$ inches and for the riser $7\frac{25}{100}$ inches.

Now for the stair-case of that for the servants, we cannot have for instance but twelve feet nine inches for the run, and of course the height will be the same as in the former; therefore we must have a less number of steps to bring it into proportion.

For example, let there be eighteen risers in number and seventeen treads.

The run which is 12 feet 9 inches, being reduced to inches, is 153, and thus

```
17) 153(9        18) 145 (8.05
    153              144
    ----             ----
                      100
                       90
                     ----
                       10
```

Which gives 9 inches for the breadth of the tread; and $8\frac{5}{100}$ for the riser, being large 8 inches. This will make a good proportion.

In laying out the rise of a stair-case, we may divide the whole height from floor to floor into equal parts on the string-boards, then when the strings are set up level at their proper places, scribe off the thickness of one tread on the lower step or curtail, and this will drop it down the thickness of one at the upper part of the landing, so when the steps are put together they will be all equal in height. A farther description of laying out and putting up stair-cases will be described hereafter.

HAND-RAILING.

"*The art of forming Hand-rails* round circular and elliptic well-holes, without the use of the cylinder, is entirely new. Mr. Price, the author of '*The British Carpenter*,' is the first who appears to have had any idea of forming a wreath-rail. Subsequent writers have contributed little or nothing towards the advancement of this most useful branch of the Joiner's profession, and have contented themselves with the methods laid down by Price, which were uncertain in their application; and, consequently, led to erroneous results in practice.

The first successful method of squaring the wreath, upon Geometrical principles, was invented and published by Peter Nicholson, in 1792, in a work called '*The Carpenter's New Guide*,' a book well known to architects and workmen. No previous author seems to have had any idea of describing the section of a cylinder through any three points in space making a mould to the form of the section, and applying it to both sides of the plank, by the principles of solid angles, so that by cutting away the superfluous wood, the piece thus formed might have been made to range over its plan.

Since the first invention of the method, the author's experience and researches have produced many essential requisites,

which were not thought of at first; so that this branch, as here presented, is now much improved.

The principle of projecting the rail, furnishes the workman with a method by which he can ascertain, with great precision, the thickness of the plank out of which the rail must be cut. To do this in the most convenient way, the diagram must be made to some aliquot portion of the full size, which will supercede the necessity of laying it down on a floor.

It must however, be observed that the thickness of stuff found by this method, is what will completely square the wreath or piece. But as the rail is reduced from the square to an oval or elliptic section, much thinner stuff may be made to answer the purpose; so that, generally for rails of common size of 2 or 2¼ inches thick, instead of requiring a four inch, when one of two and a half may be made to answer the purpose."

NOTE.—Since I commenced compiling this work, a number of young builders have requested me to make some of my drawings full size; I have therefore added two double plates, on which a part of the examples are described sufficiently large for practice.

PLATE 17.

TO DRAW THE SCROLL FOR A HAND-RAIL, AND THE MOULDS FOR EXECUTING THE SAME, AND ALSO THE CURTAIL-STEP.

Fig. 1, shows a method for drawing the scroll to a hand-rail, and curtail-step, to a full size for practice. The full lines that are drawn represent that part of the scroll, and the dotted lines the curtail-step.

To draw the scroll.—Divide the line A B the width of the scroll, into nine equal parts. Draw A D and B C at right angles with A B, and make each equal in length to one of the nine parts; join C D, and also the diagonal line B D. Draw E F perpendicular to A B, and at a distance from A of four of the nine parts into which A B is divided. Draw the semicircle E F O. Thence draw the diagonals E O G and F O H cutting the diagonal B D in the point O. Join G H, H I, I J and J K; then will E F G H I J K be the centres for describing the scroll and curtail-step.

Then from the centre E, with the distance E B, in your compasses, describe the quadrant B L. Then from the centre F, with the radius F L, describe the quadrant L D; from the centre G, with the distance G D, describe the quadrant D M; from the centre H, with the distance H M, describe the quadrant M N; from the centre I, with the distance I N, describe the quadrant N P; from the centre J, with the distance J P, describe the quadrant P Q; then from the centre K, with the distance K Q, describe the arc at R; which finishes the outer spiral of the scroll.

Thence set off the bigness of the rail from B to S on the line A B.

Again, from centre E, with the distance E S, in your compasses, describe the quadrant S T; from the centre F, with the radius F T, describe the quadrant T U; then from the centre G, with the distance G U, describe the arc at R, which will complete the inner spiral and terminate in the point R as required.

The curtail-step is described in the same manner, and from the same centres, as that of the scroll, which the lines on the face of the curtail terminates at W W, where there is two grooves cut out by running a saw through to let the ends of the veneers which bend around on the face or surface of the riser, as represented by the two dotted lines, in the diagram of the figure. X X X X shows the bigness of the newel post: and Y Y Y Y the bigness of the mortise that is made through the step, which should also run through the floor, and a mortise made through the tenon of the newel on the under side of the floor, and keyed, to draw the step down close to the floor. The circles that are described in the diagram of the figure represent the balusters; the different sections of the remaining part of the curtail-step is plainly drawn, showing the manner in which it is put together; it will not require any further explanation: therefore we will proceed to describe the face mould for the scroll of the hand-rail.

To draw the face mould, for squaring the twisted part of the scroll.—Let A A A in *fig.* 2, be the pitch board, and A the angular point as standing over the front edge of the riser of the curtail-step, at the bottom. Let the line A B A be continued to 14 until it meets the line L 14 in *fig.* 1. Then in *fig.* 1 let the line B C be produced until it meets the line L 14 also; and divide this line into any like number of parts: commencing at B, then 1, 2, 3, &c. to 14 as represented in the diagram of the figure. Then draw lines through these several divisions parallel to A B, until they meet the diagonal line A B A in *fig.* 2. Thence on the diagonal line A B in *fig.* 2, draw lines B S, 1 1 1, 222, 3 3 3, &c. at right angles with A B.

Then take B S, the bigness of the rail from *fig.* 1, and set it off from B to S in *fig.* 2; take also the distances 11 1, 222, 333, 444, &c. from *fig.* 1, and set them off from 1 1 1, 222, 333, 444, &c. in *fig.* 2. When you have got all these distances or ordinates transferred from one to the other, then with a pencil trace out the curve through the several points in *fig.* 2, which will give the form of the face mould as required.

To draw the falling mould of the scroll as in fig. 3.—This figure or mould is drawn on a smaller scale, being only one third of the size of those of *figs.* 1 *and* 2. To draw this mould let B V in *fig.* 3, be made equal in length to the stretch out, from B around to V in *fig.* 1, the scroll. Then let B V in *fig.* 3, be divided into three equal parts; and from the first division at 1, lay on the pitch board as represented at A A A. From B raise a perpendicular to C. Thence divide 1 C and 1 V each into a like number of equal parts, and form the curve by the intersection of lines. Then set down the thickness of the rail from V to D, and from A to E, and join E D. This mould should be made of thick paper, and bent around the scroll to form the curve of the upper edge of the rail. This being done, and the upper edge of the rail squared, the curve of the lower edge may be obtained by gauging.

Fig. 4, shows a method of getting a scroll out of a solid piece of wood, having the grain of the wood run in the same direction with the rail; which is far preferable to any of the other methods.

To square out this scroll. Let A A A A, be the block of wood of a suitable breadth and thickness, and let the underside of it be faced over and one edge be made square. Then let the block of wood be set up on the bench in a raking position, and place your pitch-board under it as represented at E E E, D D represents the line of the bench on which it is placed.

Then with a pair of compasses scribe through from F to G, and work off the corner or the angle A F G on the lower part. This being done, and the underside of the scroll F G being faced over true and square, place the pattern of the scroll as in *fig.* 1, on the under side, and mark all around it, and thence square up

from the under side, instead of drawing a face mould, and working by it from the upper side. This method is much quicker and as accurate. When you have got the scroll squared up around perpendicular to the plan, then bend around your falling mould, so as to have B C in *fig.* 3, stand perpendicular to B C in *fig.* 4, or stand perpendicular to B on the plan in *fig.* 1. If the scroll is required to be worked out of thinner stuff, let the joint be made and glued together at *aa* in *fig.* 1. And in this case it is not necessary to draw the face mould farther than at *aa*, in *fig.* 2. Scrolls are frequently made in this way.

Fig. 5, shows an elevation of a scroll, curtail step, balusters, &c. and two designs for brackets. The rail of stairs should be squared out, and the joints all made and cut to a length, before it is rounded or moulded. This being done, and spaced off for the balusters or plummed up, square across upon the under side of the rail, and then lay your pitch-board on as represented at A, and with a pencil mark down upon the side of the rail, which will be a guide to you in boring, or to mortice by. B shows the manner in which the base makes a finish under the string board of the stairs.

Fig. 6, shows the form of a rail-screw, which secures the joints, and screws them tight together. The screw is let into the centre of the rail, and the nuts are let in on the under side within about $1\frac{1}{2}$ inches of the joint or shoulders of the rail. The nut at A is made square, and should be let into the mortice without much play, so as not to have it turn around within the rail. B is made round with creases cut into it, for the purpose of turning the nut on or off within the rail. C shows the plan of the same nut.

PLATE 18.

TO FIND THE MOULDS FOR EXECUTING A HAND-RAIL ON A SEMICIRCULAR PLAN WITH EIGHT WINDERS; AND ALSO THE CONSTRUCTION OF THE CARRIAGE OF THE STAIR.

Let A B C, *fig.* 1, be the plan of the eight winders, as described upon the floor over which the stair is to be built. The full line that is drawn A B C represents the concave side of the string-board. And the dotted line at D E represents the concave side of the hand-rail, which projects over the vertical face of the string-board; F F the treads; G G risers; H H the falls or back risers, in which the bearers of the winders are framed; I I bearers of the winders; J the middle bearer of a step.

Fig. 2, clearly shows the section of the stretch-out of the eight winders as they would naturally appear when erected; and also one of the treads of the flyers both above and below the winders.

But before we proceed to lay out this stair-case, and to draw the moulds for executing the hand-rail, &c.; let us suppose it to be made to some aliquot portion of the full size. For instance, let the treads of the flyers be equal to $10\frac{1}{2}$ inches in breadth, and the rise or risers equal to $7\frac{3}{4}$ inches in height; and the breadth of the treads of those of the winders should be made equal to one half of the breadth of those of the flyers: that is, on the concave part of the well-hole; and thus by placing two balusters into the tread of the flyers, and one in the winders of each tread. Then the balusters in the winders and flyers will be at equal distances from each other, which will give a much better appearance. Therefore we will suppose the breadth of the winders to be $5\frac{1}{4}$ inches, on the concave part of the well-hole; and the height of the risers of course will be the same as those of the flyers. Now it becomes necessary to know how large the well-hole must be in diameter to receive the eight winders; which may be found by the following operation. Thus $5\frac{1}{4} \times 8 = 42$ inches, the stretch out of the well-hole, or the semi-circumference.

Then the circumference of the whole circle will be equal to 84 inches, and thus by (Art. 13. section 1 of mensuration)

```
as 22 : 7 :: 84 :
             7
          ----
     22) 588 (26.72
         44
         ---
         148
         132
         ---
          160
          154
          ---
           60
           44
           --
           16
```

Which we find the diameter to be $26\frac{72}{100}$ inches, being a trifle less than $26\frac{3}{4}$. Now the hand rail generally, projects over the string board from $\frac{1}{2}$ to $\frac{3}{4}$ of an inch; which renders it necessary for us to know what the diameter is, and also the circumference of the concave part of the rail, or the convex circumference of the semi-cylender, which is called the working cylinder, as represented in *fig.* 10.

Let us suppose the working cylinder of the rail to be $25\frac{1}{2}$ inches in diameter. Then we may find the circumference by (Art. 12 of mensuration.)

```
7 : 22 :: 25,5 :
          22
         ----
         510              2)80.14
         510                -----
         ----               40.07
      7)5610(80.14
        56
        ---
          10
           7
          --
          30
          28
          --
           2
```

Thus we find the semi-circumference of the cylinder to be $40\frac{7}{100}$ inches.

We will now proceed to describe the falling moulds, and also lay out the run and rise of the eight winders, and find what the length of the hypothenuse lines will be, of the string board, and that of the rail also.

First, let a line be drawn upon a floor as A B C in *fig.* 2; and make B C equal in length to the semi-circumference A B C in *fig.* 1, which is equal to 42 inches. From C raise the perpendicular C D, and make it equal in height to the eight winders; which we suppose takes 62 inches. Draw the hypothenuse line A D. From B draw B E perpendicular to A C. Then will A E and E D be the length of the hypothenuse line of the two quadrants, which is the length of the concave side of the string board A D. From A draw A F one of the treads or more; of the flyers draw D G also. And thence set off the width of the string board, from these lines down; and draw the lower line parallel to them, and whenever these lines meet in their angular point, they form the curve by the intersection of lines, as represented, (which is called by workmen the *easing of the angle.*)

To construct the falling mould for the hand-rail. In *fig.* 2 draw L m parallel to A C; and make L, n, m equal in length to the stretch out of the dotted line A B C in *fig.* 1, which is $40\frac{07}{100}$ inches (as calculated heretofore.)

From m, draw the perpendicular, m J; and equal in height to the eight winders, or C D. Draw the hypothenuse line L J, and from n, draw $n\,o$, perpendicular to L m. Thence from L draw L l N one of the flyers, or the pitch board, at the bottom, and one also at J h H, at the upper part of the windows. Then from L set off L M the bigness of the rail, and N O, J K and H I also, and draw O M, M K and K I; and wherever the angular points meet at L M and J K, form the curve by the intersection of lines, which will complete the falling mould.

To find the straight part of the rail, for the lower and upper wreath pieces or quadrants. From L let the line L m be continued across the rail until it cuts the upper side at U.

Then U L will be the straight part of the rail for the lower wreath piece, and V K the upper one, which are the same as the straight part A D in *fig.* 3, the plan, or A D in *fig.* 4.

To find the height of the rail, that is for the lower and upper wreath pieces. At o, square across the rail, and divide it into two parts, at the centre Q. Thence draw P Q perpendicular to Lm, and continue the line P Q up to R, and draw R X at right angles with R P, cutting the joint of the straight part of the rail at X. Then when you come to draw the face moulds, as in *figs.* 5 *and* 7, take the distance from R down to the under side of the rail at S, and set it off on the line R S in *fig.* 7; and for the lower wreath piece, take the distance from P up to the upper side of the rail at T, and set it off on the line from P to B in *fig.* 5.

The reader, in order to understand what has already been described, should recollect that the hypothenuse A D *fig.* 2, (the string board) is longer than L J of the hand rail.

It has been proved heretofore, that the semi-circumference of the well-hole A B C is about two inches longer than that of the rail L $n\,m$; the difference may not appear so much on the plate.

It may not be amiss to give an example here to show the learner how he may obtain the length of these hypothenuse lines A D and L J; and also ascertain what their difference will be in the lengths. This may be done by extracting the square root of the sum of the two legs, or by the properties of the *triangle* in *Trigonometry*; or it may be performed by Logarithms which is the same thing. Each method will be fully explained hereafter in their proper place. The operation will be as follows:

```
AC=42 in   CD=62 in        Lm=40.07 in   Jm=62 in
    42         62              40.07         62
    --         --              -----         --
    84        124              28049        124
   168        372            1602800        372
   ----       ----           --------       ----
   1764       3844           16057049       3844
              1764                          16056049
              ----                          --------
              56.08(74.88                   54.49.60.49(73.82
              49                            49
              --                            --
          144)708                       143)549
              576                           429
              ---                           ---
         1488)13200                    1468)12060
               11904                        11744
               -----                        -----
        14968)129600                  14762)31649
               119744                       29524
```

The length of A D we find to be $74\frac{88}{100}$ inches, and L J being $73\frac{82}{100}$ gives nearly $1\frac{1}{10}$ difference.

For further particulars, see (Art. 34 of mensuration.)

To find the face mould of the rail.—Let A B G, *fig.* 3, be the plan of the concave side of the rail, and make A G equal in diameter to D E in *fig.* 1; let A F and B C be the thickness of the rail, and let A D, the strait part of the rail, be equal in length to U L or K V in *fig.* 2. Now it is not necessary to draw or make a pattern of only one of these quadrants, as in *fig.* 3, and together with the straight part of the rail A D and E F. The most simple and the most convenient way to draw the face moulds at a full size for practice, is as follows.

Take a board of a suitable breadth and length, face over one side, and straighten one edge of it, of which the board should be of the breadth of G H in *fig.* 4, or wider and equal in length to G G on the edge of the board. From thence square across this board at G H in *fig.* 4, and place the pattern of the lower wreath piece (which you have already made as in *fig.* 3) on the board so as to have the two angular points of the pattern at B and D stand on the line G H. This pattern being tacked on the board, mark around it; you must be particular in marking the joints at B C, A F and D E; when this is done, take the pattern off the board, and then divide the quadrant A B, into any number of parts, as 1, 2, 3, &c.; the greater the number the more correct will be the operation.

And thence set a guage on the edge of the board G G, so as to draw a line from the angular point B through N P up to B in *fig.* 5. Then set it across in A, and run it from L through A M up to L in *fig.* 5. Then in F draw J K F to J; then in D draw D I to D; then in E draw H E to H also. Thence draw the lines through 1, 2, 3, 4, &c. the quadrant part of the rail.

And then square across the board at P H in *fig.* 5. Then take the distance from P up to the upper side of the rail at T. in *fig.* 2, and set it off from P up to B in *fig.* 5. And draw the hypothenuse line B H, and continue the line from B out to the edge of the board at G. Then take a thin board to make your pattern, and face it over, and straighten one edge, which should be equal in length to G H and H O the breadth. Then place the edge of the board on the hypothenuse line G H, and tack it down to the other; and thence square across this board at the several divisions H D J L I 2 3 4 5 B and G. Then take the distance of G C in your compasses in *fig.* 4, and set it off on the line from G to C in *fig.* 5. Then take the distance of B N in *fig.* 4, and set it off from B to N in *fig.* 5. Then take the distances of 5 5 5, 4 4 4, 3 3 3, 2 2 2, 1 1 1, in *fig.* 4, and set them off on the lines at 5 5 5, 4 4 4, 3 3 3, 2 2 2, 1 1 1, in *fig.* 5. Then take also L A M, J K F, D I and H E, from *fig.* 4, and transfer them to L A M, J K F, D I and H E in *fig.* 5. Then in *fig.* 5 draw the lines B C and D E, which will form the joints of the rail. And draw A D and E F the straight part of the rail. Then to draw the curve, tack in brads at the several points A F M and at 1 1, 2 2, 3 3, 4 4, 5 5, B N and C, and bend a thin slip of wood around, with a pencil trace out the curves, which will complete the face-mould for the lower wreath-piece of the rail as required.

The face-mould of the upper wreath-piece of the rail, may be found in the same manner, as described in *figs.* 6 *and* 7, only the pattern as in *fig.* 4, must be turned over so as to have the straight part of the rail come on the edge of the board as represented in the diagram of *fig.* 6. Then take the distance from R down to the under side of the rail at S in *fig.* 2; and set it off from R to R in *fig.* 7; and draw the hypothenuse line B T. Thence you may transfer the several ordinates from *fig.* 6 to *fig.* 7, as heretofore described in *figs.* 4 *and* 5, and complete it in the same manner.

Figs. 8 and 9, show the method of cutting the rail out of the plank; the plank should be from 3 to $3\frac{1}{2}$ inches thick, to square out the wreath-pieces of the rail, when the rail is from $2\frac{1}{4}$ to $2\frac{1}{2}$ inches in thickness. To do this, let *fig.* 8 be the plank; lay your pattern on the face, so as to have the angular points stand upon the edge at B D, and mark around it; then take the pattern off and turn the plank up, upon its edge, as represented in *fig.* 9. Then lay your pitch-board (of the winders) on at B L G, and draw the vertical line B L; and lay it on also at A D the straight part of the rail, and draw the vertical lines A E and D C. Thence turn your plank down upon the other side, and turn the pattern of the face-mould over also, so as to have the angular points at B D in *fig.* 8, stand on the edge of the plank at L C in *fig.* 9. When the pattern is laid upon the face as above described, mark around it, as was done in *fig.* 8; and then cut away the superfluous wood on both sides of the rail, which is generally done by screwing the plank into a vice; then set a narrow frame saw in on the line B L in *fig.* 9; two men are required to work it, one on each side of the plank, so that they may govern the saw, and work to the lines which are drawn on both sides of the plank. This being done, and the rail trimmed down smooth to the lines, take the pattern of the falling moulds as in *fig.* 2, which should be made of a thin board or thick paper, and bend it around on the concave side of the rail, and draw the upper side of the mould, and then the thickness of the rail may be obtained by gauging, after the upper side is squared around perpendicular to the plan, or the working cylinder.

It will be well, however, to leave the rail a trifle thicker at the ease-offs, that is, where the straight part of the rail joins on to the wreath-pieces. Then when the rail is screwed together, and set to its proper place, you may in a short time, with a coarse file, work it down, so as to have the curve appear natural and easy. And the string-board of the stair should also be worked in the same manner, since by thus doing you will be less liable to make cripples.

The greatest art of hand-railing, depends on finding the section of a cylinder to pass through three given points on its surface, as heretofore described; therefore the reader is requested to understand this part thoroughly, before he actually commences his study upon hand-rails; for if the principles are not comprehended, he will always be in difficulties and liable to spóil his work.

Fig. 10 shows a section of the hand-rail as formed to the *prismatic mould* or the *working cylinder*.

This method, which was formerly practised for forming the wreaths and fitting the rail together, is now of no other use than merely to help the conception of the learner; therefore it may be better for the inexperienced to form his work to a working cylinder. To do this let the cylinder be made equal in length to the height of ten steps, as represented in *fig.* 10: and the convex circumference equal to ten also; that is, the semi-circumference shall be equal to the eight winders, as A B G in *fig.* 3, together with the two fliers A H and G I, or H A B in *fig.* 10, shows one half of the plan of the vertical section; and C E D and F G I the two fliers, and J H L M N O P F the winders.

Fig. 11 shows the method of laying out the notch-board of the stair-case, in which the treads and risers enter.

Let A B and C D be the width of the board or plank, and from the under side A C gauge on one inch at the line E F; then lay your pattern or pitch-board on as represented at G H I, and draw G H and H I; then slip the pattern along to O N P and draw O N and N P, and so on until the whole are drawn. Then with a pair of dividers set off G M and H L the thickness of the riser, and draw L M; then also set off H K and I J, the thickness of the tread, and draw J K &c. The nosing of the steps may be drawn with dividers, or work a small piece in wood for a pattern, and lay it on at its proper place and mark around it; the pattern should be made a trifle less in size than that of the step, so as to have it fill up tight. If the treads and risers are calculated for wedging, let the steps be laid out as above described; then from P set off a suitable thickness for the wedges at P P and P R, and run them out to a point at N N.

In regard to this method of wedging the steps into the notch-board, there is not the necessity of wedging them in an open stair-case as in a tight one; that is, where the stair is carried between two partitions. Wherever there is to be a nice stair-case built of this description, the steps should be let into the notch-boards, or strings, upon both sides; which is done by letting the step-boards and risers in on the under side of the strings or stairs; the step-boards and risers should also be tongued and grooved, and put together with glue, and also the wedges, as described in the diagram of the figure.

The notch-boards in which the steps are let in, are generally from $1\frac{1}{2}$ to 2 inches in thickness, they should not be less than $1\frac{1}{2}$; then lowering $\frac{3}{4}$ for the thickness of the lath and plastering, there will be $\frac{3}{4}$ left for the thickness of the base; or if the plank be $1\frac{3}{4}$ inches thick, lowering $\frac{3}{4}$ for plastering, there will be 1 inch left for the thickness of the base, which generally is as thick as required. If the plank be $1\frac{1}{2}$ inches thick, the steps should be let into them at least 1 inch; and to have firm steps they should not be less than $1\frac{1}{4}$ in thickness.

To put up a tight stair-case of the above description; first lay out your notch-boards, or strings, as heretofore described, and then nail them to their proper places, and be careful also to drive the nails upon the under side of the steps, so they may not be seen. Then if your stair be all the way of equal width in the opening, square and saw off all the treads and risers, and the hollows of the nosing to their right length, and also have the wedges made, and the square blocks of wood which are glued into the internal angles of the risers and step-boards; then commence at the bottom by letting the lower riser into the floor, or scribe it down tight to it. We must remember, that this riser can neither be let in upon the under side, or wedged, nor tongued into the first step-board; therefore we must cut it one inch shorter than the rest, and so slip one end into the notch-board, the depth of one inch; then draw it back into the other, so as to divide them equal, then nail it down tight to the floor, and the notch-boards, by driving the nails obliquely on the back side of the riser. Then put in the step-boards, and when you have got them all in, and wedged firm and glued, commence putting in the risers; the tongues of the risers and step-boards should be glued into the grooves.

The hollow of the nosing must however be put into its place before the riser. When the risers are put in and wedged, nail them strong to the step-boards, upon the backside, taking care the nails do not go through the face.

Then glue in the square blocks in the internal angles of the risers, and step-boards, as represented at N N in *fig.* 11, which will strengthen the work, and prevent them in a great measure from becoming rickety; and though this stair be done with the utmost care, it can never be made so firm as not to yield to the passenger; especially those of an open stair-case.

Fig. 12, shows the method of diminishing and tracing one bracket from another, in a stair-case; A B being the breadth of those of the fliers, and A C those of the winders.

PLATE 19.

To find the Falling and Face-Moulds for an Elliptical planed Stair, *pl.* 19.

The plan and section of the rail being laid down as in *fig.* 1, *No.* 1.

The reader will observe, that the ends of the steps are equally divided off, around the elliptical wall, and also at the rail or well-hole. In order to cut the rail out to the best advantage, it should be made in three lengths, as represented at A B C.

The plan of this rail must be divided into as many equal parts as there are steps; then take the treads of as many steps as you please; suppose there be eight in A, and six in B and C; let $h\,h$ at *fig.* 1, *No.* 2, be the tread of eight steps, that is, the stretch-out of the eight steps at A, and let H h, be the stretch-out of the six steps at B and C. Thence draw the perpendiculars $h\,m$ and H k, and make $h\,m$ equal in height to the eight risers at A, and H k, equal in height to the six at B and C; and then draw the hypothenuse line $h\,m$, which will be the underside of the falling mould. Then set up the thickness of the rail $m\,n$ and $k\,l$, and draw $n\,l$ parallel to $k\,m$, which completes the falling mould. This falling mould will be a straight line, excepting a little turn at the landing, and at the scroll where the rail must bend, in order to bring it level with the landing, and to the scroll.

To construct the face-mould.—Draw the chord line for each piece to the joints at A B C; also, draw lines parallel to the chords, to touch the convex side of the plan of the rail, as $h\,h$ at A, and H h at B and C; and then from every joint draw perpendiculars to their respective chords. Then take the distance of $h\,n$ in *fig.* 1, *No.* 2, and set it off from h to n in F; and draw the hypothenuse line $n\,i$, and then take the distance of H l in *fig.* 1, *No.* 2, and set it off from H to l in D and E; also draw the hypothenuse line $l\,i$. From thence you can transfer the several ordinates in A B C to D E F, and complete the face mould as in the foregoing plate.

Fig. 2, No. 1, and fig. 2, No. 2, show a plan and elevation of a semicircular well-hole for a stair, and *fig.* 3, *No.* 1, *and fig.* 3, *No.* 2, show the method of drawing the face mould of the hand-rail, and *fig.* 4, *No.* 1, shows a method of describing a vertical scroll appertaining to the same. All of these figures are drawn at one third the size for practice. This stair-case is calculated for a straight run; and the well-hole is inserted at the landing of the floor.

To construct the well-hole as in *fig.* 2, *No.* 1, let the semicircle A B C, that part of the string-board, be worked out of two pieces of plank; and let the grain of the wood run vertically; and spring them by taking the angular corners off at D C K, E K J, and F H I, &c. Then let a groove be run in them at H, and glue them together by putting in a false tongue. Each piece should however be worked or rounded out to a pattern, before they are glued, which can be done with more facility and convenience, than when they are put together. The dotted lines at R S show the projection of the outer edge of the upper step-board of the well-hole, and P Q the hollow under the step-board, or the nosing; M N is the opening between the rough string-board and the trimmer-joist; W is the rough string; X the upper riser which mitres into the string-board of the well-hole at A. The step-board of this well-hole should be worked in one piece, that is, from S where it mitres to the upper riser, let it be

continued across to T at the centre of the trimmer joist, and let R T be a square joint, where it finishes to the level flooring; then from T let it be squared across to the centre of the trimming-joist at U. Thence let it run from U on the centre of the joist across the stair into the notch-board. The hollow of the nosing, which is worked around in the well-hole, should be worked out of a square piece of board. This may be done by cutting the well-hole off that part of the string-board, on a plane at the under side of the hollow of the nosing.

When the well-hole of the string is set in at its proper place, take a board the thickness of your hollow, and fit it in close between the rough string and trimmer joist, as represented by the dotted lines at L M N O. Then set a pair of dividers of the projection of the hollow, and from P scribe around the well-hole to Q, and this part, after it is sawed out, must be worked out around with a gouge; and let the joint at P O be square, where the hollow on the level part ends, and the joint also at L M left square, so return the hollow on the piece from Q to L, and let the hollow on the upper riser end square to it.

The upper step-board may be worked around the well-hole in the same manner as above described.

To describe the falling mould or the ease-off of the well-hole, as in *fig.* 2, *No.* 2.

The ease-off should be made where it can be at the centre, as represented at E. In order to describe it in this manner, let A B C in *fig.* 2, *No.* 2, be equal in length to the stretch-out of A B C in *fig.* 2, *No.* 1, and C D in *No.* 2, be equal to the height at the level part of the landing; then to square around within the well-hole, let a pattern of the pitch-board be made of thick paper, as represented at F G H; and place the square side of this pattern on the vertical line or edge at C D, so as to have the square corner at H come upon D, and bend it around on the plane of the well-hole. With a pencil draw the line D E to the centre; then turn the pattern over upon the other edge, as represented at F G H, and draw the line F E cutting the line D E at the centre E. Form the curve by intersecting lines, or trace the curve around with a pencil, which two inches each way from the angular point E will be sufficient to form the curve. The string-board of the stair in which the risers mitre into and finishes with that part of the well-hole at F I; the width of this string may be ascertained by drawing out one step at full size from F I.

To draw the face mould of the hand-rail, as in *fig.* 3, *No.* 1, *and fig.* 3, *No.* 2, which stands over this semicircular well-hole.

The reader will observe, this rail is not worked on a spring as is sometimes done; the quadrant part as in *No.* 1, lies on a perfect plane every way; that of *No.* 2, also lies on a plane one way, and the other, which, by running up on its true raking position brings the rail high enough upon the level landing at B D, where the joint of the rail is made.

Patterns of this face mould may be made of a thin board or thick paper. In *No.* 1, draw the quadrant E A B the inside of the rail; from A set off the bigness of your rail to C, and draw the quadrant C D; then from A C draw so much of the straight part of the rail as is necessary, and draw the line E D, which will form the joint of the rail at B D, which completes the pattern for the level part of the rail.

To draw the pattern or the face mould for the raking part of the rail as in *fig.* 3, *No.* 2. First place your pitch-board of the stair, as represented at E F G, so as to have the lower angular point come at E, and draw the raking line from E to G. Take

the pitch-board off the paper or board, then continue the line G E to H, and draw the lines E A C and H I J at right angles with G H. Then take the distance of E A in *No.* 1, and set it off from E to A in *No.* 2, and from H to I also, and draw the straight line A I. Then take the bigness of the rail, A C in *No.* 1, and set it off from A to C in *No.* 2, and at I J also, and draw the straight line C J, which will complete so much of the straight part of the rail as will be necessary. Then divide the line E D in *No.* 1, into any like number of parts, as 1, 2, 3, &c. and through these several divisions, draw the line 1 *a b*, 2 *c d*, 3 *e f*, &c. parallel to C E, until they meet the line G H in *No.* 2. Then through 1, 2, 3, &c. in *No.* 2, draw the lines 1 *a b*, 2 *c d*, 3 *e f*, &c. parallel to E C, or at right angles to G H. Thence take the distances 1 *a b* in *No.* 1, and transfer them to 1 *a b* in *No.* 2. Then take 2 *c d* in *No.* 1, and set them off at 2 *c d* in *No.* 2, and so on until you have got all of the several ordinates transferred from *No.* 1, to *No.* 2. Then with a pencil draw the curve A B in *No.* 2, through the points at *a c e*, &c. then at C D through *b d f*, &c. which completes the face mould.

In order to work this rail out, let us suppose *No.* 2, to be the plank, and the pattern of the face mould being laid on upon the face of it as represented in the diagram of the figure; mark around on the inside of the pattern at B A I, and on the outside also at D C J. Take the pattern off, and let the plank be turned up upon its edge as represented on the plate, at the right of *No.* 2; then place your pitch-board on the edge as represented at A B C, and draw the vertical line A D; slip the pitch-board along at E, and draw E F, to G also, and draw G H. Then let the plank be turned down upon the other side, and turn the pattern of the face mould over so as to have the edge E B D, lie upon the edge at D F H, and mark around it as above described. Then let this plank be screwed into a vice, and set your compass-saw in on the line E F upon the edge of the plank as represented at the right of *No.* 2, and saw the quadrant part around to A D, by following the lines which are drawn upon both sides of the plank.. Then set the saw in on the line G H, and let this be worked around in the same manner. Thence if your rail be round, draw the circle as represented at E F, G H, which does not require the plank to be thicker than that of the level part, as represented at the left of *No.* 1.

To draw the vertical scroll of the hand-rail. This is generally made about eight inches in height; therefore we will suppose the height on the line A B as in *fig.* 4, *No.* 1, to be eight inches. Then let the line A B be divided into eight equal parts, then each division will be equal to one inch, and bisect the fifth division at the centre 1, from A down; from 1 draw 1, 2, at right angles to A B, which make 1, 2, equal in length to one inch or one of the eight parts, (see *fig.* 4, *No.* 2, on which the centres are drawn to the full size;) bisect A B at the centre H.

Draw the lines B G C and H E D, at pleasure, and at right angles to A B. Take the distance of B H in your compasses, and set it off from B to G, and draw the line E F G parallel to A B. Bisect E G at the centre F. Then take the distance of G F and set it off from G to C, and draw C D parallel to G E. Then will A B C D and E be the centres as represented at 1, 2, 3, 4 and 5, in *No.* 1.

To describe the scroll as in *No.* 1. Take the distance of 1 A in your compasses, and on 1 as a centre draw the quadrant A C. Then on 2 as a centre, with the distance of 2 C draw the quadrant C D. Then on 3 as a centre, with the distance of 3 D draw the quadrant D E. Then on 4 as a centre, with the distance 4 E draw the quadrant E F. Then on 5 as a centre, with the distance of 5 F draw F G. Then from A set down the thickness of your rail at H; and then in 1 as a centre, with the distance of 1 H draw the quadrant H G, cutting the former at G: which completes it.

To draw the ease-off of the falling mould. Let a line be drawn from A at I, at right angles to the perpendicular A B. From A set off $2\frac{1}{2}$ inches to J. Then place your pitch-board on as represented at I J K, and draw the raking line J K, and set off about 3 inches from J to L, and form the curve by the intersecting lines.

In putting up a continued rail of the above description, there should be one or more iron balusters inserted near the well-hole at the landing, to support the rail and prevent it from trembling; or you may let an iron run from the under side of the raking rail, off into the trimmer joist, as represented at A B C, in *fig.* 6.

To describe a scroll for a hand-rail, as in *fig.* 5, *No.* 1. The reader will observe the scroll that has already been described on this Plate, is suitable for one of 8 inches; and the one that has been described on Plate 17, is suitable for one of 10 inches; which is drawn at full size. And the one that I am now about to describe, will be suitable for one of 12 inches; though either of the above mentioned examples, may be increased, or diminished, as choice may direct.

To describe this scroll as in *fig.* 5, *No.* 1. Suppose the line A B, the width of the scroll, to be 12 inches. Then divide the line A B into eleven equal parts, and from 1, at the sixth division from A, draw 1, 2, at right angles to A B, and make 1, 2, equal in length to one of the eleven divisions. We will now turn to *fig.* 5, *No.* 2, where the centres are drawn at full size. Let A B be equal in length to 1, 2, or one of the eleven divisions, as in *No.* 1. Bisect A B as in *No.* 2, at the centre K; from K draw K H at right angles to A B, and make K H equal to K A or K B; join the diagonals H A and H B. From B draw B J C at pleasure, and at right angles to A B; from H draw H J parallel to A B; and bisect H J at the centre G; then bisect H G at the centre I; and from I draw I L parallel to B C, which makes I L equal in length to I J; from L draw L C parallel to I J; produce the line C L to D and make L D equal in length to L C. From D draw D E parallel to L I or B C, until it meets the diagonal H A at E. From E draw E F parallel to A B, until it meets the diagonal H B at F. From F join F G. Then will A B C D E F and G be the centres, which is the same as 1 2 3 4 5 6 and 7, in *No.* 1. Thence in *No.* 1, take the distance of 1 A in your compasses, and one foot in 1 as a centre, draw the quadrant A C. Then in 2 as a centre, with the distance of 2 C draw the quadrant C D. Then in 3 as a centre, with the distance of 3 D draw the quadrant D E. Then in 4 as a centre, with the distance of 4 E, draw the quadrant E F. Then in 5 as a centre, with the distance of 5 F draw the quadrant F G. Then in 6 as a centre, with the distance of 6 G draw the quadrant G H. Then in 7 as a centre, with the distance of 7 H draw the arc to I. Then set off the bigness of your rail from A to J. And again, with one foot of your compass in 1 as a centre, with the distance of 1 J draw the quadrant J K. Then in 2 as a centre, with the distance of 2 K draw the quadrant K L. Then in 3 as a centre, with the distance of 3 L draw the arc at I, intersecting the former at I, which completes it.

Fig. 7, shows a section of a *dog-legged stair* with newel-posts. The proportions are figured off in feet and inches.

The shortest newel, as represented at the landing, stands

opposite to the other. The hand-rail of the stair finishes with a *knee* at the bottom, and is mitred into a cap, which is turned on the upper part of the newel, the cap being turned in the form of the rail. And the rail mitres into the cap of the newel-posts at the upper part, and is ramped up in the form of a *Swan-neck*, so called, being concave below and convex on the top, and terminating at the newel, so as to be parallel to the horizon.

This kind of stair, is substantial when finished, but its erection requires double the labor necessary for the completion of continued rails, as described on this plate.

COUNTRY VILLAS.

PLATE 20.

This plate exhibits the plans and elevations for Country Villas, after the Castellated or Gothic style; or more properly called, British Architecture. The designs are by M. A. Nicholson.

Fig. 1.—*Elevation of a Castellated Gothic Villa,* with buttresses, &c. The whole length of this building is 78 feet; and its height from the surface of the ground to the top of the battlements, 34 feet 2 inches. The battlements are continued all around the building, and the height of them is 2 feet 6 inches. The buttresses are 29 feet 3 inches high, with two water tables, on the top of which is a cornice. The cornice is continued all round the building. The windows on the ground floor are 4 feet 3 inches from the ground, their height nine feet, and width 4 feet. The top of each window is crowned with a tablet, which reaches a little below the top of the window, on each side. The chamber-floor windows are 19 feet from the ground, their height is 8 feet, and width 4 feet, and they are crowned with a tablet, as below. The entrance is on the flank to the left, raised 1 foot 6 inches above the level of the ground, and ascended by three steps; it is enclosed within a porch of 12 feet in front, and 8 feet deep; the openings of the front and sides of the porch are 8 feet, and 4 feet 10 inches. The height to the springing of the arches is 8 feet, and to the top of the arch 3 feet 10 inches, over which runs a band, and is of the same height as that in the octagonal front of the building; on the right flank is a green-house, which will have a very beautiful effect on entering, as seen at one extremity of the passage through a sash-door.

Fig. 2.—*Ground Plan of the Principal Story.* A, *Porch,* 8 feet by 6 feet; B, *Passage,* communicating with the different apartments, 74 feet long by 6 feet wide; C, *Staircase* to the bedchambers, steps 3 feet 6 inches long, treads 11 inches, risers rather more than 6 inches; *Breakfast-room,* 20 feet by 17 feet 6 inches; *Dining-room,* 20 feet by 17 feet 6 inches; *Drawing-room,* 30 by 30 feet; *Library,* 20 feet by 17 feet 6 inches; E, *Parlor,* 20 feet by 17 feet 6 inches; D, *Waiting-room* or *Dressing-room,* 19 feet by 15 feet; F, *Water-closet,* which is entered by a door under the staircase; *Green-house,* 42 feet 6 inches by 12 feet 6 inches. The *Servants' apartment,* &c. are on the basement, which is entered by the staircase, C.

Fig. 3.—*Elevation of a Castellated Gothic Villa,* with buttresses and pinnacles, on a straight front. The extent of this building, from the extremity of one wing to that of the other, is 60 feet; extent of each of the wings, 11 feet 10 inches. The body of the building, 36 feet 4 inches. The entrance, 3 feet 4 inches wide, with a Gothic head, receding from the central part of the front, 3 feet 6 inches, forming a Porch, and raised above the level of the ground 1 foot 6 inches; ascended by 3 steps of 6 inches rise. The entrance to the Porch is 4 feet 10 inches wide, and it rises 6 feet to the springing of the arch; the arch is 4 feet high, and is ornamented with mouldings and crockets on each side. The windows of the ground floor on each side of the Porch, are 6 feet 6 inches high by 4 feet 6 inches wide; those in the bed-chamber 4 feet 3 inches high and 4 feet wide.

Fig. 4.—*Ground Plan of the Principal Story.* A flight of three steps to the Porch, L, 7 feet 7 inches by 2 feet 6 inches. I, *Hall,* 9 feet 6 inches by 7 feet 7 inches. N, *Staircase,* steps 3 feet 3 inches, their rise is 6½ inches, and the tread 10½ inches. P P, *Passage* to the different apartments, 57 feet 8 inches by 4 feet. B, *Breakfast-room,* 14 feet 3 inches by 12 feet. A, *Dining-room,* with folding doors, 14 feet 3 inches by 12 feet. F, 12 feet 3 inches by 10 feet 7 inches. D, *Parlor,* 12 feet by 10 feet 4 inches. C, *Library,* 12 feet by 10 feet 4 inches. H, *Servants' Waiting-room,* 7 feet 6 inches by 6 feet 3 inches. E, *Kitchen,* 12 feet 3 inches by 10 feet 7 inches. G, *Wash-house,* 10 feet by 7 feet 6 inches. O, *Water-closet,* 4 feet 2 inches by 3 feet.

PLATE 21.

Ground Plan and Elevations of a Church in the Grecian Style. The design is by M. A. Nicholson.

References to the plan. The Portico *a* of four columns, projects out from the front of the wall, 8 feet 6 inches. The *Vestibule b* leads to the body of the chapel and side stair-cases; diameter, 19 feet 9 inches within the columns.

Stair-cases of an elliptical form *c c* leading to the gallery, 22 feet 8 inches by 20 feet 9 inches; length of treads, 4 feet 10 inches; breadth in the middle, 11 inches; risers rather more than 6 inches. Side entrances, *f f.*

The body of the Church may be 89 feet long and 54 feet 3 inches broad, and it will contain sixteen hundred sittings, (including free seats,) exclusive of seats for children in front of the organ.

h h h h, represents Pews 3 feet wide; seats 1 foot; book-desk 5½ inches; *o o, larger Pews; p p,* spaces between the free seats. The *Pulpit n,* of an hexagonal form; *i, Stairs* ascending to it. *Reading-desk m,* with clerk's seat in front; stairs *i,* ascending to it. The *Communion-place* of a circular form, with four three-quarter columns, and two antæ; betwixt the columns are two niches and a window in the centre.

The *Vestry-room s,* is 22 feet 2 inches by 15 feet 4 inches. The entrance to Circular Stairs *y,* leading to library above. The *Anti-room* under the Portico *u,* 10 feet 5 inches by 8 feet. The *Robing-room t,* 22 feet 2 inches by 15 feet 4 inches. The *Ante-room v,* 10 feet 5 inches by 8 feet. X, *Entrance* to the Catacombs. W, *Back-Portico* of four columns, projecting out from the wall 5 feet 6 inches.

Description of the *Front Elevation.* The extent of the front of this building to the extremities of the antæ, is 64 feet 9 inches; the breadth of the portico at the top of the columns, is 37 feet. The columns are raised 1 foot 7½ inches above the level of the ground, and are designed from the Monument of Lysicrates, as described on Plate 46; their diameter is 3 feet, and height, including the base and capital, 29 feet. The height of the entablature is 7 feet 4 inches; the architrave 2 feet 9 inches; the frieze, 1 foot 10 inches; and the cornice, 2 feet 9 inches. The

ornament which stands on the top of the cornice is 13 inches high, and is continued all round the building. The antæ are of the same width as the top of the upper diameter of the columns, and do not diminish. The capitals of the antæ are a composition, as there are no antæ to be found in this style of Grecian architecture. The principal entrance is ascended by three steps in front of the portico, of 6½ inches rise; tread 1 foot. Its width at the bottom is 6 feet 11 inches; it diminishes to the top, and its height is 14 feet. The side entrances are each 6 feet 7 inches at the bottom, and diminish to the top; their height is 12 feet 9 inches, with an architrave round them and a cornice at the top, supported at each extremity by a console. The niches on each side of the principal entrance are 4 feet 3 inches wide, and 9 feet 10 inches high, and diminish on each side, parallel to the sides of the columns. The Attic, which stands over the cornice of the entablature, is 5 feet 9 inches high, with a dentil cornice and three fascias below. The height of the pediment is 7 feet 10 inches from the top of the cornice on the attic. The height of the pedestal, from the bottom of the pediment to the top of the columns round the belfry, is 7 feet 10 inches. The columns and entablature round the belfry are 20 feet 10 inches high, and are similar to those in the portico; the wall which is seen between the columns, is rusticated above the two plinths. The apertures in the belfry for letting out the sound, are 4 feet 2 inches wide, and 11 feet 3 inches high.

The part where the dials of the clock are placed, is of an octagonal form; its height, including the two circular steps from the top of the cornice, round the entablature of the belfry, to the top of the cornice above the dials, is 9 feet 10 inches. There are four dials in it at right angles to each other, and four small apertures in the diagonal faces, each 3 feet wide and 4 feet high, filled in with perforated luffer boarding in the form of scales.

The part over the dials above the two circular steps, is of an octagonal form, with eight columns supporting an entablature. Its height, including the two circular steps at the top of the entablature, is 15 feet eight inches. The diameter of each column is 1 foot 5½ inches, and 11 feet 7 inches high; the entablature 2 feet 7 inches. The height of the small pediments above the entablature is 1 foot 9 inches, with a honey-suckle betwixt each.

The height of the spire above the top of the pediments to the top of the cross, is 44 feet 9½ inches, and this portion is ornamented with scales to the height of 28 feet 10 inches. The whole height of the steeple, from the ground to the top of the cross, is 152 feet.

Descriptton of the Flank Elevation.—The whole extent of this front, including the projecting porticos on the bottom line of the entablature, is 166 feet 3 inches. That between the two extreme half antæ, on each side of the bows, is 146 feet 8 inches; and the plain part between the bows, is 88 feet 2 inches. Each of the bows is 26 feet 3 inches. The height, from the top of the steps to the top of the sills of the lower windows, is 3 feet 8 inches. The lower windows are 5 feet 2 inches wide, diminishing a little at the top, and their height is 4 feet 10 inches. The height between the under side of the lintel of the lower windows and the top of the sill of the upper windows, is 6 feet 7 inches. The height of the windows above is 9 feet 6 inches; and the breadth, at the bottom, 5 feet, diminishing to the top about 3½ inches. The height from the under side of the lintel of the upper windows, to the lower line of the entablature, is 4 feet 5 inches. The height from the ground to the top of the roof is 50 feet 7½ inches. The frames of the windows to be of metal. All the ornaments on the exterior of this building may be of terra-cotta, or of stone, if built in a country where both labor and stone are cheap.

PLATE 22.

Ground Plan and Elevations of a Chapel.—A, represents the *Porch*, recessed within two columns, 26 feet 4 inches by 4 feet 6 inches. B, an elliptical *Vestibule*, with pilasters and niches, lighted from the top, 27 feet 3 inches by 16 feet 10 inches. D and C, *Side Staircases* to gallery, 25 feet by 13 feet; with a circular staircase in one corner, leading to the children's gallery and tower.

The size of the interior body of the chapel is 83 feet by 58 feet. The principal passage, representing the free seats, is 8 feet within the clear of the pew-doors. The side passages are each 3 feet to the front of the seats next the walls. The pews are 3 feet wide; the seats 1 foot; the book-desk 5½ inches; and the doors 1 foot 7 inches.

The *Pulpit n* is of an hexagonal form, with stairs ascending up to it; *o*, *Reading Desk*, with clerk's seat in front.

The little black circles represent the columns which support the gallery. The *Communion-Place*, H is of an elliptical form, and raised one foot high. E, *Vestry-Room*, 18 feet 6 inches by 13 feet, with a fire-place, and small closets in the angles. The *Strong Closet*, *e* 6 feet two inches by 5 feet 2 inches. *Water-Closet*, *g* 6 feet by 3 feet 9 inches, the mean proportion. F, *Robing-Room*, with fire-place, and closets of the same dimensions as Vestry-Room. G, entrance to the vaults.

Description of the Front Elevation.—This building is in the style of the *Grecian Doric*. The extent of its front, to the extremities of the pilasters, is 66 feet. Its height, from the ground to the top of the cross, is 112 feet. The entrance, or door, is raised 2 feet 8 inches above the level of the ground, ascended by 5 steps of rather more than 6 inches rise, which are continued all around the building. The opening of the door is 7 feet 3 inches in the clear at bottom, and 6 feet 10 inches at the top; diminishing about one-seventeenth part of the breadth. The door is of oak; it is divided into eight pannels, and opens in two halves, to the height of the bead betwixt the third and fourth panel; and is hinged to a vertical bead, which runs up by the side of the architrave of the door. The architrave is about two-ninths of the breadth of the door. That part of the architrave which extends across the top of the door, is a little less. Over the architrave is a cornice and pediment, with an ornament at each corner, supported at each extremity of the cornice by a console. Over the door is a panel, which may be filled in with bas-relief, or an inscription.

The columns are 31 feet 5 inches high; their diameter at bottom 5 feet 2½ inches, and at top 4 feet. The pilasters are of the same width as the top of the column. The mouldings of the caps are similar to those of the Monument of Thrasyllus. The lower windows, betwixt the pilasters, are 5 feet 5 inches wide at bottom, and at top 5 feet 1¼ inches. The windows above are of the same width at bottom as those below, and at top 5 feet 3½ inches. The architraves are 1 foot 1½ inches, with a break at top, of about 2 inches. The bars of the windows are of metal. The sills of the windows are 11½ inches.

The height of the architrave, frieze, and cornice, is 8 feet. The breadth of the triglyphs is 2 feet 5 inches. The height of the pediment, from the top of the cornice to the top of the Cym-

atium, is 11 feet 6 inches: on the top, and at each extremity of which are placed *Acroteriæ.*

That part which projects beyond the bottom of the cupola, is to admit light into the vestibule by means of six small windows in the faces of the pedestal of the cupola, which is concealed within it. The windows in the belfry are 4 feet 5 inches wide, and 11 feet 6 inches high, to the top of the arch. The aperture of the latter is filled in with horizontal luffer-boarding. The pilasters round the belfry are 16 feet 6 inches high, and 1 foot 11 inches wide; the moulding in the caps, are the same as those in the front; the bars are similar to the attic base; the height of the entablature is 4 feet 2 inches, with wreaths in the frieze, and ornaments above the cornice.

The part above the belfry, which contains the clock-work, is of an octagonal form, with a cornice and continued ornament above, similar to that on the top of the cornice of the Monument of Lysicrates,—*pl.* 46. The faces of the octagonal part is filled in with four dials, at right angles to each other, and four small windows, 3 feet 6 inches wide, and 3 feet high: the apertures of which are filled in with luffer-boarding, in the form of scales. Above the octagonal part is a circular dome raised upon a step, with a ball and cross over it.

Description of the Flank Elevation.—The whole length of this elevation is 142 feet between the two outer pilasters. That part between the pilasters, wherein the windows are, is 79 feet. The heights of the doors, pilasters, entablature, and cupola, are the same as those in the front elevation. The lower windows are 5 feet 4 inches wide, and 4 feet 7 inches high, and diminish at the top one inch and a half: the windows above these are 5 feet 4 inches wide, and 9 feet 8 inches high, and diminish $3\frac{3}{4}$ inches at top. The architraves are 1 foot $1\frac{1}{2}$ inch, with a break at top of about 2 inches on each side: over the top of the architrave is a cornice and a pediment, with a honey-suckle at each extremity.

GOTHIC ARCHITECTURE.

Although much has been said against this style of building, yet it must be acknowledged that we are indebted to Gothic Architects for many improvements in our present mode of construction. We find a lightness in Gothic designs, and a boldness in their execution, which the Greeks and Romans never attained, or the moderns duly appreciated till within the last century. Formerly every design which did not perfectly accord with Grecian or Roman models, was censured as barbarous and unworthy the attention of modern architects: but an abatement of that enthusiastic zeal for classic structure, which for a time universally prevailed in Europe, opened the way for a revolution in architecture. Within the last century, many Gothic buildings have been erected in Great Britain, that are admirable both for the art with which they were designed, and the taste with which they were executed. The English Architect has studied the antiquities of his own country, as well as those of Greece and Rome; and the Gothic abbeys, cathedrals, and baronial castles of his ancestors, are no longer considered as void of ornament or convenience. The ancient architecture of England is at the present period held in high esteem, and though the edifices of "olden times," are fast falling to ruin, yet a remembrance of them is preserved in the beautiful buildings which are erected in every part of the country, for worship, education, benevolence, and the accommodation of man.

It would afford me much satisfaction to speak of Gothic Architecture at considerable length, but the limits of the present work will not permit me to enter its history. If any person desires to be thoroughly acquainted with this style of building, he should examine the works of A. Pugins, where the subject is extensively treated of, and from which, the specimens here given were taken.

PLATE 23.

Fig. 1.—Plan and Elevation of the Open Parapet and Turrets, over the Western entrance of Hampton Court Palace.

A succession of three gates with towers over them, leads from the western front to the interior of the place, where king William's buildings join to the ancient courts.

The embattled parapet here represented, has a very light, airy effect; the tracery being all pierced, as is shown on the plan. The pinnacles, formed into slender copies of the turrets, instead of shooting up into pointed spires, as in earlier buildings, are peculiar to the latest period of Gothic taste. The same sort of pinnacle is seen upon the battlements of the hall, and in other parts of the palace.

Fig. 2.—Plan and Elevation of an Oriel Window, John of Gaunt's Palace, Lincoln.

The curious investigator of domestic antiquities, will not fail to appreciate this remnant of a once splendid habitation. In delineating its form and enrichments, most scrupulous care has been taken to give a full and exact portrait; such an interesting *specimen* being very rarely seen. The elevations of the front and profile, exhibit no more than what actually exists, except the tops of the pinnacles, which being broken off level with the foliage between them, are here restored in style corresponding with the ornaments. It may also be proper to notice, that three lights, which, no doubt, were once *cloised well with roiall glass*—[Old Romance of the Squire of Low Degree] are now blocked up, and the mouldings partly obscured by plastering. The bracket which sustains the frame of the window, is covered with sculpture, divided by plain mouldings into four tiers. The lowest of these consists of a single figure, representing an angel, serving as a bracket. The next has three marks, or faces; viz. at the right, a queen; in front, a king; on the left, a bearded man, rather defaced. Above these runs a course of foliage, displayed in large leaves. The uppermost division has six figures, one beneath each of the little abutments, which guard the angles of the window. Against the wall on the right hand, is a man covered with hair, and with a long beard, holding a bird in one hand, in the other a branch; next to him, an angel playing upon a cithern: then a king with a long beard; on his left hand, an old man clothed in a mantle; beyond this figure, a youth in a close robe; and lastly, against the wall, a bearded man, rather disfigured. A plan, or horizontal section, taken at two different heights, is drawn in the upper part of the plate, D, E; below is an enlarged section of the bracket, showing the projection of all its mouldings, with their several measurements. These details are also represented separately, with letters referring to the elevations B. Head upon the little bracket of one of the niches, in the two blank lights. C, a panel, with section, of those beneath the lights. F, loping of a buttress. H, finial rising from the crockets over every light. All examination of the interior

of the oriel is unfortunately obstructed by a modern chimney, built up within it.

PLATE 24.

Turret and Gable of King's College Chapel, Cambridge.

The Chapel of King's College, Cambridge, has been as much celebrated as any Gothic building in Europe, so that nothing need be said here respecting the general character of its architecture. At the left of this plate represents the upper part of one of the four lofty turrets which adorn its angles; with a portion of the adjoined gable. The turrets are corniced up without any ornament as high as the battlements of the roof, above which they are beautifully decorated, as is shown in the plate. The character of these decorations deserve a particular examination; the projections and recesses are bold and decisive, producing a clear and distinct effect even at the great height they are placed. The fretted compartments in the sides are pierced quite through the walls, giving light to the interior, and making the turrets appear very rich on the outside. The armorial badges and crowns refer to Henry VII., who contributed very largely to the completion of the structure, though it was not effected in his day. A, plan, taking in the lower compartments of the elevation. B, one corner of the same on a larger scale. C, mullion. An enlarged elevation of the cross upon the crest of the gable.

To the right of this plate are designs of buttresses and one battlement, as taken from Oxford.

PLATE 25.

Exhibits a number of designs of windows, as taken from different churches and other edifices from Oxford.

PLATE 26.

Figures 1, 2, 3, 4, 5 and 6, are cornices from Westminster Abbey and Henry VII's Chapel.

The sections of these six specimens are drawn on a larger scale than the front views, the better to show the turns of the mouldings.

Figs. 1, 2, and 4, have *crests* of small battlements above the cornices, and their casements are studded with small ornaments of *entail*, set at intervals. Figs. 3 and 5 have *crests* of leaves, arranged to a pattern of great elegance, and which was very frequently used in the 15th century. The *crest* of *fig.* 6, appears to have been broken off. This specimen being of wood, the *entail* is worked on a thin piece, inserted afterwards into the *casement*.

Figs. 7, 8, 9, and 10, are specimens of parapets from St. George's Chapel, Windsor.

The upper roof of this magnificent structure is guarded by a straight parapet pierced in compartments; whilst the aisles have an embattled parapet, which is also pierced. These last four examples that are exhibited on this plate, the cornices are studded with heads, grotesque and ludicrous, agreeably to the fashion of the age in which the building was erected, when exhibitions of masques and mummeries entertained the gravest and most polished characters, no less than the lowest classes of society.

The elevation and corresponding section of each of these *specimens* seem to require no explanation.

PLATE 27.

Exhibits a number of designs for capitals and bases of pillars, brackets and sculptured ornaments, &c.

Nos. 1, 2, and 3, show three specimens of foliated capitals, with their respective bases. It may be useful to observe, that in designing a capital of this sort, the *corps* or solid part, ought to be proportioned before any ornaments of leaves, flowers, &c. are applied; a small *neck-mould* is required to distinguish the capital from the shaft, and over the leaves a *hood-mould*, such as that marked *e* in the second specimen. By comparing the letters on the sections, with the corresponding ones on the elevations, the whole will be clearly explained. Nos. 1 and 2, are of the latter end of the 14th century; No. 3, of the early part of the same, or the end of the 13th.

Nos. 4 and 5, exhibit two specimens of capitals and bases, of a plain description, they being finished with mouldings only, without foliage. The mouldings are expressed in feet and inches by figures, and they clearly show the manner in which the arches are set upon the pillars, and will be found carefully marked, and the size and form of each pillar.

Nos. 6, 7, 8, 9, and 10, are enrichments of cornices.

No. 11, a design of a corble; which is sculptured with leaves, after the form of the capital of a column.

Nos. 12 and 13, are specimens of running foliage, fruit, &c.

Nos. 14, 15, 16, and 17, are specimens of *knots* on the intersections of ribs, in roofs; these are all shown in profile, as well as in front. In 17, the letters I H S, an abbreviation of the sacred name Jesus, are wrought amongst the foliage.

DECIMAL FRACTIONS.

DECIMAL FRACTIONS.

Fractions, or *Vulgar Fractions*, are expressions for any assignable part of an unit; they are usually denoted by two numbers, placed one above the other, with a line between them: thus, $\frac{1}{4}$ denotes the fraction one-fourth, or one part of four of some whole quantity, considered as divisible into four equal parts. The lower number 4 is called the *denominator* of the fraction, showing into how many parts whole or integer is divided; and the upper number 1, is called the *numerator*, and shows how many of those equal parts are contained in the fraction. And it is evident that if the numerator and denominator be varied in the same ratio, the value of the fraction will remain unaltered: thus, if the numerator and denominator of the fraction $\frac{1}{4}$ be multiplied by 2, 3, or 4, &c., the fractions arising will be $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, &c. which are evidently equal to $\frac{1}{4}$.

Decimal Fraction, is a fraction whose denominator is always an unit with some number of ciphers annexed, the numerators of which may be any numbers whatever: as, $\frac{2}{10}$, $\frac{3}{100}$, $\frac{15}{1000}$, &c. And as the denominator of a decimal is always one of the numbers 10, 100, 1000, &c., the inconvenience of writing these denominators may be avoided, by placing a point between the integral and the fractional part of the number; thus, $\frac{3}{10}$ is written 3, and $\frac{14}{100}$ is written 14; the *mixed* number 3, $\frac{14}{100}$, consisting of whole numbers and fractional ones, is written 3.14.

In setting down a decimal fraction, the numerator must consist of as many places as there are ciphers in the denominator; and if it has not so many figures, the defect must be supplied by placing ciphers before them; thus, $\frac{16}{100} = .16$, $\frac{16}{10000}$ .0016, &c. And as ciphers on the right hand side of integers, increase their value in a ten-fold proportion, as .2, 20, 200, &c. so when set on the left hand of decimal fractions, they decrease their value in a ten-fold proportion, as .2, .02, .002, &c.; but ciphers being set on the right hand of these fractions, can make no alteration in their value, neither in their increase or their decrease; thus .2 is the same as .20 or .200, The common arithmetical operations are performed the same way in decimals, as they are in integers; regard being had only to the particular notation, to distinguish the integral from the fractional part of a sum.

ADDITION OF DECIMALS.

Addition of decimals is performed exactly like that of whole numbers, placing the numbers of the same denomination under each other, in which case the decimal separating points will range straight in one column.

EXAMPLES.

	Miles.	Feet.	Inches.
	34.6	4.30	4.53
	45.18	3.44	127.05
	126.306	12.25	.047
	.004	1.346	35.6
Sum	206.090	21.336	167.227

SUBTRACTION OF DECIMALS.

Subtraction of decimals is performed in the same manner as in whole numbers, by observing to set the figures of the same denomination and the separating points directly under each other.

EXAMPLES.

From	35.345	54.46	2.340	1246.2
Take	3.23	.032	.286	25.164
	32.115	54.428	2.054	1211.036

MULTIPLICATION OF DECIMALS.

Multiply the numbers together the same as if they were whole numbers, and point off as many decimals on the right hand as there are decimals in both factors together; and when it happens that there are not so many figures in the product as there must be decimals, then prefix as many ciphers to the left hand as will supply the defect.

EXAMPLE I.

Multiply 4.35 by 6.6
4.35
5.6

2.610
21.75

Answer 24.360

In one of the factors is one decimal, and in the other, two; their sum 3, is the number of decimals of the product.

EXAMPLE II.

Multiply 0.6 by 0.8
0.6
0.8

Answer 0.48

EXAMPLE III.

Multiply 4.33 by .04
4.33
.04

Answer 17.32

EXAMPLE IV.

Multiply .12 by .09
.12
.09

Answer .0108

In each of the factors are two decimals, the product ought therefore to contain 4, and there being only 3 figures in the product, I prefix a cipher.

EXAMPLE V.

Multiply .16 by .26
.16
.26

96
32

Answer 4.16

EXAMPLE VI.

Multiply 33.2 by 2.5
33.2
2.5

16.60
66.4

Answer 83.00

DIVISION OF DECIMALS.

Division of Decimals is performed in the same manner as in whole numbers; only observing that the number of decimals in the quotient, must be equal to the excess of the number of decimals of the dividend above those of the divisor. When the divisor contains more decimals than the dividend, ciphers must be affixed to the right hand of the latter to make the number equal or exceed that of the divisor.

EXAMPLE I.

Divide 14.625 by 3.25

```
3.25)14.625(4.5
     1300
     ----
      1625
      1625
      ----
```

In this example there are 2 decimals in the divisor, and 3 in the dividend, hence there is one decimal in the quotient.

EXAMPLE II.

Divide 0.35 by 0.7

```
7).35(.5
   .35
```

EXAMPLE III.

Divide 3.1 by .0062.

Previous to the division I affix a number of ciphers to the right hand of 3.1, which does not alter its value.

```
.0062)3.100000(500.00
      310
      ---
       0000
```

Therefore the answer is 500.00 or 500.

EXAMPLE IV.

Divide 9.6 by .06

```
06)9.60
   ----
```

Answer 160

Here by affixing a cipher to 9.6 it becomes 9.60, and has then 2 decimals in it, which is the same number as is in the divisor, therefore the quotient is an integer number.

EXAMPLE V.

Divide 17.256 by 1.16

```
1.16)17.25600(14.875
     116
     ---
      565
      464
      ---
      1016
       928
       ---
        880
        812
        ---
         680
         580
         ---
         100
```

EXAMPLE VI.

Required the value of 3.25 yards.

```
3.25
   3
----
 .75
  12
----
9.00
```

Answer 3 yards, 0 feet, 9 inches.

EXAMPLE VII.

Required the value of 7.231 days.

```
 7.231
    24
 -----
   924
  462
 -----
 5.544
    60
 -----
32.640
    60
 -----
38.400
```

Answer 7 days, 5 hours, 32 minutes, and 38 seconds.

REDUCTION OF DECIMALS.

If you wish to reduce a vulgar fraction to a decimal, you may add any number of ciphers to the numerator, and divide it by the denominator, the quotient will be the decimal fraction; the decimal point must be so placed that there may be as many figures to the right hand of it as you added ciphers to the numerator: if there are not as many figures in the quotient, you must place ciphers to the left hand to make up the number.

EXAMPLE I.

Reduce $\frac{1}{5}$ to a decimal.

```
5)1.0
  ---
```

Answer .2

EXAMPLE II.

Reduce $\frac{3}{8}$ to a decimal.

```
8)3.000
  -----
```

Answer 375

EXAMPLE III.

Reduce 3 inches to the decimal of a foot.

Since 12 inches = 1 foot, this fraction is $\frac{3}{12}$.

```
12)3.00
   ----
```

Answer .25

EXAMPLE IV.

Reduce $3\frac{1}{2}$ inches to the decimal of a foot.

$3\frac{1}{2}=\frac{7}{2}$. this divided by 12 is $\frac{7}{24}$.

```
Answer 24)7.000(.291
          48
          --
          220
          216
          ---
           40
           24
           --
           16
```

EXAMPLE V.

Reduce 1 foot and 6 inches to the decimal of a yard.

Here 1 foot 6 inches = 18 inches.

And 1 yard = 36 inches, therefore this fraction is $\frac{18}{36}$.

```
36)18.0(.5 Answer.
   180
```

If you have any decimal fraction, it is easy to find its value in the lower denominations of the same quantity; thus if the fraction was a decimal of a yard, by multiplying it by 3 we have its value in feet and parts; if we multiply this by 12, the product is its value in inches and parts; and in the same manner the values may be obtained in other cases.

MULTIPLICATION OF FEET, INCHES AND PARTS; OR DUODECIMALS.

The multiplication of *feet* and *inches* is generally called *duodecimals,* because every superior place is 12 times its next inferior in this scale of notation. This way of conceiving an unit to be divided, is chiefly in use among *artificers,* who generally take the linear dimensions of their work in *feet* and *inches:* it is likewise called *cross multiplication,* because the *factors* are sometimes multiplied crosswise.

RULE I.

1. Under the multiplicand write the corresponding denominations of the multiplier; that is, feet under feet, inches under inches, parts under parts, &c.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier; write each result under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.

3. In the same manner multiply every term in the multiplicand by the inches in the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand.

4. Work in a similar manner with the parts in the multiplier, setting the result of each term removed two places to the right hand of those in the multiplicand. Proceed in like manner with the rest of the denominations, and their sum will be the answer required.

EXAMPLE 1. Let 7 feet 9 inches be multiplied by 3 feet 6 inches.

	F.	I.	
Multiplicand	7 ..	9	
Multiplier	3 ..	6	
	23 ..	3	Parts,
	3 ..	10 ..	6
Product	27 ..	1 ..	6

First multiply 9 inches by 3, saying, 3 times 9 is 27 inches, which make 2 feet 3 inches; set down 3 under inches, and carry 2 to the feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the feet.

Then begin with 6 inches, saying, 6 times 9 is 54 parts, which is 4 inches and 6 parts; set down 6 parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 inches, which is 3 feet 10

inches; which set down, and add all up together, and the product is 27 feet 1 inch 6 parts.

Example 2. Let 7 feet 5 inches 9 parts be multiplied by 3 feet 5 inches 3 parts.

```
              F.      I.      P.
Multiplicand  7   ..  5   ..  9
Multiplier    3   ..  5   ..  3
             ---------------------
             22   ..  5   ..  3   S.
              3   ..  I   ..  4   9   T.
                      1   ..  10 .. 5 .. 3
             ---------------------------
     Product 25   ..  8   ..  6 .. 2 .. 3
```

In this example, I first begin with 3 feet, and thereby multiply 7 feet 5 inches and 9 parts: first, I say, 3 times 9 is 27 parts, that is 2 inches and 3 parts; set down 3 under the parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 foot 5 inches; set down 5 inches, and carry 1, and say, 3 times 7 is 21, and 1 I carry is 22; set down 22 feet: then begin with 5 inches, saying, 5 time 9 is 45, which is 45 *seconds*, which makes 3 parts and 9 seconds; set down 9 seconds in a place towards the right hand, and carry 3 parts, saying, 5 times 5 is 25, and 3 I carry is 28, which is 2 inches and 4 parts, set down 4 parts and carry 2, saying, 5 times 7 is 35, and 2 I carry is 37, which is 3 feet 1 inch; set down 3 feet 1 inch; and begin to multiply by 3 parts, saying, 3 times 9 is 27 *thirds*, that is 2 *seconds* and *thirds;* set down 3 thirds, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 part and 5 seconds; set down 5 seconds, and carry 1, saying, 3 times 7 is 21, and 1 I carry is 22, which is 1 inch and 10 parts, which set down and add all up, and the product is 25 feet 8 inches 6 parts 2 seconds 3 thirds.

RULE II.

When the feet in the Multiplicand are expressed by a large number.

Multiply first by the feet in the multiplier, as before. Then, instead of multiplying by the inches and parts, &c. proceed as in the Rule of Practice, by taking such aliquot parts of the multiplicand as correspond with the inches and parts, &c. of the multiplier. Then the sum of them all will be the product required.

Example 3. Let 75 feet 7 inches be multiplied by 9 feet 8 inches.

```
In  |     |   F.      I.
4 } |     |   75  ..  7  Multiplicand.
4 } | 1/3 |    9  ..  8  Multiplier.
    |     |  -----------
    |     |  680  ..  3
    |     |   25  ..  2  ..  4
    |     |   25  ..  2  ..  4
    |     |  -----------------
    | Product 730 ..  7      8
```

Multiply by 9 feet first, as above directed; then instead of multiplying by the 8 inches, let them be divided into aliquot parts of a foot, as 4 and 4, because 4 is the third part of 12. So if you take the third part of 75 feet 7 inches, and set it down twice, and add all together, the sum will be 730 feet 7 inches 8 parts. To take the third part, say, how often 3 in 7, which is twice; set down 2; then because twice 3 is 6, say, 6 out of 7, and there remains 1, for which you must add 10 to the 5, and it makes 15; then the threes in 15 are 5 times; set down 5; and because three times 5 is 15, there is 0 remains. Then go to the 7 inches, saying, the three in 7 are twice; set down 2 in the inches; and because twice 3 is 6, take 6 out of 7, and there remains 1 inch, which is 12 parts; threes in 12 are 4 times, and 0 remains. So the third part of 75 feet 7 inches is 25 feet 2 inches

4 parts; which set twice over, and add them together as in the example.

Example 4. Let 37 feet 6 inches 5 parts, be multiplied by 4 feet 8 inches 6 parts.

```
I.      P.   |     |   F.      I.      P.
4  ..   0 }  |     |   37  ..  7  ..  5  Multiplicand.
4  ..   0 }  | 1/3 |    4  ..  8  ..  6  Multiplier.
             |     |  ---------------------
             |     |  150  ..  5  ..  8      S
I.      P.   |     |   12  ..  6  ..  5  ..  8
0  ..   6    | 1/8 |   12  ..  6  ..  5  ..  8      T
             |     |    1  ..  6  ..  9  ..  8  ..  6
             |     |  ----------------------------------
             Product  177  ..  1  ..  5  ..  0  ..  6
```

In this example I first multiply by 4 feet as usual. Then for the 8 inches I say 4 inches is the third of a foot, therefore I take the third part of 37 feet 7 inches 5 parts, which is 12 feet 6 inches 5 parts 8 seconds, and set it down twice. Then for 6 parts, I say, 6 parts are the eighth of four inches, because 12 parts make 1 inch; hence it follows, that whatever be the value, or product, by 4 inches, the value of 6 parts will be one-eighth thereof; therefore I take one-eighth of 12 feet 6 inches 5 parts 8 seconds, and find it to be 1 foot 6 inches 5 parts 8 seconds 6 thirds; so that the sum of the whole is 177 feet 1 inch 5 parts 6 thirds.

EXTRACTION OF THE SQUARE ROOT.

If a square number be given.

To find the root thereof, that is, to find out such a number, as being multiplied into itself, the product shall be equal to the number given; such operation is called, *The Extraction of the Square Root;* which to do, observe the following directions.

1. You must point your given numbers; that is, make a point over the unit's place, another over the hundred's and so over every second figure throughout.

2. Then seek the greatest square number in the first period towards the left hand, placing the square number under that point, and the root thereof in the quotient, and subtract the said square number from the first point, and to the remainder bring down the next point, and call that the resolvend.

3. Then double the quotient, and place it for a divisor on the left hand of the resolvend; and seek how often the divisor is contained in the resolvend, (reserving always the unit's place,) and put the answer in the quotient, and also on the right hand side of the divisor; then multiply by the figure last put up in the quotient, and subtract the product from the resolvend, (as in common division,) and bring down the next point to the remainder, (if there be any more,) and proceed as before.

A Table of Squares and their Roots.

Root	1	2	3	4	5	6	7	8	9
Square	1	4	9	16	25	36	49	64	81

Example 1. Let 4489 be a number given, and let the square root thereof be required.

```
       . .
       4489(67
       36
      -----
  127) 889  Resolvend.
       889  Product.
      -----
       ...
```

First, point the given number, as before directed; then by the little table foregoing, seek the greatest square number in 44, (the

first point to the left hand,) which you will find to be 36, and 6 the root; put 36 under 44, and 6 in the quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other point 89, placing it on the right hand, so it makes 889 for a resolvend; then double the quotient 6, and it makes 12; which place on the left hand for a divisor, and seek how often 12 in 88, (reserving the unit's place,) the answer is seven times; which put in the quotient, and also on the right hand side of the divisor, and multiply 127 by 7, as in common division, and the product is 889, which subtracted from the resolvend, there remains nothing; so is your work finished; and the square root of 4489 is 67; which root if you multiply by itself, that is 67 by 67, the product will be 4489, equal to the given square number, and proves the work to be right. Had there been any remainder it must have been added to the square of the root found.

Example 2. Let 106929 be a number given, and let the square root thereof be required.

```
 . . .
106929(327
 9
--------
62)169     Resolvend.
   124     Product.
--------
647)4529   Resolvend.
    4529   Product.
    ----
    ....
```

First point your given number, as before directed, putting a point over the units, hundreds, and tens of thousands; then seek what is the greatest square number in 10, (the first point,) which by the little table you will find to be 9, and 3 the root thereof; put 9 under 10, and 3 in the quotient; then subtract 9 out of 10, and there remains 1; to which bring down 69, the next point, and it makes 169 for the resolvend; then double the quotient 3, and it makes 6, which place on the left hand of the resolvend for a divisor, and seek how often 6 in 16; the answer is twice, put 2 in the quotient, and also on the right hand of the divisor making it 62. Then multiply 62 by the 2 you put in the quotient, and the product is 124; which subtract from the resolvend, and there remains 45; to which bring down 29, the next point, and it makes 4529 for a new resolvend. Then double the quotient 32, and it makes 64, which place on the left side of the resolvend for the divisor, and seek how often 64 in 452, which you will find 7 times; put 7 in the quotient, and also on the right hand of the divisor, making it 647, which multiplied by the 7 in the quotient, it makes 4529, which subtracted from the resolvend, there remains nothing. So 327 is the square root of the given number.

Note.—The root will always contain just so many figures, as there are points over the given number to be extracted: and these figures will be whole numbers or decimals respectively, according as the points stand over the whole numbers or decimals. The method of extracting the square root of a decimal is exactly the same as in the foregoing examples, only if the number of decimals be odd, annex a cipher to the right hand to make them even before you begin to point. The root may be continued to any number of figures you please, by annexing two ciphers at a time to each remainder, for a new resolvend.

EXTRACTION OF THE CUBE ROOT.

To extract the Cube Root, is nothing else but to find such a number, as being first multiplied into itself, and then into that product, produceth the given number; which to perform, observe the following directions.

1. You must point your given number, beginning with the unit's place and make a point, or dot, over every third figure towards the left hand.

2. Seek the greatest cube number in the first point, towards the left hand, putting the root thereof in the quotient, and the said cube number under the first point, and subtract it therefrom, and to the remainder bring down the next point, and call that the resolvend.

3. Triple the quotient, and place it under the resolvend; the unit's place of this under the ten's place, of the resolvend; and call this the triple quotient.

4. Square the quotient, and triple the square, and place it under the triple quotient; the units of this under the ten's place of the triple quotient, and call this the triple square.

5. Add these two together, in the same order as they stand, and the sum shall be the divisor.

6. Seek how often the divisor is contained in the resolvend, rejecting the unit's place of the resolvend, (as in the square root,) and put the answer in the quotient.

7. Cube the figure last put in the quotient, and put the unit's place thereof under the unit's place of the resolvend.

8. Multiply the square of the figure last put in the quotient into the triple quotient, and place the product under the last, one place more to the left hand.

9. Multiply the triple square by the figure last put in the quotient, and place it under the last, one place more to the left hand.

10. Add the three last numbers together, in the same order as they stand, and call that the subtrahend.

11. Subtract the subtrahend from the resolvend, and if there be another point, bring it down to the remainder, and call that a new resolvend, and proceed in all respects as before.

Note.—To square a number is to multiply that number by itself. And to cube a number is to multiply the square of the number by the number itself.

A Table of Cubes and their Roots.

Roots	1	2	3	4	5	6	7	8	9
Cubes	1	8	27	64	125	216	343	512	729

Example I. Let 314432 be a whole number, whose root is required.

```
  .  .
314432(68 Root.
216
-------
98432 Resolvend.
-----
   18 Triple quotient of 6.
 108  Triple square of the quotient 6.
-----
 1098 Divisor.
-----
   512 Cube of 8, the last figure of the root.
 1152  The square of 8, by the triple quotient.
864    The Triple square of the quotient 6 by 8.
-----
98432 The Subtrahend.
-----
.....
```

After you have pointed the given number, seek what is the greatest cube number in 314, the first point, which, by the little table annexed to the rule, you will find to be 216, which is the nearest that is less than 314, and its root is 6; which put in the quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next point, 432, and annex it to 98; so will it make 98432 for the resolvend. Then triple the quotient 6, it makes 18, which write down the

unit's place, 8, under 3, the ten's place of the resolvend. Then square the quotient 6, and triple the square, and it makes 108, which write under the triple quotient, one place towards the left hand; then add those two numbers together, and they make 1098 for the divisor. Then seek how often the divisor is contained in the resolvend, (rejecting the unit's place thereof) that is, how often 1098 in 9843, which is 8 times; put 8 in the quotient, and the value thereof below the divisor, the unit's place under the unit's place of the resolvend. Then square the 8 last put in the quotient, and multiply 64, the square thereof, by the triple quotient 18; the product is 1152; set this under the cube of 8, the units of this under the tens of that. Then multiply the triple square of the quotient by 8, the figure last put up in the quotient, the product is 864; set this down under the last product, a place more to the left hand. Then draw a line under these three, and add them together, and the sum is 98432, which is called the subtrahend; and being subtracted from the resolvend, the remainder is nothing; which shows the number to be a true cubic number, whose root is 68; that is, if 68 be cubed, it will make 314432.

For if 68 be multiplied by 68, the product will be 4624; and this product, multiplied again by 68, the last product is 314432, which shows the work to be right.

Example 2. Let the cube root of 5735339 be required.

After you have pointed the given number, seek what is the greatest cube number in 5, the first point, which by the little table, you will find to be 1, which place under 5, and 1, the root thereof, in the quotient; and subtract 1 from 5, and there remains 4; to which bring down the next point, it makes 4735 for the dividend. Then triple the 1, and it makes 3; and the square of 1 is 1, and the triple thereof is 3; which set one under another, in their order, and added, makes 33 for the divisor. Seek how often the divisor goes in the resolvend, and proceed as in the last example.

5735339 (179	Root.
1	
4735	
3	Triple of the quotient 1, the first figure.
3	The triple square of the quotient 1.
33	The divisor.
343	The cube of 7, the second figure of the root.
147	The square of 7, multiplied in the triple quotient 3.
21	The triple square of the quotient multipled by 7.
3913	The subtrahend.
822339	The new resolvend.
51	The triple of the quotient 17, the two first figures.
867	The triple square of the quotient 17.
8721	Divisor.
729	The cube of 9, the last figure of the root.
4131	The square of the 9, multiplied by the triple quotient 51.
7803	The triple square of the quotient 867 by 9.
822339	The subtrahend.
....	

In this example, 33, the first divisor, seems to be contained more than seven times in 473, the dividend, after the unit's place has been rejected; but if you work with 9, or 8, you will find that the subtrahend will be greater than the dividend.

MENSURATION.

MENSURATION OF SUPERFICES AND SOLIDS.

OF SUPERFICES.

SECTION 1.

THE areas or superfices of any plane figure is estimated by the number of squares contained in its surface; the side of those squares being either an inch, a foot, a yard, a link, or a chain according to the measures peculiar to different artists.

Our common measures of length are given in the tables below the right hand table; it is the square measure as taken from the other by squaring the several numbers.

1. LINEAL MEASURE.		2. SQUARE MEASURE.	
12 Inches	1 Foot.	144 Inches	1 Foot.
3 Feet	1 Yard.	9 Feet	1 Yard.
6 Feet	1 Fathom.	36 Feet	1 Fathom.
16½ Feet }	{ 1 Pole or	272¼ Feet }	{ 1 Pole or
5½ Yards }	{ Rod.	30¼ Yards }	{ Rod
40 Poles	1 Furlong.	1600 Poles	1 Furlong.
8 Furlongs	1 Mile.	64 Furlongs	1 Mile.

Land is generally measured by a chain of 4 rods, or 66 feet, in length, called Gunter's chain, from the name of the inventor.

This chain is made up of 100 links, and every tenth link, from either end, is marked by a small brass plate attached to it and notched to designate its number from the end. This chain being divided into one hundred equal parts is a convenient one, since the divisions, or links, are decimals of the whole chain, and in the calculations may be treated as such.

The length of the chain being 4 poles or 66 feet, is equal to 792 inches, which being divided by 100, gives 7.92 inches for the length of each link, a mile being equal to 320 rods; therefore 80 chains are equal to 1 mile, and 40 chains are equal to half a mile, and 20 chains are equal to one-fourth of a mile. And ten square chains, or ten in length and one in breadth, make an acre, or 160 square poles or 100,000 square links, each being the same in quantity. Forty perches or square poles make a rood, and 4 roods make an acre.

The length of lines which are measured with a chain, are in general way set down in links, as whole numbers, every chain being 100 links in length. Therefore after the dimensions are squared or the superfices found, it will be in square links; when this is the case, it will be necessary to cut off five of the figures on the right hand for decimals, and the rest will be acres. These decimals must be then multiplied by 4, for roods, and the decimals of these again, after 5 figures are cut off by 40 for poles, and the five decimals again by 272¼ for feet.

ARTICLE 1. To measure a Square, having equal sides.

Rule.—Multiply the side of the square into itself, and the product will be the area or superficial content, of the same name with the denomination taken, either in inches, feet, or yards, respectively.

Let A B C D *fig.* 1, *pl.* 28, which represents a square whose sides are 12 inches, or 12 feet. Multiply the side 12 by itself thus:—

	12 inches,	12 feet.
	12 inches,	12 feet.
Area,	144 inches.	144 feet.

ARTICLE II. To measure a Parallelogram or long square.

Rule.—Multiply the length by the breadth, and the product will be the area, or superficial content.

Let A B C D, *fig.* 2, represent a parallelogram, whose length is 16 feet, and breadth 10 feet.

Length,	16
Breadth,	10
Area,	160 Ans.

To demonstrate these examples, let two sides of the given squares be divided into equal parts as represented in the figures, and draw lines through the several divisions parallel to A B and A D, which will divide the figures into as many little squares as it contains inches, feet, respectively.

ARTICLE III. To measure a Rhombus.

Rule.—Multiply the side by the length of a perpendicular let fall from one of the obtuse angles to the opposite side.

What is the area of a rhombus A B C D, *fig.* 3, the length A B being 5 feet 9 inches, and the perpendicular height A E 5 feet 9 inches.

By Duodecimals.		By Decimals.
5-9	5-9	5-75
5-9	5-9	5-75
4-3-9	28-9	2875
28-9	4-3-9	4025
		2875
Ans. 33-0-9	Ans. 33-0-9	Ans. 33,0625
f i ii	*f i ii*	

ARTICLE IV. To find the area of a Rhomboid.

Rule.—Multiply one of the longest sides by the perpendicular let fall from one of the obtuse angles to the opposite side.

What is the area of a rhomboid A B C D, *fig.* 4, whose length A B is 10f. 9 and the perpendicular height A E 4f.-6i.

10-9	10-9	10-75
4-6	4-6	4-50
5-4-6	43-0	53750
43-0	5-4-6	4300
Ans. 48-4-6	48-4-6	48,3750
f i ii	*f i ii*	

Demonstration of the two last figures; let B F in *fig.* 3 *and* 4, be drawn perpendicular to A B, and produce the line C D to F, then will the angles A D E be equal to the angles B C F and A B E F will be a square in *fig*, 3, and *fig*. 4 will be a rectangle.

Article V. To find the area of a Trapezoid.

Rule.—Multiply the half sum of the parallel ends, by the length, and the product will be the area.

What is the area of the trapezoid A B C D in *fig*. 5, the end A D being 4 f-6 and the end B C 3 f-8 and 12 f 3 inches long.

End A D=4-6	12-3	12-3
End B C=3-8	4-1	4-1
divide 2) 8-2	1-0-3	49-0
	49-0	1-0-3
4-1-half sum		
Ans.	50-0-3	50-0-3
	f i ii	

Article VI. To find the area of a Triangle.

Rule.—If it be a right-angled triangle multiply the base by half the perpendicular, or half the base by the perpendicular, and the product will be the area; but if it be an oblique, obtuse, or acute-angled triangle, multiply the base by half of the perpendicular let fall on the base from the angle opposite to it, and the product will be the area.

What is the area of the triangle A B C, *fig*. 6, whose base A B is 24 f. and the perpendicular B C 18 f. 6 i.

Base A B=24-0	Or perp. B C=18-5
½ Perp. B C= 9-3	½ Base A B=12-0
6-0-0	3700
216-0	185
Ans. 222-0-0	222-00

To demonstrate this figure let A G be drawn perpendicular to A B; at E, the half of the perpendicular height, draw E F G parallel to A B, draw C H parallel to E G, and make equal in length to E F, and through F draw H F D parallel to B C (Geometry, Theorem 1,) and thus, the triangle C E F is equal to the triangle A G F, and if the triangle C E F be taken and placed upon the angle A G F, then will A B E G become a rectangle, which proves the first method. Again let the triangle A D F be taken away and placed upon the angle C H F, then B C D H becomes a rectangle, which proves the second method.

The succeeding figures may also be demonstrated in the same manner.

To find the area of the triangle A B C, *fig*. 7, whose base A B is 16 *f.* and the perpendicular C D is 10 *f.* 2 *i.*

Base A B=16.0	or Perp C D=10.2
½ Perp. C D= 5.1	½ Base A B= 8.0
1.4.0	14.0
80.0	80
81.4.0	81.4.0
f i ii	

Article VII. There is another method of finding the area of triangles, the three sides being given.

Rule.—Add the three sides together, then take the half of that sum, and out of it subtract each side severally; and multiply the half of the sum and these remainders continually, and the square root of this product will be the area of the triangle.

What is the area of the triangle A B C *fig*. 7, the side A B being 16 *f.* and the side A C 14 *f.* and the side B C is 12 *f.*

side A B=16	21	21	21
side A C=14	16	14	12
side B C=12	5 first diff.	7 sec. diff.	9 third diff.

2) 42 sum	
21=half the sum	6615 (81.33=area
×.5	64
105	161) 215
×.7	161
735	1623) 5400
×.9	4869
6615	16263) 53100
	48789

If any two sides of a right-angled triangle be given, the third side may also be found; this part will be fully explained hereafter in trigonometry.

Article VIII. To find the area of a Trapezium.

Rule.—Draw a diagonal line from one of the angles to the opposite angle, as A C in *fig*. 8, and then will the trapezium be divided into two triangles, of which the diagonal is the common base; then letting fall perpendiculars from the opposite angles as B F and D E to the diagonal line A C: and then add these perpendiculars B F and D E together, and multiply half that sum into the diagonal, or half of the diagonal into the perpendiculars, and that product will be the area of the trapezium.

What is the area of the trapezium A B C D, *fig*. 8, the base A C is 18 *Ch* 74 *Li*: the perpendicular B F 6 *Ch* 90 *Li* and D E 7 *Ch* 56 *Li*.

Perp. B F=6.90	Base A C=18.74
Perp. D E=7.56	½ Perpendicular=7.23
Divide by 2) 14.46	5622
	3748
7.23	13.118
	acres.=1354902
	4
	roods=2.19608
	40
	rods=7.84320
	272
	168640
	590240
	168640
	feet=229.35040

Article IX. To find the area of any irregular figure.

Rule.—Divide the figure into triangles, by drawing diagonals from one angle to another, then measure all the triangles by either of the rules, already taught, at Articles 6, 7, or 8, and the sum of the several areas of all the triangles, will be the area of the figure.

What is the area of the irregular figure A B C D E F in *fig*. 9. Draw the diagonal F C, and let fall the perpendiculars F G, F H, and B I and D K, then we will commence on the base A B, which is 12 *Ch* 8 *Li*, and the base F C 10 *Ch* 86 *Li*, the base

E D 8 *Ch* 68 *Li*, the perpendiculars F G 6 *Ch* 49 *Li*, F H 4 *Ch* 95 *Li*. B I 6 *Ch* 80 *Li*, and D K 5 *Ch* 90 *Li*.

Perp. F G=6.49
Perp. F H=4,95
Perp. B I =6.80
Perp. D K=5.90
Divide by 2) 24.14
12.07

Base A B=12.08
Base F C=10.86
Base E D= 8.68
31.62
½ Perp.=12.07
22134
63240
3162
acres=38.16534
4
roods=.66136
40
rods=26.45440
272
90880
318080
99880
feet=123.59680

Article X. To find the area of a Trapezoid.

Rule.—Add the parallel sides together, and multiply half that sum by the perpendicular breadth, and the product will be the area.

What is the area of the trapezoid A B C D, *fig.* 10, the side A B being 9 *f*, the side C D 12 *f* 6 *i*, and the perpendicular height A E 9 *f* 8 *i*.

By Duodecimals.
side A B= 9.0
side C D=12.6
21.6=sum
10.9=½ half of sum
height A E= 9.8
96.9
7.2.0
ans. 103.11.0
f i ii

By Practice.
10.9
9.8
7.2.0
96.9
103.11.0

Article XI. To find the area of any regular Polygon.

Rule.—Multiply the whole perimeter or sum of the sides by half of the perpendicular, or multiply half of the perimeter by the whole perpendicular let fall from the centre to the middle of one of the sides.

Example. 1. What is the area of the pentagon A B C D E, *fig.* 11, the sides A B, B C, &c. being 6 *f* 1 *i*, and the perpendicular height from F to the centre of the figure or polygon, is 4 *f*, 1 *i*.

Sides=30.0.0
½ Perp. F G=2.0.6
60.0.0
1.3.0
area 61.3.0
f i ii

or 2)30
15.0
Perp.=4.1
1.3.0
60.0
61.3.0

To demonstrate this figure by the first method, let a line be drawn F, F in *fig.* 12, and make F, F equal in length to the whole perimeter of the stretch-out of the five sides in *fig.* 11, which is 30 feet, then take half of the perpendicular F G in *fig.* 11, and set it up from F, F to G, G in *fig.* 12, and draw the line G, G which will be parallel to F F; then take the several trapezoids from *fig*, 11, and transfer them to *fig.* 12, commencing at F B the half side, then the sides B C, C D, D E, E A, and A F, &c., and place them on the line F, F in *fig.* 12; then take the five small angles *n*, *n*, *n*, &c. in *fig.* 11, and transfer them to *n*, *n*, *n*, &c. in *fig.* 12, which completes the figure, and now we can see that any regular polygon can be formed into a parallelogram. Again, by the second rule; let F O or O F in *fig.* 13, be drawn equal in length to half of the perimeter, (which is 15 feet,) and make F O and O F equal in height to the whole perpendicular, which being 4 feet 1 inch, and draw O F parallel to F O, then will this figure contain the same quantity of space as *figs.* 11 and 12, and it is made up of the five equal angles from *fig.* 11, that is four wholes and two halves; as may be seen in the diagram of the figure, therefore these parallelograms are equal to the pentagon, which proves the rules correct.

Example 2.—What is the area of the hexagon A B C D E F, *fig.* 14, each side being 14 feet 6 inches, and the perpendicular height from G to the centre of the polygon, is 12 feet 8 inches?

half of the length of the side is = 43-6
the perpendicular is = 12-8
29-0-0
522-0
Answer 551-0-0
f i ii

This figure may also be demonstrated in the same way as *fig.* 11, for every regular polygon is equal to the parallelogram or long square whose length is equal to half the sum of the sides, and breadth equal to the perpendicular of the polygon as has been proved heretofore; this figure is made up of six equilateral triangles, and the parallelogram H I J K is also composed of six equilateral triangles, that is five whole ones and two halves, as may be seen in the diagram of the figure; therefore the parallelogram is equal to the hexagon.

But for finding more readily the area of a regular polygon, and also the perpendicular and the radius, the following table is introduced; containing the multipliers for all regular figures from the triangle to the duodecagon.

1. No. of Sides	2 Name of the Polygon.	3 Area multipliers.	4 Perpendicular multipliers.	5 Radius multipliers.	6 No. of degrees.
3	Trigon	0.433013	0.2886751	0.5773603	120
4	Tetragon	1.000000	0.5000000	0.7071068	90
5	Pentagon	1.720477	0.6881910	0.8506508	72
6	Hexagon	2.598076	0.8660254	.1side=rad.	60
7	Heptagon	3.633912	1.0382617	1.1523825	51$\frac{4}{10}$
8	Octagon	4,828427	1.2071068	1.3065630	45
9	Nonagon	6.181824	1.3737387	1.4619022	40
10	Decagon	7.694209	1.5388418	1.6186340	36
11	Undecagon	9.365640	1.7028437	1.7747329	32$\frac{7}{10}$
12	Duodecagon	11.196152	1.8660254	1.9318516	30

Rule 2.—Multiply the square of the side by the tabular area and the product will be the area of the polygon. Or multiply the side of the polygon by the tabular perpendicular, and the product will give the perpendicular from F to the centre of the polygon as in *fig.* 11. And by multiplying the side of the polygon also by the tabular radius, and the product will give the length from A B C, &c. to the centre of the polygon.

Examples.—1. If the side of a pentagon be 6 feet what is the area?

6
6
36=square

1.720477 tabular area.
36
10322862
5161431
answer 61.937172

2. What is the area of an octagon whose sides are 8 feet 3 inches?

```
   8.25              4.828427
   8.25                   680
  ------            ----------
   4125             386274160
  1650             28970562
 6600               ----------
 -------  answer  328.3330360
 680625
```

3. If the sides of a pentagon be 6 feet, what is the length of the perpendicular?

```
  6881910
        6
 ---------
 4.1291460
```

4. If the sides of a pentagon be 6 feet what is the length of the radius?

```
         8506508
               6
        ---------
answer  5.1039048
```

And in case it is required to find the tabular numbers answering for the perpendicular, where the polygon has more than twelve sides, they may be found by Trigonometry. There may be instances where the sides of a polygon may be obtained, and the perpendicular cannot, and if we can have the length of the sides and perpendicular, it is all we want to find the area.

Note.—There is no letter described at the centre of the polygon in the diagram of the figure; therefore we will suppose that the angles at the centre of the polygon, as in *fig.* 11, to be designated by the letter O.

To find the tabular number; thus, find the angle of the centre of the polygon as O in *fig.* 11, by dividing the number of sides by 360°; then suppose each side of the pentagon in *fig.* 11, be 1 or A B the log of 10, then A F would be 5 the half log.

Example.—Divide 360° by 5, (the number of sides in the pentagon,) and the quotient is 72° for the angle A O B, the half of which is 36°, the angle F O A, whose complement to 90° is 54°, the angle O A F. Then say,

As cosecant F O A 36° - - - - - -	10.230701
Is to 5 the half side F A log - - - -	0.698970
So is sine O A B 54° - - - - - -	9.907958
To the perpendicular F O 0.6881910 -	0.837709

Article XII.—The diameter of a circle being given to find the circumference; or the circumference being given to find the diameter.

Rules.—As 7 is to 22, so is the diameter to the circumference nearly; or as 22 is to 7, so is the circumference to the diameter nearly.

2. Or more exactly as 113 is to 355, so is the diameter to the circumference; or as 355 is to 113, so is the circumference to the diameter.

Examples.—What is the circumference of a circle, whose diameter is 6 feet 6 inches?

```
as 7 : 22 :: 6 : 5 :                      as 113 : 355 :: 6 : 5 :
       65                                          65
      ----                                        ----
      110                                         1775
     132                                         2130
     ----                                        ----
   7)1430(20.42 the circum. nearly           113)23075(20.42
     14                                          226
     ----                                        ----
      30                                          452
      28                                          475
      --                                          ----
      20                                          230
      14                                          226
```

Article XIII.—What is the diameter of a circle whose circumference is 20.$\frac{42}{113}$ feet?

```
as 22 : 7 :: 20.42 :                 as 355 : 1130 :: 20.42
             7                                       2042
           ----                                      ----
       22)14294(6.5 diam. nearly                     2260
          132                                        4520
          ----                                      22600
           109                           355)2307460(6-5 dm. nearly.
           110                               2130
                                             ----
                                             1774
                                             1775
```

We now see the proportions of these examples are nearly correct; the latter however is the most accurate.

Circles like all other similar plain figures are in proportion to one another, as the squares of their diameters and their circumferences, are to one another as their diameters of circles, or radii.

The proportion of the diameter of a circle to its circumference has never yet been exactly determined. This problem has engaged the attention and exercised the abilities of the greatest mathematicians for ages; no squares or any other right lined figures, has yet been found that shall be perfectly equal to a given circle. But though the relation between the diameter and circumference has not been accurately expressed in numbers, it may be approximated to any assigned degree of exactness. Archimedes, about two thousand years ago, discovered the proportion to be nearly as 7 to 22; other and nearer ratios have since been successively assigned, viz. as 106 to 333, or 113 to 355; this last proportion is useful, for being turned into a decimal, it agrees with the truth to the 6th figure inclusively.

Article XIV.—To find the area of a circle.

Rules.—1. Multiply half the diameter by half the circumference, and the product will be the area.

2. Multiply the square of the diameter by .7854, and the product will be the area.

3. Multiply the square of the radius by 3.1416 and the product will be the area.

4. Multiply the square of the circumference by .07958, and the product will be the area.

If the diameter be given, find the circumference by Art. XII.

If the circumference be given, find the diameter by Art. XIII.

What is the area of a circle whose diameter is 6 feet 6 inches, and circumference 20 feet 4 inches.

```
        By Rule I.                          By Rule II.
½ 20 feet 4 inches = 10-2                 diameter 6-5
½  6 feet 6 inches =  3-3                          6-5
                     ----                          ---
                    2-6 6                          325
                   30-6                           390
                   -----                          ----
          Answer 33-0-6                           4225 sq. of diameter.
                  f i s                           7854
                                                  ----
                                                 16900
                                                21125
                                               33800
                                              29575
                                              --------
                                     Answer 33.183150
```

To demonstrate the first rule, let A B C D, *fig.* 15, be the circle, and A B C half of the circumference; and B E the semidiameter. Let a line be drawn F G parallel to the diameter A C; and divide the quadrants B A and B C into any like number of equal parts; or take the stretch out of B A and B C and produce

them to F and G; and draw H F and G I parallel to B E; and produce the diameter A C to H and I. Then will the parallelogram H I G F contain the same area as the circle A B C D; for every circle may be conceived to be a polygon of an infinite number of sides; now the semi-diameter, must be equal to the perpendicular of such a polygon; and the circumference of the circle equal to the periphery of the polygon; therefore half of the circumference multiplied by half of the diameter will give the area of a circle.

Demonstration of Rule II. All circles are to each other as the squares of the diameters, and the area of a circle whose diameter is 1, is 7854 (by the second example;) therefore as the square of 1 which is 1 to .7854, so is the square of the diameter of any circle to its area.

Article XV.—Having the diameter, circumference or area of a circle given, to find the side of a square equal in area to the circle, and the side of a square inscribed in the circle, or having the side of a square given to find the diameter of its circumscribing circle, and also of a circle equal in area, &c.

Rules.—1. The diameter of any circle multiplied by 8862269 will give the side of a square equal in area, as in *fig.* 16.

2. The circumference of any circle multiplied by 2820948 will give the side of a square equal in area.

3. The diameter of any circle multiplied by 7071068 will give the side of a square inscribed in that circle.

4. The circumference of any circle multiplied by 2250791 will give the side of a square inscribed in that circle.

5. The area of any circle multiplied by 6366197 and the square root of the product extracted will give the side of a square inscribed in that circle.

6. The side of any square multiplied by 1.414236 will give the diameter of its circumscribing circle.

7. The side of any square multiplied by 4.4428829 will give the circumference of its circumscribing circle.

8. The side of any square multiplied by 1.1283791 will give the diameter of a circle equal in area to the square.

9. The side of any square multiplied by 3.5449076 will give the circumference of a circle equal in area to the square.

Article XVI.—To find the area of a semicircle.

Rule.—Multiply the fourth part of the circumference of the whole, (that is half the arc line,) by the semi-diameter, the product is the area.

What is the area of the semicircle A B C, *fig.* 17, the arc A B being 17 feet $4\frac{1}{2}$ inches, and the semi-diameter B D is 11 feet 3 inches?

By Duodecimals.	By Practice.
Cir. A B = 17-4-6	17-4-6
11-3-0	11-3-0
191-1-6	4 4-1-6-0
4-4-1-6	191-1-6
Answer 195-5-7-6	195-5-7-6-0
f i p s	

Article XVII.—To find the area of a quadrant.

Rule.—Multiply half the arc line of the quadrant, (that is the eighth part of the circumference of the whole circle,) by the semi-diameter, and the product is the area of the quadrant.

What is the area of the quadrant A B C, *fig.* 18, half of the arc B C is 8 feet 9 inches, and the semi-diameter A C is 11 feet 3 inches?

By Duodecimals.	By Decimals.
11-3	11-25
8-9	8-75
90-0	5625
8-5-3	7875
	9000
Answer 98-5-3	98.4375
f i p	

Article XVIII.—The chord and height of a segment being given to find the chord of the half arc.

Rule.—To the square of the half chord add the square of the height, and the square root of the sum will be the length of the chord of half the arc.

Example—The chord A C, *fig.* 19, being 48 feet, and the height B D 18 feet, what is the length of the chord of half the arc?

```
2)48                 18
                     18
  24                ---
  24                144
 ---                18
  96                ---
 48                 324
 ---
Add { 576 square of half the chord.
    { 324 square of the height.
     ---
     900(30, length of half the chord.
     9
     --
     00
```

Article XIX.—To find the length of any arc of a circle, the half chord and chord of the whole arc being given.

Rule.—Subtract the chord of the whole arc from double the chord of the half arc; add one third of the remainder to the double chord of the half, and the sum will be nearly equal to the length of the arc.

Examples.—What is the length of the arc A B C, *fig.* 19, whose chord A C is 48, and the half chord A B is 30.

```
2×30=60 the double chord of the half arc.
     48 the chord of the whole arc.
    ---
  3) 12
      4
     60 the double chord of the half arc.
    ---
     64 the length of the arc required.
```

Article. XX.—The chord and height of a segment being given to find the radius of the circle.

Rule.—To the square of the half chord, add the square of the height, and divide the sum by twice the height of the segment, and the quotient will be the radius of the circle, when it is less than a semicircle.

The chord A C, *fig.* 19, of a segment being 48 feet, and the height B D 18 feet, what is the radius of the circle?

```
÷2) 48
    --                      18 the height.
    24 half the chord.      18
    24                     ---
   ---                     144
    96                      18
   48                      ---
   ---                     324
Add { 576 square of the half chord.
    { 324 square of the height.
   ---
36)900(25 the radius required.   18
   72                             2
   ---                           ---
   180                           36 twice the height.
   180
```

Article XXI.—Given any two parallel chords in a circle, and their distance, to find the distance of the greater chord from the centre.

Rule.—To the square of the distance between the chords, add the square of half the lesser chord. The difference between this sum, and the square of half the greater chord divided by twice the distance of the chords, will give the distance of the centre from the greatest chord.

Example.—Suppose the greater chord C D, *fig.* 20, is 48 feet, and the lesser A B 30, their distance E G 13 feet, what is the distance E F from the centre to the greater chord C D?

```
 13      ½ 30 = 15              ½ 48 = 24
 13             15                     24
 ---            ---                    ---
 39             75                     96
13             15                     48
 ---            ---                    ---
169            225 sq. of the lesser ch.   576 sq. of the greater ch.
               169 sq. of the distance.    394
               ---                         ---
               394          2 × 13 = 26) 182 (7 = E F dist. req'd.
                                         182
                                         ---
```

Article XXII.—Given a chord of a circle and its distance from the centre to find the radius of the circle.

Rule.—To the square of the half chord, add the square of the distance from the centre, and the square root of the sum will be the radius required.

Example.—Given the chord C D, *fig.* 20, 48 feet, and its distance E F from the centre 7 feet, required the radius of the circle.

```
½ 48 = 24 and 24 × 24 = 576
                7 × 7 =  49
                        ---
                        625(25 the radius.
                        4
                        ---
                    45) 225
                        225
```

Article XXIII.—Given any two parallel chords in a circle, and the distance between them, to find the perpendicular height from the middle of either chord to the circumference.

Rule.—Find the nearest distance of the greater chord from the centre, by Art. 21, and find the radius of the circle by Art. 22, add the distance between the two parallel chords, and the distance between the greater chord and the centre of the circle together; this sum being taken from the radius, will give the perpendicular height from the middle of the lesser chord, to the circumference or height of the lesser segment; to the lesser segment add the distance between the parallel chords and the sum will be the height of the greater segment.

Examples.—Given the greater chord C D, *fig.* 20, 48 feet, and the lesser chord A B, 30 feet, their distance E G, 13 feet, required the distance G H perpendicular from the middle of A B to the circumference. The distance from the centre to the greater chord will be found to be 7 feet by Art. 21, and the radius 25 feet by Art. 22.

Thus 13 + 7 = 20 and 25 — 20 = 5 feet height of the lesser segment.
Then 13 + to 5 = 18 the height of the greater segment.

Article XXIV.—To find the area of a sector of a circle.

Rule.—Multiply the radius or half the diameter by half the length of the arc of the sector, and the product will be the area.

Example 1.—To find the area of the sector A B C, *fig.* 21, first find the half arc B D, by Art. 19, which is 3 feet, 6 inches, and the radius or semi-diameter A B being 12 feet 4 inches.

```
     Radius A B is = 12-4
The half arc B D is =  3-6
                     ------
                      6-2-0
                     37-0
                     ------
              Answer 43-2-0
                      f i ii
```

Example 2.—What is the area of the sector A B C, *fig.* 22, which is greater than a semicircle, half of the arc A B being 72 feet 6 inches, and the radius A D is 56 feet 3 inches?

```
         72-50
         56-25
        -------
         36250
        14500
       43500
      36250
     ----------
Area 4078,1250
```

Article XXV.—To find the area of a segment of a circle, the chord and height of the arc being given.

Rule 1.—Find the length of the arc A B C, *fig.* 23, by Art. 19, and the radius of the circle by Art. 20, the area of the sector A B C E by Art. 24. Subtract the area of the triangle A E C as found by Art. 6, from the area of the sector, and the remainder will be the area of the segment.

Example 1.—What is the area of the segment of a circle A B C, *fig.* 23, the chord A C being 48 feet, and the height B D 18 feet? The length of the arc will be found to be 64 feet, and the radius 25 feet, then

```
2)64                          25
 ---                         —18
 32                          ---
×25                            7 perpendicular D E.
 ---                          48
 160                          ---
 64                         2) 336
 ---                           168 area of the triangle A C E.
 800 area of the sector.
 168
 ---
 632 area of the segment.
```

Rule 2.—To two thirds of the product of the base multiplied by the height, add the cube of the height divided by twice the length of the segment and the sum will be nearly the area.

Example 2.—What is the area of a circular segment, the chord being 48 feet, and the height 18 feet?

```
   48                            18
  ×18                           ×18
  ---                           ---
  384                           144
  48                            18
  ---                           ---
3)864                           324
  ---                           ×18
  288                           ---
   ×2                          2592
  ---                          324
  576                          ---
+60-75               2×48=96) 5832(60-75
  ---                          576
636-75 the area required.      ---
                                 720
                                 672
                                 ---
                                 480
                                 480
```

To find the area of a segment of a circle which is greater than a semicircle as A B C, *fig.* 24.

Rule 3.—Find the length of the arc A B C by Art. 19, and the radius A D by Art. 20, thence find the whole area of the sector A B C D, by Art. 24, Example 2: and then find the area of the triangle A C D by Art. 6, and add the area of the triangle to the area of the sector, and it will give the whole area of the segment.

Example 3.—What is the area of a segment of a circle A B C, *fig.* 24? Suppose the half arc A B to be 86 feet, the radius A D or C D 40 feet, and the chord A C 68 feet.

```
                                      chord A C = 68
½ arc  A B = 86                    side A D = 40    74    74    74
radius A D = 40                    side G D = 40    68    40    40
           ----                              ----    --    --    --
      3440=area of the sector A B C         2)148     6    34    34
       716=area of triangle A C D           -----
      ----                                     74
      4156=whole area of seg. A B C             6     513264(716 area.
                                              ---     49
                                              444     ---
                                               34  141)233
                                             ----      141
                                             1776     ----
                                             1332 1426)9164
                                             ----      8556
                                            15096     ----
                                               34      608
                                            -----
                                            60384
                                            45288
                                           ------
                                           513264
```

Article XXVI.—To find the area of a circular zone as in *fig.* 25, which is that part of a circle laying between two parallel chords, and the parts of the circle intercepted by the chords.

Rule.—First find the area of the whole circle by Art. 14, then find the area of the two segments of the circle by Art. 25, and subtract the areas of the two segments from the area of the whole circle; and the remainder will be the area of the zone.

Article XXVII.—To find the area of a Lune or Crescent. A Lune is a figure made by two circular arcs which intersect each other, as A B C D, in *fig.* 26.

Rule.—If it be a semicircle, first find the whole area of the semicircle A B C, by Art. 16, then find the area of the segments A D C by Art. 25, and subtract the area of the segment from the area of the semicircle, and the remainder will be the area of the Lune A B C D. But if it be a segment of a circle, first find the area of the whole segment A B C, by Art. 25, then find the area of the lesser segment A D C, (by the same Art.) and subtract the area of the lesser segment from the area of the greater, and the remainder will be the area of the Lune A B C D, &c.

Article XXVIII.—To find the area of Compound Figures. Mixed or compound figures are such as are composed of rectilineal and curvilineal figures together, as *fig.* 27.

Rule.—First find the area of the trapezium A B C D, by Art. 8, then find the area of the segments A E D and B F C, by Art. 25, and add the areas all together and you will have the area of the whole figure.

Article XXIX.—To find the circumference of an ellipsis, the transverse and conjugate axis being given.

Rule.—Multiply half the sum of the two axis by 3; to the product add ⅐ part of the sum of the two axis, and this sum will give the circumference near enough for most practical purposes.

What is the circumference of an ellipsis whose transverse axis A B, *fig.* 28,) is 24 feet, and the conjugate C D 18 feet.

```
   24
  +18
  ---
2) 42
  ---
   21
  ×3⅐
  ----
   63
   +3
  ----
   66 feet the circumference.
```

Article XXX.—To find the area of an ellipsis, the transverse and conjugate axes being given.

Rule.—Multiply the transverse axis by the conjugate, and the product by .7854, will give the area required.

What is the area of an ellipsis whose transverse axis A B, *fig.* 28, is 30 feet, and the conjugate C D 20 feet.

```
  30             7854
 ×20             ×600
 ---        ---------
 600   Area 471,2400
```

Note.—Ellipsis' of a large size, are frequently laid out in gardens of which they can be accurately drawn, by driving two pins into the ground at the foci of the ellipsis, for the chord to revolve around, (as described in Geometry Problem 38.)

Article XXXI—To find the area of an Elliptical Ring, or the space included between the circumference of two concentric and similar Ellipsis.*

Rule.—First find the area of the greater ellipsis A B C D, *fig.* 29, by Art. 30; then find the area of the lesser E F G H, (by the same Art.) and subtract the area of the lesser from the area of the greater, and the remainder will be the area of the ring; or from the product of the two diameters of the greater ellipsis, subtract the product of the two diameters of the lesser; the remainder multiplied by .7854 will be the area of the ring.

Note.—This rule will also serve for a circular ring; for when the diameters of each ellipsis become equal to the square of the diameter of the greater circle diminished by the square of the diameter of the less, and the remainder multiplied by .7854 is the area of the circular ring, or, multiply the sum of the diameters by their difference, and that product into .7854 for the area of a circular ring.

Article XXXII.—To find the area of a parabola, the base or double ordinate being given, and the axis or height.

Rule.—Multiply the base by the height, and two thirds of this product will be the area required.

What is the area of the parabola A B C, *fig.* 30, the axis C D being 12 feet 3 inches, and the double ordinate A B 18 feet 6 inches?

```
      12-25
     ×18-50
     ------
      61250
     9800
    1225
   --------
  3)2266250
   --------
     755416
         ×2
    -------
Area 151,0832
```

*It is here supposed that the difference between the conjugate diameters is equal to the difference between the transverse diameters, but it is well known, that in this case the elliptic arc will not be every where equi-distant; the difference between the semi-transverse or semi-conjugate diameters being the least distance between the arc.

Article XXXIII.—To find the area A B, C D, *fig.* 31, of the frustum of a parabola whose parallel ends A B and C D are given, also their distance E F.

Rule.—To the square of the greatest end, add the square of the lesser end, to the product of the ends; divide the sum by the sum of the ends, and the quotient multiplied by the distance of the ends, two thirds of the product will be the answer.

Suppose the end A B 24, the end C D 20, and their distance E F 5; required the area A B C D.

```
 24                 20     33            24
×24                ×20      5            20
 —                 ———    ———            ———
 96                400  3) 165           480
48                 576    ———
———                480     55
576               ————      2
       24+20=44)1456(33   ———
                132        110 Answer nearly.
              ————
                136
                132
                ———
                  4
```

Article XXXIV.—To find the area of a cylinder.

Rule.—Multiply the circumference by the length of the cylinder and the product will be the area.

What is the area of the cylinder A B, C D, *fig.* 32, whose diameter A B being 24 inches, and the perpendicular height or length E F is 128 inches?

First I find the circumference by Art. 12, of which is $75\frac{42}{100}$ inches, this multiplied by 128 inches, the length, and divided by 144, gives $67\frac{04}{100}$ feet the area.

```
as 7 : 22 :: 24                       75.42
           22                           128
          ———                         ———————
           48                          60336
          48                          15084
         ————                         7542
       7) 528(75,42 circumference.   ————————
          49                  144) 965376(67,04 Area.
         ————                      864
           38                      ———————
           35                      1013
         ————                      1008
           30                      ————
           28                         576
         ————                         576
            20
            14
```

There is to be a circular stair-case built around this cylinder containing 15 treads and 16 risers; I demand what the width of these treads and risers will be, and also the length of the hand rail.

First I divide the 15 treads by $75\frac{42}{100}$ inches, which gives $5\frac{028}{1000}$ inches for the width of the treads, being a trifle over 5 inches, then I divide the 16 risers by 128 inches, and it gives 8 inches for the height of the risers, and by squaring the circumference and perpendicular height, then extract the square root of the two sums, and it gives the length of the rail as required.

```
15)76,42(5.028     16(128(8       128
   75                 128         128
  ———                ————        ————
   42                            1024
   30                             256
  ————                           128
   120                          ——————
   120                          16384
                                 5688 in parts.
                                ———————
            75,42               2,20,72(148,56 length of rail.
            75,42               1
           ———————             ————
            15084            24)120
           30168                 96
          37710                ————
         52794              288)2472
        ——————————              2304
         5688,1764             ——————
                           2965)16800
                                14825
                               ———————
                          29706)197500
                                178236
```

MENSURATION OF SOLIDS.

SECTION II.

Solid measure is the finding the number of cubic inches, feet, yards, &c. contained in any thing that consists of length, breadth and thickness. The least solid measure is a cubic inch, and all solids are measured by cubes whose sides are inches, feet, yards, &c., and the solidity of a body is said to be so many cubic inches, feet, yards, &c., as will fill the same space as the solid, or as the solid will contain; that is, $12\times12\times12=1728$ cubic inches, which makes one cubic or solid foot.

A Table of Cubic Measure.

1728	cubic or solid	inches,	make	1	solid	foot.
27	"	" feet,	"	1	"	yard.
$166\frac{3}{8}$	"	" yards,	"	1	"	pole.
64000	"	" poles,	"	1	"	furlong.
512	"	" furlongs,	"	1	"	mile.

Article XXXV.—Of a Cube. A Cube is a solid of 6 equal sides, each of which is an exact square.

To find the solidity.

Rule.—Multiply the side of the cubic into itself, and that product again by the side; and the last product will be the solidity.

What is the solidity of a cube as A B C D, *fig.* 33, *pl.* 28, whose sides are 12 inches, or 12 feet.

```
  12                  12
  12                  12
 ————                ————
 144                 144
  12                  12
 ————                ————
 1728 inches.        1728 feet.
```

Article XXXVI.—To find the solidity of a parallelopipedon. A parallelopipedon is a solid, having six rectangular sides, every opposite pair of which are equal and parallel.

Rule.—Multiply the breadth by the depth, and that product by the length, and it will give the solid contents.

Example 1.—How many cubic feet is there in a rectangular box as A B C D E, *fig.* 34, whose breadth A B, is 2 feet 4 inches, depth A C, is 3 feet 8 inches, and length D E, 12 feet 3 inches?

```
       AC=3-8                 Or thus AC=44 inches.
       AB=2-4                         AB=28 inches.
          ------                        ----
          1-2-8                          352
          7-4                             88
          ------                        ----
          8-6-8 area of base.           1232 area of base.
   DE=12-3-0                     DE=147 inches.
          ------                        ----
       2-1-6-0-0                         8624
     102-8-0                            4928
     ----------                        1232
Answer 104-9-6-0-0                     -------
       f  i p                   1728) 181104 (104.80 Answer.
                                       1728
                                       ----
                                        8304
                                        6912
                                       -----
                                        13920
                                        13824
                                        -----
                                           96
```

Example 2.—How many feet of wood is there in a load whose breadth is 3 feet 10 inches, and depth 4 feet 3 inches, and 8 feet long?

```
              3-10
              4- 3
             -----
             11- 6
          15- 4
          --------
          16- 3- 6 area of the end.
           8
          --------
Answer   130- 4- 0
          f   i  p
```

Article XXXVII.—To find the solidity of a prism. A prism is a body with two equal or parallel ends, either square, triangular or polygonal, and three or more sides which meet in parallel lines, running from the several angles at one end to those of the other.

Rule.—Prisms of all kinds, whether square, triangular or polygonal, are measured by one general rule, viz. first find the area of the end or base, by Art. 11, and then multiply the area of the end by the perpendicular height or length of the prism, it will give the solid contents.

Example 1.—How many feet of timber is there in a stick which is hewn three square, as A B C, *fig.* 35, the sides being 16 inches, and the perpendicular E F is 7 inches, and the length C D is 14 feet?

```
 AB=16 inches          or thus, area of base = 112 inches
 EF= 7 inches                   length CD = 168 inches
     ----                                      ----
     112 area of base                           896
 CD=14 feet                                     672
     448                                        112
    112                                        -----
    ----                               1728) 18816 (10.80 ans.
144)1568 (10.80 ans.                          1728
    144                                       -----
    ----                                      15360
     1280                                     13824
     1152                                     -----
     ----                                      1536
      128
```

Example 2.—How many feet is there in the octagon, or eight squared stick, as A B C D E F G H, *fig.* 36, the sides being $6\frac{5}{8}$ inches wide, and half of the perpendicular height is 4 inches, the length 18 feet 6 inches?

```
          the sides =53 inches
          ½ perp. =  4 inches
                    ----
                    212 area of the end
     length EK=185 inches
                    ----
                    1060
                   1696
                   212
                  ------
              144)39220(27.23 ans.
                  288
                  ----
                  1042
                  1008
                  ----
                   340
                   288
                   ---
                    520
                    432
                    ---
                     88
```

Article XXXVIII. To find the solidity of a Cylinder.

Rule.—The diameter of a base being given, find the area of the end by Article 14, then multiplying the area of the base by the length, that product will be the contents of the cylinder.

What is the solidity of a cylinder as *fig*, 37, whose height C D is 12 feet, and the diameter A B of the base 2 *f* 6 *i*.

```
   2.5              7854
   2.5               625
   ----            ------
   12.5             39270
   50.             15708
   ----           47124
   62.5           --------
                  4908750 area of the base.
                       12
                ----------
           ans. 58.905000
```

Note.—If the circumference of a cylinder be given, multiply the square of the circumference by 07958, and the product will be the area of the base, then multiply the area of the base by the length, and it will give you the solid content.

Article XXXIX. To find the solidity of a Cylindroid.

Rule.—The diameter of the base being given, find the area of the end by Article 30, then multiplying the area of the end by the length, that product will be the content of the cylindroid.

What is the solidity of a cylindroid, as A B C D E, *fig.* 38, the transverse axis A B is 3 *f* 9 *in*, and the conjugate axis C D is 2 *f* 3 *in*, the length D E 10 *f*.

```
 AB=375             84375
 CD=225              7854
    ----            ------
    1875            337500
    750            421875
   750            675000
   -----         590625
   84375         ---------
                 662681250 area of base
                        10
                -----------
           ans. 66.26812500
```

Article XL. To find the solidity of a Pyramid.

Rule.—Find the area of the base, whether triangular, square, polygonal, or circular, by the rules already mentioned in superficial measure; then multiply this area by one-third of the height, and the product will be the solid content of the pyramid.

Example 1. In a triangular pyramid A B C, *fig.* 39, the height D F being 15 *f* 9 *i* and each side of the base is 6 *f* 6 *i* the perpendicular D E 2 *f* 10 *i*

```
      side A B=6.6              or thus A B=6.5
    perp. D E=2.10                          6.5
              ----                          ---
             5.5.0                          325
              13.0                          390
              ----                          ----
            18.5.0                         4225
   ½ of D F= 5.3.                 tab. area=4330
             -----                       ------
             4.7.3                       126750
             92.1                        12675
             ----                        16900
    Ans.  96.8.3                        -------
                                     18.29.4250
                              ⅓ of D F=5.25
                                         ----
                                         9145
                                        3658
                                       9145
                                       -------
                                 Ans. 96.0225
```

Example 2. What is the solidity of a quadrangular pyramid, the height E F *fig.* 40, being 30 *f* 6 *i* and each side of the base is 12 *f* 7 *i*

```
        Sides=12.7
              12.7
              ----
             7.4.1
            151.0
            ------
           158.4.1  area of base.
    ⅓ E F=  10.2
           -------
          26.4.8.2
        1583.4.10
        ---------
   Ans. 1609.9.6.2
```

Example 3. What is the solidity of a hexagon pyramid as *fig.* 41, the height G H being 39 feet, the sides A B C &c. is 14 *f* 6 *i* and the perpendicular I G 12 *f* 6 *i*

```
   ½ of sides=43.6
   perp. I G=12.6
             -----
            21.9.0
           522.0
           ------
           543.9.0  area of base.
 ⅓ of height=13
           -------
           1638.9
           543
           ------
     Ans.  7068.9
```

Example. 4. What is the solidity of a cone, the diameter A B, *fig.* 42, being 9 feet, the height C D 27 feet.

```
            9
            9
            --
           81  square of the diameter.
         7854  tab. multiplier.
         ----
         7854
        62832
        -------
      63.6174  area of base.
            9=⅓ of height.
      --------
     572.5568=content.
```

Article XLI. To find the solidity of the Frustum of a Pyramid. A frustum of a pyramid is what remains after the top is cut off by a plane parallel to the base.

Rule.—If it be a frustum of a square pyramid, multiply the side of the greater base by the side of the less; to this product add one-third of the square of the difference of the sides, and the sum will be the mean area between the bases; but if the base be a triangular, polygon, or any regular figure, multiply this sum by the proper multiplier of its figure in the table, (Art. 11,) and the product will be the mean area between the bases; lastly, multiply this by the height, and it will give the content of the frustum.

Example 1. What is the solidity of the triangular frustum of a pyramid, the sides of the greater base A B, *fig.* 43, being 9 feet, the side of the less C D, 6 feet, and the height E F, is 10 feet.

```
  A B=9                                       9
  C D=6                                       6
  --                                          -
     3  difference of the sides.             54
    ×3                                    Add 3
    -                                        --
  3) 9=square of the difference.             57
  --                                        433  tab. multiplier.
     3=⅓ of the square.                    ----
                                            171
                                            171
                                           228
                                           ------
                                           24.681=mean area.
                                               10=height.
                                           -------
                                          246.810=content.
```

Example 2. What is the solidity of a frustum of a square pyramid, the side of the greater base A B, *fig.* 44, being 9 feet, the side of the less C D 6 feet, and the height E D is 10 feet.

```
  9                                     A B=9
  6                                     C D=6
  -                                        --
  3=difference.                            54
 ×3                                     add 3
 --                                        -
 3) 9=square of the difference.            57  mean area.
 --                          height  E D=10
  3=⅓ of the square.                      ---
                                          570=content.
```

Example 3. What is the solidity of a frustum of an octagonal pyramid, whose sides A B, *fig.* 45, of the greater base is 9 feet (as before,) the lesser C D 6 feet, and the height E D 10 feet.

```
  A B=9                                   A B=9
  C D=6                                   C D=6
  --                                         --
     3=difference of the sides.              54
    ×3                                    Add 3
    --                                       --
  3) 9=square of the difference.             57
  --                                       4828 tab. multiplier.
     3 =⅓ of the square.                  -----
                                          33796
                                          24140
                                          ------
                                          275196=mean area.
                                              10=height.
                                         -------
                                         2751.960 content.
```

In measuring square timber where the sticks run tapering, the best method is as follows: take the girth of it in the middle; square ¼ of the girth, or multiply it by itself in inches. Then say as 144 inches is to that product, so is the length taken in feet to the contents in feet.

Example 4. What is the solidity of a tapering square stick

of timber, whose sides of the largest end is 14 inches, the least end 10, and the length 40 feet.

One-fourth of the girth in the middle $=12$ and $12\times12=144$, the area in the middle; then as 144 : 144 :: 40 feet : 40 feet the content.

```
or thus, 12×12=144
               40=length.
          ----
     144)5760(40 content.
         576
         ---
           0
```

There is more timber of this description measured by the following rule, than any other, viz: By adding the area of the two ends together, and one half of the sum multiplied into the length and divided by 144 for the solid content; but it is not so correct as the former; see the following example.

```
14×14=196
10×10=100
      ---
    2)296
      ---
      148
      ×40=length.
     ----
144)5920(41.11 content.
    576
    ---
     160
     144
     ---
      160
      144
      ---
       160
       144
       ---
        16
```

There is more than a foot's difference in the two examples; but in measuring small timber of not much value, this rule may apply; to find the area of a board that runs wedging, this rule is correct, as may be seen by Article 5. And if it be a tapering three square stick of timber, you may find the area midway from the end; then as 144 is to the area, so is the length taken in feet, to the content in feet. Or by multiplying the area of the two bases together, and to the square root of the product add the two areas, that sum, multiplied by one-third of the length, will give the solidity of any frustum.

Example 5. What is the solidity of a tapering square stick of timber, the greater end being 14 inches, (as in example 4) the lesser end 10, and the length 40 feet.

```
14×14=196                Area=196
10×10=100                     100
      ---                     140
   1.96.00(140                ---
   1                          436
   --                 ⅓ of 40=13.33
 24)96                       ----
    96                       1308
    --                      1308
    00                     1308
                           436
                          ------
                   144)581188(30.46=content.
                       576
                       ---
                         518
                         432
                         ---
                          868
                          864
                          ---
                            4
```

Article XLII. To find the solidity of a Wedge, as A B C D E F, *fig.* 46.

Rule.—Multiply the area of the base A B C, by the length B E, and half of the product will give the solidity.

Required the solidity of a wedge A B C D E F, the side A B being 9 inches, and A C 18 inches, the length B E 4 feet.

```
A C=18                 2)4.50
A B= 9                   ----
    --                   2.25=content.
   162
B E ×4
   ---
144)648(4.50
    576
    ---
     720
     720
     ---
```

Article XLIII. To find the solidity of a Frustum of a Cone.

Rule.—Multiply the diameters of the two bases together, and to the product add one-third of the square of the difference of the diameters; then multiplying this sum by .7854, it will be the mean area between the two bases; which being multiplied by the length of the frustum, will give the solid content.

Note.—In general way in measuring or guaging a frustum, it is best to find the mean area in inches. Then by multiplying the inches by the height in feet, and by dividing the product by 12, the quotient will be the answer in board measure, or divide by 144, the quotient will be in square measure; or if you multiply the mean area in inches, by the height in inches, and divide the product by 1728, gives it also in square feet; and when we have the solid content in cubic inches, we can easily find the number of gallons and bushels, &c. by dividing by 231 for wine gallons, 282 for ale gallons, and by 2150.4 for bushels; of guaging, this part will be fully explained hereafter.

Example 1. What is the solidity of a frustum of a cone, whose greater base A B, *fig.* 47, being 60 inches, the lesser C D, 51 inches, and the perpendicular height E F 6 feet 6 inches.

```
A B=60                      60
C D=51                      51
    --                      --
    60                       9=difference.
   300                      ×9
   ---                      --
  3060                   3) 81 square of the difference.
Add 27                      --
  ----                      27=⅓ of the square.
  3087
  7854              144)157560(109.41 feet content.
  ----                  144
  12348                 ---
 15435                   1256
24696                    1296
21609                    ----
--------                  600
2424.5298=mean area.      576
   6.5 feet=length.       ---
--------                  240
12120                     144
14544                     ---
------                     96
157560=content.
```

Example 2. What is the solidity of a mast or spar, whose diameter is 30 inches at one end, and 18 inches at the other, and 80 feet long.

30	7854 tab. multiplier.	30
18	×588	18
240	62832	12=difference.
30	62832	×12
540	39270	3) 144 square of the difference.
Add 48	46181.52=mean area.	48=⅓ of the square.
588	80=length.	

144) 3694480 (256.56 feet content.
288
814
720
944
864
808
720
880
864
16

Article XLIV. To find the solidity of an ellipsis frustum of a cone.

Rule.—Find the area of the two bases by Art. 30, and thence multiply the areas of the two bases together, and to the square root of the product add the two areas; that sum multiplied by one third of the height. This rule will give the solidity of any frustum; for it is plain when figures run uniformly taper, but not to a point, they being considered as portions of the cone or pyramid, we may find the solidity by supplying what is wanting to complete the figure, and then deducting the part cut off.

A general rule for completing every straight sided solid whose ends are parallel and similar: As the difference of the top and bottom diameters is to the perpendicular height, so is the longest diameter to the altitude of the whole cone or pyramid.

Example.—What is the solidity of a frustum of a cone, as ABCD, EFGH, *fig.* 48 whose transverse axis AB of the greater base is 60 inches, the conjugate axis CD 42 inches, and the transverse axis EF of the lesser base 40 inches, and the conjugate GH 28 inches, the height DH 6 feet.

Area of the greater base ABCD = 1979 inches. 1979 = area.
Area of the lesser base EFGH = 879 inches. 879 = area.

17811	1318 = sq. root of prod.
13853	4176
15832	2 = ⅓ of height.
1.73.95.41 (1318	144)8352(58 feet content.
1	720
23)73	1152
69	1152
261)495	
261	
2628)23441	
21024	
2417	

Art. XLV. To find the solidity of a parabolic conoid; the diameter AB *fig.* 49 of the base being given, and the perpendicular height CD.

Rule 1.—Multiply the square of the diameter of the base by .3927, and the product by the height will give the solidity.

Example 1.—What is the solidity of a parabolic conoid, whose diameter AB is 30 feet, and the height 50 feet.

3927 multiplier.
AB 30×30=900
3534.300
50=height.
17671.700 the solidity required.

Rule 2.—Multiply the area of the base by the height, and half the product will be the solid content.

Example 2. AB 30×30=900 (as before.
7854
900
7068600=area.
50=height.
2) 353430000
17671.5=content.

Article XLVI. To find the solidity of the frustum of a parabolic conoid; the greater diameter AB, *fig.* 50 the lesser CD, and the perpendicular height EF, being given.

Rule.—To the square of the diameter of the greater end AB, add the square of the diameter of the lesser end CD; multiply the sum by .3927, (being one half of .7854,) and the product by the height EF will give the solidity required.

What is the solidity of a parabolic frustum, the diameter of the greater end AB being 5 feet, the lesser end CD 4 feet, and the distance of the ends EF 4 feet.

AB 5×5=25	3927
CD 4×4=16	41
41 sum.	3927
	15708
	161007
	4=the distance EF.
	64.4028=content.

Article XLVII. To find the solidity of a parabolic spindle.

Rule.—Multiply the square of the middle diameter by .41888 (being $\frac{8}{15}$ of .7854) and that product by its length; the last product is the solid content.

What is the solidity of a parabolic spindle whose middle diameter AB *fig.* 51 is 30 inches, and its length CD 5 feet.

AB=30	41888
30	900
900=square.	37699200
	5=length.
	144)1884960(13.09 content.
	144
	444
	432
	1296
	1296

Article XLVIII. To find the solidity of a sphere or globe.

Rule.—Multiply the cube of the diameter by .5236 and the product is the solidity.

What is the solidity of a globe whose diameter AB *fig.* 52 is 4 feet.

AB=4	5236
4	64
16	20944
4	31416
64 cube.	33.5104 content.

NOTE.—If it be a sphere of a semi-circle as in *fig.* 53, first find the whole solidity as in *fig.* 52, and divide the product by 2, and the quotient will be the answer.

ARTICLE XLIX. To find the solidity of a segment of a globe.

Rule.—To three times the square of the semi-diameter of the base, add the square of the height, then multiply that sum by the height, and then the product multiplied by .5236 will give the solidity.

What is the solidity of a spherical segment; the diameter of the base A B *fig.* 54 being 18 feet, and the height of the segment C D is 4 feet.

$\frac{1}{2}$ of 18 is=9 the semi-diameter.
9
81=square.
×3
243
4×4=16
259
×4=height.
1036

5236
1036
31416
15708
52360
542.4496=solid content.

ARTICLE L. To find the solidity of a spherical zone, the radius A B *fig.* 55, and C D of the two parallel circles at the end being given and their distance B C.

Rule.—To the square of the two radii add one third of the square of the height; multiply the sum by the height, and the product by 1.5708, will give the solidity.

What is the solidity of a spherical zone, whose greater radius A B is 10 feet, the lesser C D 8 feet, and the height or distance of the ends B C is 6 feet.

A B 10×10=100
C D 8× 8= 64
Add 12
176
×6=height.
1056

6
6
3) 36=square.
12=$\frac{1}{3}$ of the sq.

1.5708
1056
94248
78540
157080
1658.7648=content.

ARTICLE LI. To find the solidity of a spheroid, the fixed axis and the revolving axis being given.

Rule.—Multiply the square of the revolving axis by the fixed axis, and that product by .5236 for the solidity.

What is the solidity of a prolate spheroid whose transverse axis C D *fig.* 56, is 10 feet, and the conjugate A B 6 feet.

A B=6
6
36
C D=10
360

5236
360
314160
15708
188.4960=content.

ARTICLE. LII. To find the solidity of an annulus or a cylindric ring whose thickness and inner diameter are known.

Rule 1.—To the thickness of the annulus, add the inner diameter; multiply the sum by the square of the thickness, and the product by 2.4674 will give the solidity sought.

What is the solidity of an annulus, whose inner diameter A B *fig.* 57, is 8 inches, and the thickness of the annulus B C is 3 inches.

8
3
11
3×3=9
99

2.4674
99
222066
222066
244.2726=content.

Rule 2.—Multiply the circumference round the middle of the annulus, or that circle generated by the centre of the generating circle, by the area of the generating circle, and the product will give the solidity.

NOTE.—This last method will give the solidity of any part of an annulus or ring comprehended between any two planes passing through the fixed axis.

OF THE FIVE REGULAR SOLIDS.

DEFINITIONS I. A regular solid, is a body that either may be inscribed or circumscribed by a sphere, in such a manner as to be contained under equal and similar planes; alike posited, and equally distant from the centre of the sphere.

II. The *Tetraedron,* is contained under 4 equilateral triangles.

III. The *Hexaedron,* is contained under 6 equal squares.

IV. The *Octaedron,* is contained under 8 equilateral triangles.

V. The *Dodecaedron,* is contained under 12 equilateral and equiangular pentagons.

VI. The *Icosaedron,* is contained under 20 equilateral triangles.

TO FIND THE SUPERFICES, AND SOLIDITY OF ANY OF THE FIVE REGULAR BODIES.

To find the superfices. Multiply the area (taken from the following Table) by the square of the linear edge of the solid, for the superfices.

To find the Solidity. Multiply the tabular solidity by the cube of the linear edge, for the solid content.

A Table of the Surfaces and Solidities of the five regular Solids.

No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64573	7.66312
20	Icosaedron	8.66025	2.18169

EXAMPLE 1.—If the linear edge or side of a tetraedron be 3, required its superficial and solid content.

Thus 1.73205× 9=15.58845 superficies.
And 0.11785×27= 3.18195 solidity.

EXAMPLE 2. What is the surface and solidity of a hexaedron whose linear side is 2?

Answer $\begin{cases}\text{superficies}=24\\ \text{solidity}\ =8\end{cases}$

EXAMPLE 3.—Required the superficies and solidity of the octaedron, whose linear side is 2.

Answer $\begin{cases}\text{superficies}=13.8564\\ \text{solidity}\ =3.7712\end{cases}$

EXAMPLE 4. What is the superficies and solidity of the dodecaedron, whose linear side is 2?

Answer $\begin{cases}\text{superficies}=82.58292\\ \text{solidity}\ =61.30496\end{cases}$

EXAMPLE 5. What is the superficies and solidity of an icosaedron, whose linear side is 2?

Answer $\begin{cases}\text{superficies}=34.641\\ \text{solidity}\ =17.45352\end{cases}$

For finding Convex surfaces of solids.

ARTICLE LIII. To find the convex surfaces of a cube, or a square box as in *fig.* 33, *Pl.* 28.

Rule. Find the surface or area of one of the sides as in section first, and multiply it by 6, and the product will be the whole surface.

What is the convex superficies of a cube or a box whose sides are 2 feet 4 inches.

2-4
2-4
———
9-4
4-8
———
5-5-4
6
———
32-8-0 = the whole surface.

Article LIV.—To find the convex surfaces of a parallelopipedon, as in *fig.* 34.

Rule. Find the surfaces of the depth, breadth, and one of the ends, and double it, or multiply it by 2, which is the same thing.

What is the convex superficies of a parallelopipedon whose depth is 2 feet 8 inches, and breadth 3 feet 2 inches, and 4 feet 7 inches in length?

2-8
3-2
———
5-4
8-0
———
8-5-4 = the surface of one end.

2- 8
3- 2
———
5-10 = depth and breadth.
4- 7 = length.
———
3-4-10
23-4
add 8-5-4
———
35-2-2 = whole surface.

Article LV.—To find the convex surfaces of any prism, as in *figs.* 35 *and* 36.

Rule.—Multiply the circumference of the base by the length of the prism, the product will be the upright surface, to which add the area of the bases; the sum will be the whole surface.

Article LVI.—To find the convex surfaces of a cylinder.

Rule.—Multiply the circumference by the length of the cylinder, as in *figs.* 37 *and* 38, the product will be the surface of the length, thence add the area, or the surfaces of the two ends, and the sum will give the whole surface.

Article LVII.—To find the convex superficies of a pyramid or a right cone, as in *figs.* 39, 40, 41 *and* 42, the circumference and slant side being given.

Rule.—Multiply the circumference of the base by the slant side of the cone, and half the product will be the area; or multiply the slant height by half the circumference of the base, and the product will be the upright surface; to which the area of the base may be added, for the whole surface.

Article LVIII.—To find the convex surface of a frustum of a right cone, as in *figs.* 43, 44, 45, 47, and 48, the circumferences of both ends being given, and the slant side of the cone.

Rule. Multiply the sum of the circumferences by the slant side of the cone, and half the product will be the area.

Or multiply half the sum of the perimeters of the two bases by the slant height, and to the product add the areas of the two bases for the whole surface.

Article LIX.—To find the superficies of a sphere or globe as in *fig.* 52, the greatest circumference being given.

Rule.—Multiply the square of the circumference by .3183, and the product will be the superficies.

Example.—What is the superficies of a globe, the greatest circumference being 10.6 feet?

Thus 10.6×10.6×.3183=35.764188 the superficies required.

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Article LX.—To find the convex superficies of the segment, sphere, or globe, as in *fig.* 54, the diameter of the base of the segment, and its height, being given.

Rule.—To the square of the diameter of the base, add the square of twice the height, and the sum multiplied by .7854 will give the superficies.

Example.—What is the convex surface of the segment of a globe, the diameter of the base being 17.25 feet, and the height 4.5 feet?

2×4.5 = 9 twice the height.
9× .9 = 81 square of twice the height.
———2
17.25 = 297.5825 square of the diameter of the base.

Then 297.5625+81×.7854 = 297.3229975 the superficies required.

Article LXI.—To find the convex surfaces of a spherical zone, the diameters of the ends and their distance being given, as in *fig.* 55.

Rule.—Find the diameter of the sphere by Arts. 21, and 22, in section first of superficies; then multiply the diameter of the sphere, and the distance of the parallel ends of the zone together, and the product by 3.1416, will be the superficies required.

Example.—In a spherical zone the distance of the parallel ends being 4 inches, the diameter of the greater end 24 inches, and that of the lesser end 20 inches, what is the convex superficies, when the centre of the sphere is without the zone?

The distance of the greater chord from the centre, will be found to be 3.5 inches by Art. 21.

The radius will be found to be 25 inches, by Art. 22, or the diameter 50 inches.

Then 50×4×3.1416–628.92 the answer.

Note.—If the diameter is given, find the circumference, and proceed as before.

Article LXII.—To find the convex superficies of an annulus, or ring, as in *fig.* 57, whose thickness and inner diameters are known.

Rule 1.—To the thickness of the ring add the inner diameter; multiply the sum by the thickness, and the product by 9.869 will give the superficies required.

Rule 2.—Multiply the circumference of the generating circle by the circumference round the middle of the ring, or that line generated by the centre of the generating circles, and the product will be the area.

Note.—This last method will give the convex superficies of any part of an annulus or ring, comprehended between two planes passing through the fixed axis.

Example.—What is the convex superficies of an annulus or ring, whose inner diameter is 8 inches, and the thickness 3 inches?

Thus 3+8×3×9.869–325.677 the superficies required.

Of measuring irregular surfaces and solids.

Definition.—An irregular surface or solid, is such a surface or solid as have their bounds by lines or surfaces in any manner whatever, of no particular kind of form or shape, but merely accidental, according as they are to be found or given.

Article LXIII.—To measure any irregular surface whatever by means of equidistant ordinates.

Rule. 1. To the half sum of the two outside ordinates, add the sum of all the other remaining ordinates; multiply the whole

sum by the distance between any two ordinates, and the product will be the superficial content.

Example 1. Let *fig.* 58, *pl.* 28, be the curve proposed, whose equidistant ordinates, A B, C D, E F, G H, I K, L M, and N O, are respectively 5 feet, 5 feet 6 inches, 6 feet, 7 feet, 9 feet, 10 feet, and 8 feet, and the distance of A C, C E, E G, &c. is 3 feet, required the area of the curve.

A B = 5
N O = 8
2) 13
6.6 = half the sum of the outside ordinates.
C D = 5.6
E F = 6.0
G H = 7.0
I K = 9.0
L M = 10.0
44.0
3
132 = superficies.

Example 2. Let A B C D, *fig.* 59, be a circle whose diameter A C or B D is 10 feet, it is required to find the area by means of equidistant ordinates, marked 3 feet, 4 feet, 4.5 feet, 4.9 feet, and 5 feet, being at the distance of 1 foot from each other.

0
5
2) 5
2.5 half sum of the outside ordinates.
3
4
4.5
4.9
18.9 area of one quarter.
4
75.6 feet, area of the whole.

If the diameter, which is 10 feet, be multiplied by .7854, the product, 78.54, will be the area. From hence it appears that the mode of operation by means of equidistant ordinates, is very near the truth in measuring irregular planes; for it will produce the area of a circle, which is one of the most oblique curves possible, as the ends raise quite perpendicular to the axis, from only 10 equidistant spaces within the $\frac{1}{26}$ part of the truth; and would be still nearer when applied to measuring any plane surface, where it is bounded partly by concave and partly by convex curves; because, if wholly bounded by a convex curve, or curves, the area will be something less than the truth, but if bounded by a concave curve, or curves, the area will be something greater than the truth; and if the extremities of the ordinates are joined by straight lines, the area so found will be exactly true; but the following is a method of approximation still nearer the truth, whether the curve be convex or concave to the axis.

Rule 2.—Divide the given curve, by ordinates, into any even number of equal parts, then add into one sum four times the sum of all the even ordinates; twice the sum of all the odd ordinates except the first and last, and also the first and last ordinates; and if one third of that sum be multiplied by the common distance between any two ordinates, the product will be the answer.

Example 1.—Let *fig.* 58 be a curve of any kind, whose equidistant ordinates A B, C D, E F, G H, I K, L M, and N O, are respectively 5 feet, 5 feet 6 inches, 6 feet, 7 feet, 9 feet, 10 feet, and 8 feet, and the distance between the ordinates is 3 feet, required the area of the curve.

C D, G H, and L M, will be the even ordinates; that is, the second, fourth, and sixth; E F and I K, the odd ordinates; that is, the third and fifth; A B and N O, the first and last.

f i	*f*
C D = 5.6	E F = 6
G H = 7.0	I K = 9
L M = 10.0	
	15
22.6	2
4	
	30

90.0 four times the sum of the even ordinates.
30.0 twice the sum of the odd ordinates.
5.0 first ordinate.
8.0 last ordinate.
3) 133.0 sum
44.4
3
133.0 the area or superficial contents.

Now by comparing this area, viz. 133 feet, with the area found in Rule 1, Example 1, viz. 132 feet, there appears to be a difference of 1 foot; but the last method is the most correct.

Example 2.—Let A C E G I L N, *fig.* 60, be a concave curve, whose equidistant ordinates A, B C, D E, F G, H I, K L, and M N, are respectively 0, 1, 3, 6, 10, 15, 21, and the common distance 2, required the area.

By Example 1.
0
21
2) 21
10.5
1
3
6
10
15
45.5
2
91.0 the area greater than the truth.

By Example 2.

3	1
10	6
	15
13	
2	22
	4
26	

88 four times the sum of the even ordinates.
26 twice the sum of the odd ordinates.
0 first ordinate.
21 last ordinate.
3) 135
45
2 common distance.
90 the area very near the truth.

Example 3.—Let A H I, *fig.* 61, be a parabola, whose ordinates A, B C, D E, F G, and H I, are respectively 0, 7, 12, 15, and 16, and their common distance 6, required the area of the curve.

```
  7          12
 15           2
 ——          ——
 22          24
  4
 ——
 88 four times the sum of the even ordinates.
 24 twice the sum of the odd ordinates.
 16 sum of the end.
 ——
3)128
  42⅔
   6
 ———
256 the true area.
```

Article LXIV.—To find the superficial contents of a mixed figure, partly a curve and partly right lined.

Rule.—Find the area of the curve part of the figure by the last example, by dividing it into equidistant ordinates; divide the right lined parts of the figure by ordinates drawn through every angle, which will divide the right lined parts of the figure into trapezoids and triangles; find the area of each part, according to their respective rules; add the areas of all the parts together, and the sum will give the area of the whole figure.

Example.—Let A B K N O R S, *fig.* 62, be the figure proposed, to find its area.

As the end A B turns round nearly perpendicular to the base A S, draw the ordinate C B in such a manner, as it may cut off the most perpendicular part of the curve A B at the end, and divide it by ordinates, which are respectively 1, 2, $1\frac{1}{2}$, 1, 0, at the distance of 3 from each other; the part C B K L of the equidistant space is also divided into four equal parts, between the first and last ordinates B C, K L, by the ordinates E D, G F, H I, and L K, which are respectively 12, 13, 12, 10, 9, and their common distance 4; the other parts of the figure are divided into 3 trapezoids, K L M N, M N O P, O P Q R, and the triangle Q R S, by ordinates from the angles at K, N, O, and R; the whole figure being thus prepared, by dividing it into curvilineal parts, trapezoids, and triangles, each part will be measured according to their respective rules. The measures or dimensions are marked on their respective places on the figure; the contents of each part is computed separately, as is shown in the following operation.

```
 1½        2          13     12     9
 2         1          10      2    16
 —         —          —      —     -
 3         3          23     24  2)25
           4           4           —
           —           —           12½
          12          92            4
           3          24           —
           1          12           50 { area of the tra-
           —           9              { pezoids K L M N.
        3)16         ———
           —      3)137
           5⅓        45.6
           3          4
           —         ——
          16 { area of the   182.4 area of the part C B K L.
             { part A B C.
            16              13            2) 14
  16.0      13              14                7
 182.4      —               —                 —
  50.0    2)29            2)27                7
  43.5      14.5            13½               —
  27.0       3               2               49 { area of the tri-
  49.0     ———               —                  { angle Q R S.
  ————     43.5 area of M N O P  27 area of O P Q R.
 367.9 sum of the areas, or contents of the whole figure.
```

Article LXV.—To find the superficies of a groin.

Rule 1.—When the sides of the groin are semicircles, to the area of the base add $\frac{1}{7}$th part of itself, and the sum will give the superficies required.

Example 1.—What is the curve superficies of a circular groin, each side of the square, base being 14 feet, as A B C D, *fig.* 63.

```
    14                    7) 196(28 = to 1/7
    14                       14
    —                        ———
    56                       56
   14                        56
   —
   196 area of the base.
add 28
  ———
   224 area of the groin.
```

Rule 2.—When the groin stands upon a rectangular plan, the sides being either segments of circles or segments of an ellipsis.

The area of each two opposite parts of the surfaces of the cylinders, or cylindroids may be computed in the following manner, viz. let A B C D, *fig.* 63, be the plan of the groin; A C and B D are the intersection of the planes of the diagonals; M N Q O is one of the cylindrical, or cylindroid surfaces stretched out on a plane; H V I is one of the side arches. Then to find the area of any two opposite quarters of the cylindrical, or cylindroidal surfaces to the arch line H V I, standing over B C on the plan; that is, N M when stretched out on the plane, add four times the arch standing over F G on the plan taken in the middle between the end B C, and the vertex at E, that is, Q R when stretched out on the plane; multiply one third of the sum by E S, or P O, which is equal to it, and the product will be double the area of the two opposite cylindrical or cylindroidal surfaces, standing over A E D and B E C.

Example 2.—Let A B C D be the plan of a groin, the sides A B and B C are each equal to 8 feet; let M N, the length of the arch H V I, standing over B C on the plan, be 10 feet, and P O, equal to E S, be 4 feet; that is the distance measured along from the vertex at E, to either of the ends at S, and Q R the length of the arch over F G, be 4 feet; required the superficies of the groin.

```
   4
   4
   —
  16 four times Q R.
  10 the end.
  —
3)26
 ———
   8-8 inches.
   4
  ———
  34-8 area of the two opposite parts A E D, and B E C.
   2
  ———
  69-4 area of the whole groin standing over ABCD on the plan.
```

Article LXVI.—To find the Tonnage of a Ship.

"By a law of the Congress of the United States of America, the tonnage of a ship is to be found in the following manner.

"If the vessel be double-decked, take the length thereof from the fore part of the main stem to the after part of the stern post above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel; and deduct from the length three-fifths of the breadth, multiply the remainder by the breadth, and the product by the depth; divide this last product by ninety-five, and the quotient will be the true contents or tonnage of such vessel.

"If the vessel be single-decked, take the length and breadth

as above directed, in respect to a double-decked vessel, and deduct from the length three-fifths of the breadth, and taking the depth from the under side of the deck plank to the ceiling in the hold; multiply and divide as aforesaid, the quotient will be the true contents or tonnage of such vessel."

Example.—Suppose a double-decked vessel is 98 feet, and the breadth 30 feet; what is her tonnage?

```
  30                 Length=98          95)36000(378-94 tonnage.
  ×3                    18=3/5              285
 ----                  -----               -----
 5)90                    80                 750
 ----                   ×30 = breadth.      665
  18=3/5 of breadth.    -----              -----
                        2400                850
           1/2 of 30 = 15                   760
                       -----               -----
                       12000                900
                       2400                 855
                       -----               -----
                       36000                450
                                            380
                                           -----
                                             70
```

Carpenters, in finding the tonnage, multiply the length of the keel by the breadth of the main beam, and the depth of the hold in feet, and divide the product by 95; the quotient is the number of tons. In double-decked vessels, half the breadth is taken for the depth.

OF GAUGING.

Gauging is the art of measuring and finding the contents of all kinds of vessels, in gallons or cubic inches; such as casks, brewer's vessels, &c.

Having found the number of cubic inches in any body by the rules given in (section 2 of solids.) You may thence determine the contents in gallons, bushels, &c. by dividing that number of cubic inches in a gallon, bushel, &c. respectively.

A *wine gallon*, by which most liquors are measured, contains 231 cubic inches. A *beer gallon*, by which beer, ale, and a few other liquors are measured, contain 282 cubic inches. A bushel of corn, malt, &c. contains 2150.4 cubic inches.

Note.—In all the following rules, it will be supposed that the dimensions of the body are given in inches and decimal parts of an inch.

Article LXVII. To find the number of gallons or bushels in a body of a cubic form, (see *fig.* 33, *pl.* 28.)

Rule.—Divide the cube of the side by 231, the quotient will be the answer in wine gallons, or by 282, and the quotient will be the answer in beer gallons; or by 2150.4, and the quotient will be the number of bushels.

Example.—Required the number of wine gallons contained in a cubic cistern, the length of whose side is 60 inches. Multiplying 60 by itself, and the product again by 60 gives the solidity 216000; which divided by 231, gives the content $935.\frac{6}{100}$ wine gallons.

In the like manner the content of any other figure may be found, by finding the number of cubic inches in the body by the rules already taught in section second of mensuration; then bring it into gallons, bushels, &c. by dividing the number of cubic inches found in the body by 231 for wine gallons, and by 282 for ale gallons; and by 2150.4 for bushels; or you may multiply the number of cubic inches found in the body by 004329 for wine gallons, and the product will be the number of gallons; and by 003546 for ale gallons; though the shortest and best method of gauging a frustum of a cone, and a body in a cylindrical form will be found by the following examples.

Article LXVIII.—To find the number of gallons or bushels contained in a body of a cylindrical form, (see *fig.* 37, *pl.* 28.

Rule.—Multiply the square of the diameter by the height of the cylinder, and divide the product by 294, the quotient will be the number of wine gallons; if you divide by 359, the quotient will be the number of ale gallons; and if you divide by 2738, the quotient will be the number of bushels.

Note—These divisors are found by dividing 231,282 and 2150.4 by 7854.

Example.—How many wine gallons is there in a cylinder, whose diameter at its base is 30 inches, and the length 50 inches?

```
         30 = diameter.
        ×30
        ----
        900 square of the diameter.
        ×50 = height.
   294)45000(153.06 gallons.
       294 etc.
```

Article LXIX.—To find the number of gallons or bushels contained in a body of the form of a frustum of a cone, (see *fig.* 47, *pl.* 28.)

Rule.—Multiply the top and bottom diameters together, and to the product add one third of the square of the difference of the same diameters: multiply this sum by the perpendicular height, and divide the product by 294 for wine gallons, by 359 for ale gallons, and by 2738 for bushels.

Example.—How many wine gallons is there in a frustum of a cone whose greatest diameter is 70 inches, the lesser 61, and the height 72?

```
  70               61=the lesser diameter.    294)309384(1052.32=galls.
  61              ×70-the greater diameter.       294
  --              ----                           ----
   9 difference.  4270                           1538
  ×9          add   27                           1470
 ---              ----                           ----
3)81(27=1/3 of sq. 4297                           684
  6                ×72=height.                    588
  --              ----                           ----
  21              8594                            960
  21             30079                            882
                 ------                          ----
                 309384                           780
                                                  588
                                                 ----
                                                  192
```

Note.—It may be proper here to remark, that cisterns, frustums, &c. built either of wood, brick or stone, are generally made by the hogshead; and in gauging the above, we bring it first into wine gallons, then allow one hundred gallons for a hogshead.

Cisterns that are built of brick, are mostly built in the form of a cylinder, and a cylindroid, (see *figs.* 37 *and* 38, *pl.* 28,) and are arched over. And in gauging the above, if it be in the form of a cylinder, find the contents of the straight part, by Art. 38, or by Art. 67; and if the arch be a semicircle, find the contents by Art. 48; but if it be a segment of a circle, find the contents by Art. 49, and add them together, which will give the whole content.

But if it be in the form of a cylindroid, find the contents of the straight part by Art. 39; and the contents of the arch by Art. 52, then add them together, which will give the whole content; and when you have obtained the content in cubic inches, thence you may determine the contents in gallons, bushels, &c. (by the rules given at Art. 64,) by dividing the number of cubic inches by 231, the quotient will be the answer in wine gallons, or by 282, the quotient will be the answer in beer gallons, or by 2150.4, and the quotient will be the number of bushels, &c.

TRIGONOMETRY.

Trigonometry is that branch of the general science of Geometry, which treats of the properties and relations of certain straight lines, drawn in and about a circle, and also teaches to compute the sides and angles of triangles. It is divided into two parts, plane and spherical.

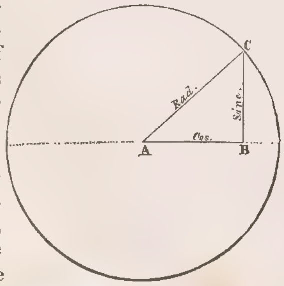
Plane Trigonometry treats only of *Rectilinear* Triangles; while Spherical Trigonometry treats of triangles, formed by the intersections of three great circles upon the surface of a sphere.

Scarcely any department of mathematics is more important, or more extensive in its application to the useful purposes and business of life. By Trigonometry, the builder determines the length of braces, rafters, beams, the projection of roofs, and whatever appertains to the architecture of bridges or arches, and to the proper delineations of plans and drawings. There is no branch of science so essential to the practical architect, as that of trigonometry. The science of architecture is, indeed, the science of trigonometry reduced to examples. Without the latter, the former would have remained, at best, but as an art without rules, exactness, or order.

The *terms* of this science, such as *Sines*, *Tangents*, *Secants*, &c. have already been explained, among the definitions of Geometry. It remains now to show their application, and the manner of computing them. Each of these terms, under different circumstances, is but another name for the perpendicular, the base, and the hypothenuse of a triangle. In one triangle, for instance, the perpendicular becomes the *Sine*, and in another perfectly similar to the former, it is as often the *Tangent* of an angle. The same thing is true of the base, also. It becomes, first of all, necessary therefore to know, under what circumstances the respective sides of a triangle become sines, tangents, and secants, or co-sines, co-tangents, and co-secants.

This depends on which side of the triangle the circle is described: for if any side whatever be made radius, each of the other sides will be the sine, tangent or secant, of the arc described by this radius.

Fig. 1.

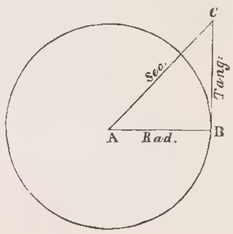


Case 1. In every right-angled triangle, if the hypothenuse be made radius, one of the sides will be a *sine* of its opposite angle, and the other side a *co-sine* of the same angle. Thus:

The triangle A B C being constructed, and a circle described around the centre A, with the radius A C, then the perpendicular B C will be the *sine* and the base A B the *co-sine* of the angle at A. But as the sine of either of the acute-angles of a right-angled triangle is the co-sine of the other, and the contrary; therefore the perpendicular B C, being the *sine* of the angle at A, is also *co-sine* of the angle at C.

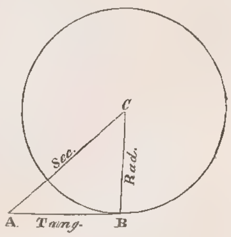
Case 2. If either the base or perpendicular be made radius, the other will be a *tangent* of its opposite angle, and the hypothenuse will be a *secant* of the same angle; that is, of the angle between the secant and the radius. Thus:

Fig. 2.



Let an arc or circle be described around A, with the base A B, as radius, then the perpendicular B C will be the *tangent*, and the hypothenuse A C the *secant* of the angle at A. And because the side which is the sine, tangent, or secant, of one of the acute angles of a right-angled triangle, is the *co-sine*, *co-tangent*, or *co-secant* of the other angle, therefore the perpendicular B C, being tangent of the angle at A, is also *co-tangent* of the angle at C. And the hypothenuse A C, being secant of the angle at A, is, also, *co-secant* of the angle at C.

Fig. 3.



Case 3. If the perpendicular B C be made radius, with the centre at C, then the base A B will be the *tangent*, and the hypothenuse A C, (as before) the *secant* of the angle at C. And, because every side of a right-angled triangle, which is the sine, tangent, or secant of one of the acute angles, is the *co-sine*, *co-tangent*, or *co-secant*, of the *other* acute angle, therefore, the base A B being *tangent* of the angle at C, is also *co-tangent* of the angle at A. And the hypothenuse A C, being *secant* of the angle at C, is also co-secant of the angle at A.

The solution of all the cases in right-angled trigonometry, depends upon the principle that the radius of one circle, is to the radius of any other, as the sine, tangent or secant, in one, is to the sine, tangent or secant, of the same number of degrees, in the other. It is also plain, that if the side of any triangle, which is made radius should remain unchanged, the other two sides would vary according to the size of the angle; that is, they would be *longer* or *shorter*, according as they subtended a greater or less number of degrees. Upon this principle, the table of sines, tangents or secants, was computed, to radius 1, for every 5′ of a degree.

In every triangle, there are six parts, three sides, and three angles. Of these parts *three* must be given, including a side, to enable us to find the rest. To find a *side,* we begin the statement of the problem with an *angle.* To find an *angle,* we must begin with a *side,* as in the following rules.

1. *To find a Side.*—Call any one of the sides of a triangle radius, and write upon it the word radius; observe whether the other sides become sine, tangent, or secant, and write these words on them, or suppose them to be written, as in some one of the three last figures. Consider the word thus written upon each side, as the *tabular name* of that side. Then institute the following proportion:

As the *tabular name** of the given side,
Is to the *length* of the side,
As is the tabular name* of the required side,
To the length of the required side.

2. *To find an Angle.*—One of the given sides must be made radius, then institute the following proportion:

As the length of the given side made radius,
Is to its tabular name,* that is, radius;
So is the length of the other given side,
To its tabular name.*

Rule.—Add together the logarithms of the second and third terms, and from their sum subtract the logarithm of the first term. The remainder will be the logarithm of the answer.

Note.—From the property of a plane triangle, that the three angles are together equal to two right angles, or 180 degrees, the following useful corollaries arise:

1. When two angles of a triangle are given, the third is also said to be given; for it is the supplement of the other two, and may be found by subtracting their sum from 180 degrees.

2. When one angle of a triangle is given, the sum of the other two may be found, by subtracting the given angles from the two right angles, or 180 degrees.

3. If one angle of a triangle be right, the other two are acute, and together make another right-angle; and if one of the acute angles be given, the other is also given, being the complement of the other given one, or what it wants of 90 degrees.

4. The sine or tangent of any angle is to the side opposite to it, as the sine or tangent of any other angle is to its opposite side, and the contrary.

PROBLEM I.

GIVEN THE ANGLES AND THE HYPOTHENUSE OF A RIGHT-ANGLED TRIANGLE, TO FIND THE BASE AND PERPENDICULAR.

Example 1. In the triangle A B C, right-angled at B, suppose the angle at A=50 degrees 30 minutes, and the hypothenuse, A C=125 feet, or yards; required the sides A B and B C.

Here, the hypothenuse being the *given* side, must be made radius; and B C will then become the sine of the angle at A, and A B the co-sine of the same angle; *see fig.* 1.

To find the perp. B C		To find the base A B	
As rad. or sine of 90°	10.000000	As radius, sine of 90°	10.000000
Is to hypothenuse 125	3.096910	Is to hyp. A C 125	2.096910
So is the sine of ∠A 50° 30′	9.827406	So is Cos. of ∠ A 50° 30′	9.803511
	10.984316		11.900421
(Subtract 1st term)	10.000000	(Subtract 1st term)	10.000000
To perp. B C 96.4	1.984316	To base A B 79.51	1.900421

* Which will be either Sine, Tangent, or Secant, &c.

Note.—Where the first term is radius, it may be subtracted from the sum of the other two, by merely rejecting 10 in the index, without the trouble of setting it down a second time.

To find the angle at C, we have only to subtract the angle at A from 90°. Thus 90°—50° 30′=39° 30′ which is the angle at C.

2. *Making the base radius,* the perpendicular B C becomes the tangent of the angle A, and A C becomes the secant of A; *see fig.* 2. Thence,

To find B C		To find A B	
As secant of it 50° 30′ =	10.196489	As secant of A 50° 30′ =	10.196489
Is to hyp. A C 125	2.096910	Is to hyp. A C 125	2.096910
So is tangent of A 50° 30′	10.083896	So is radius 90°	10.000000
	12.180806		12.096910
(Subtract 1st term)	10.196489	(Subtract 1st term.)	10.196489
To perp. B C 96.45	1.984317	To base A B 79.51	1.900421

3. *Making the perpendicular radius,* the base A B will be the tangent of the angle at C, or co-tangent of A; and the hypothenuse A C, will be the secant of C, or the co-secant of A; *see fig.* 3. Hence,

To find B C		To find A B	
As co-secant A 50° 30′ =	10.112594	As co-secant of A 50° 30′=	10.112594
Is to hyp. A C 125	2.096910	Is to hyp. A C 125	2.096910
So is radius 90°	10.000000	So is cotang. of A 50° 30′	9.916104
	12.096910		12.013014
(Subtract 1st term)	10.112594	(Subtract 1st term)	10.112594
To perp. B C 96.45	1.984316	To base A B 79.51	1.900420

PROBLEM II.

BY GUNTER'S SCALE.

1st. Extend the compasses, from 36° 52′ the complement of A to 90, on the line of sines; that extent will reach from 288 to 48°= the hypothenuse A C.

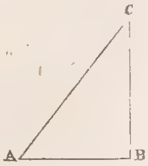
2d. Extend the compasses from 36° 52′ to 53° 8′ on the line of sines; that extent will reach from 288 to B C 384, on the line of numbers.

In working the several cases of right-angled trigonometry by Gunter's scale, we shall always suppose the hypothenuse radius (where it can be done) because it is the most simple of the three.

GIVEN THE ANGLES AND THE BASE, TO FIND THE HYPOTHENUSE AND THE PERPENDICULAR.

In the triangle A B C, right-angled at B, given the angle at A, 53° 8′, and the base A B = 288, to find the hypothenuse A C, and the perpendicular B C.

Fig. 4.



1. *Making the hypothenuse radius,* the perpendicular B C, will be the sine of the angle at A, and the base A B the co-sine. *See fig.* 1. Hence,

To find A C.		To find B C.	
As co-sine A 53° 8′ =	9.778119	As cos. A 53° 8′ =	9.778119
Is to base A B 288	2.459392	Is to base A B 288	2.459392
So is radius 90°	10.000000	So is sine A 53° 8′	9.903108
	12.459392		12.362500
(Subtract 1st term)	9.778119	(Sub. 1st. term)	9.778119
To hyp. A C 480	2.681273	To perp. B C 384	2.584381

2. *Making the base radius*, the perpendicular B C will be the tangent of the angle at A, and the hypothenuse will be secant of the same angle. *See fig.* 2. Hence,

To find A C		To find B C	
As radius 90° =	10.000000	As radius 90° =	10.000000
Is to the base A B 288	2.459392	Is to the base A B 288	2.454392
So is secant of A 53° 8′	10.221881	So is tang. of A 53° 8′	10.124990
To hyp. A C 480	2.681273	To perp. B C 384	2.584382

3. *Making the perpendicular radius*, the base A B will be the tangent of C, or the co-tangent of A; and the hypothenuse A C will be the secant of C, or co-secant of A. *See fig.* 3. Hence,

To find A C		To find B C	
As co-tangent A 53° 8′ =	9.875010	As co-tangent A 53° 8′ =	9.875010
Is to the base A B 288	2.459392	Is to the base A B 288	2.459392
So is co-secant A 53° 8′	10.096892	So is radius 90°	10.000000
	12.556284		12.459392
(Subtract 1st term)	9.875010	(Subtract 1st term)	9.875010
To hyp. A C 480	2.681274	To hyp. A C 384	2.584382

BY GUNTER'S SCALE.

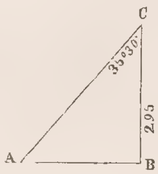
1. Extend the compasses from 90° to 50° 30′ on the line of sines, and then that extent will reach from 125 to 96.45 = B C on the line of numbers.

2. Extend the compasses from 90° to 39° 30′ the complement or co-sine of the angle A, and that extent will reach from the hypothenuse A C = 125, to the base A B = 79.51, on the line of numbers.

PROBLEM III.

GIVEN THE ANGLES AND THE PERPENDICULAR TO FIND THE HYPOTHENUSE AND THE BASE.

Fig. 5.



EXAMPLE. 1. In the triangle A B C, right-angled at B, let the angle at C be 35° 30′, and the perpendicular B C 295 feet; required the base A B and the hypothenuse A C.

1. *Making the hypothenuse radius*, the perpendicular B C becomes the sine of A, and the base A B the co-sine. *See fig.* 1. Hence,

To find A B		To find A C	
As co-sine of C 35° 30′ =	9.910686	As co-sine of C 35° 30′	9.910686
Is to perp. B C 295	2.469822	Is to perp. B C 295	2.469822
So is sine C 35° 30′	9.763954	So is rad. 98	10.000000
	12.233776		12.469822
(Sub. 1st term)	9.910686	(Sub. 1st term)	9.910686
To base A B 210.4	2.323090	To hyp. A C 362.3	2.559136

NOTE.—It is customary to subtract the 1st term from the sum of the other two, *where it stands*, without writing it down the second time, as, for the sake of plainness, we have done in the foregoing examples.

2. *Making the base radius*, B C will be the tangent of A, and A C the secant thereof. *See fig.* 2. Hence,

To find A B		To find A C	
As co-tang. of C 35° 30′ =	10.146732	As co-tangent C 35° 30′ =	10.146732
Is to perp. B C 295	2.469822	Is to perp. B C 295	2.469822
So is radius 90	10.000000	So is co-secant C 35° 30′	10.236046
	12.469822		12.705868
To base A B 210.4	2.323090	To hyp. A C 352.3	2.559136

3. *Making the perpendicular radius*, the base will be the tangent of C, or the co-tangent of A; and the hypothenuse will be the secant of C, or co-secant of A. *See fig.* 3. Hence,

To find A B		To find A C	
As radius or sine 90° =	10.000000	As rad. or sine 90° =	10.000000
Is to perp. B C 295	2.469822	Is to perp. B C 295	2.469822
So is tangent C 35° 30′	9.853268	So is secant C 35° 30′	10.089314
	12.328090		12.559136
To base A B 210.4	2.323090	To hyp. A C 362.3	2.559136

BY GUNTER'S SCALE.

1. Extend the compasses from 35° 30′ to its co-sine 54° 30′ on the line of sines; and that extent will reach from the perpendicular 295 to the base 210.4 on the line of numbers.

2. Extend the compasses from 54° 30′ the complement of C, to 90°, on the line of sines; and that extent will reach from the perpendicular 295, to the hypothenuse 362.3 on the line of numbers.

PROBLEM IV.

GIVEN THE HYPOTHENUSE AND THE BASE TO FIND THE ANGLES AND THE PERPENDICULAR.

In the right-angled plane triangle A B C, *fig.* 4, given the hypothenuse A C = 480, and the base A B = 288, to find the angles A and C, and the perpendicular B C.

1. *Making the hypothenuse radius*, B C will be the sine of the angle A, and A B the co-sine of the same angle. *See fig.* 1. Hence,

To find the angle A.		To find the perpendicular B C.	
As hyp. A C 480 =	2.681241	As radius 90° =	10.000000
Is to radius 90	10.000000	Is to hyp. A C 480	2.681241
So is base A B 288	2.459392	So is sine A 53° 8′	9.903108
	12.459392		12.584349
To co-sine A 53° 8′	9.778151	To perp. B C 384	2.584349

2. *Making the base radius*, B C will be the tangent of A, and A C the secant of A. *See fig.* 2. Hence,

To find the angle A.		To find the perpendicular B C.	
As the base A B = 288	2.459392	As radius 90°	10.000000
Is to the radius 90°	10.000000	Is to the base A B 288	2.459392
So is hyp. A C 480	2.681241	So is tang. A 53° 8′	10.124990
	12.681241		12.584312
To secant A 53° 8′	10.221849	To perp. B C = 384	2.584312

BY GUNTER'S SCALE.

1st. Extend the compasses from the hypothenuse 480, to the base 288, on the line of numbers; that extent will reach from 90 to 36° 52′, on the line of sines, the complement of the angle A.

PROBLEM V.

GIVEN THE HYPOTHENUSE AND PERPENDICULAR, TO FIND THE ANGLES AND THE BASE.

In the right-angled plane triangle A B C, *fig.* 4, given the hypothenuse A C = 480, and the perpendicular B C = 384; required the angles and the base.

1. *Making the hypothenuse radius*, B C will be the sine of the angle A, and A B the co-sine thereof. *See fig.* 1. Hence,

To find the angle A.		To find the base A B.	
As hyp. A C = 480	2.681241	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to hyp. A C = 487	2.681241
So is perp. B C 384	2.584331	So is co-sine A 53° 8′	9.778119
To sine A 53° 8′	9.903090	To base A B 288	2.459360

2. *Making the perpendicular radius,* A B will be the tangent of C, or the co-tangent of A; and A C will be the secant of C, or the co-secant of A. *See fig.* 3. Hence,

To find the angle A.		To find the base A B.	
As perp. B C = 384	2.584331	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to perp. B C = 384	2.584331
So is hyp. A C 480	2.681241	So is co-tangent A 53° 8′	9.875010
To co-secant A 53° 8′	10.096910	To base A B = 288	2.459341

BY GUNTER'S SCALE.

1st. Extend the compasses from the hypothenuse 480 to the perpendicular 384, on the line of numbers, and that extent will reach from 90° to 53° 8′ on the line of sines.

2d. Extend the compasses from 90° to 36° 52′ the complement of A, on the line of sines, and that extent will reach from 480 to 288, on the line of numbers.

PROBLEM VI.

GIVEN THE BASE AND PERPENDICULAR TO FIND THE ANGLES AND THE HYPOTHENUSE.

In the right-angled plane triangle A B C, *fig.* 4, given the base A B = 288, and the perpendicular B C = 384, to find the angles at A and C, and the hypothenuse A C.

1. *Making the base radius,* B C will be the tangent, and A C the secant of the angle A. *See fig.* 3. Hence,

To find the angle A.		To find the hypothenuse A C.	
As base A B = 288	2.459392	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to base A B = 288	2.459392
So is perp. B C 384	2.584331	So is secant A 53° 8′	10.221881
To tangent A 53° 8′	10.124939	To hypoth. A C 480	2.681273

2. *Making the perpendicular radius,* A B will be the tangent of C, or the co-tangent of A; and A C will be the secant of C, or the co-secant of A. *See fig.* 3. Hence,

To find the angle A.		To find the hypothenuse A C.	
As perp. A C = 384	2.584331	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to perp. B C = 384	2.584331
So is base A B 288	2.459392	So is co-sec. A 53° 8′	10.096892
To co-tangent A 53° 8′	10.124961	To hyp. A C 480	2.681223

BY GUNTER'S SCALE.

1st. Extend the compasses from 384 to 288 on the line of numbers, and that extent will reach from 45° to 53° 8′ on the line of tangents.

2d. Extend the compasess from 58° 8′ to 90°, on the line of sines, and that extent will reach from 384 to 480, on the line of numbers.

The foregoing examples embrace all the variety of cases that can be solved in right-angled trigonometry. We are now about to treat of

OBLIQUE-ANGLED TRIGONOMETRY.

RULE I.—WHEN TWO OF THE THREE GIVEN PARTS ARE A SIDE, AND ITS OPPOSITE ANGLE.

Any one side of a triangle,
Is to the sine of its opposite angle;
As any other side
Is to the sine of its opposite angle.
And, the sine of any angle,
Is to its opposite side;
As the sine of any other angle,
Is to its opposite side.

An *angle,* found by this rule, is sometimes *ambiguous;* for trigonometry gives us only the sine of an angle, and not the angle itself, and the sine of every angle is also the sine of its supplement. When the *given* side, opposite to the *given* angle, is greater than the other *given* side, then the angle opposite to that other given side is always acute. But when the *given* side opposite to the *given* angle is less than the other given side, then the angle opposite that other given side, may be either acute or obtuse, and consequently it is *ambiguous.*

RULE II.—WHEN TWO SIDES AND THEIR INCLUDED ANGLE ARE GIVEN.

As the sum of the two given sides,
Is to their difference;
So is the tangent of half the sum of the opposite angles,
To the tangent of half their difference.

This half difference between the two required angles being *added* to half their sum, gives the *greater* angle, and *subtracted* from half their sum, gives the *less.*

The remaining side of the triangle is then found by Rule 1.

RULE III.—WHEN THE THREE SIDES ARE GIVEN TO FIND THE ANGLES.

Assume the longest of the three sides as base, upon which suppose a perpendicular to be let fall from the opposite angle. Then,

As the base or longest side,
Is to the sum of the two other sides,
So is the difference of those sides,
To the difference of the segments of the base.

Then *half* the base, or longest side, added to the said difference, gives the greater segment of the base, and subtracted, gives the less.

The triangle being thus divided into two right-angled triangles, each of which contains two given sides, the remaining angles may be found by Rule 1. It is to be observed, that the greater segment is always adjacent to the greater side.

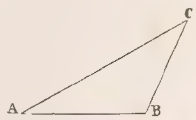
To enable us, therefore, to find the sides and angles of an oblique-angled triangle, *three* of them must be given. These may be either

1. Two angles and a side, or
2. Two sides and an angle *opposite* to one of them, or
3. Two sides and the *included* angle, or
4. The three sides.

PROBLEM I.

GIVEN THE ANGLES AND ONE SIDE TO FIND THE OTHER ANGLE AND REMAINING SIDES.

Fig. 6.



In the plane triangle A B C, given the angle at A = 32° 15′, the angle at B = 114° 25′, and the side A B = 98, to find the angle C, and the sides A C and B C.

To the angle A = 32° 15′, add the angle B = 114° 25′, and the sum will be 146° 40′, which subtract from 180° and it leaves the angle C = 33° 20′. Then,

BY RULE 1.

To find the side A C.		To find the side B C.	
Sine of C = 33° 20′	9.739975	Sine C = 33° 20′	9.739975
Is to side A B = 98	1.991226	Is to side A B 98	1.991226
As sine B 114° 25′, or 65° 35′	9.959310	As sine A 32° 15′	9.727228
	11.950536		11.718454
To side A C = 162.39	2.210561	To side B C, 95.17	1.978479

NOTE.—In all cases where the angle is *obtuse*, or grearer than 90°, subtract it from 180, and with the remainder take out the sine, tangent, or secant, from the tables. Thus, if we subtract 114° 25′ (as in the above example,) from 180, it leaves 65° 35′, the tabular sine of which is 9.959310, and is exactly the same for 114° 25′. And generally, the tabular sine, tangent, or secant of an obtuse angle, is *the same* as that of its supplement, and the contrary.

BY GUNTER'S SCALE.

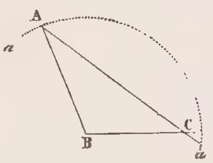
1. Extend the compasses from 33° 20′ to 65° 35′ the supplement of B, on the line of sines, and that extent will reach from 98 to 162, on the line of numbers.

2. Extend the compasses from 33° 20′ to 32° 15′, on the line of sines, and that extent will reach from 98 to 95, on the line of numbers.

PROBLEM II.

GIVEN TWO SIDES AND AN ANGLE OPPOSITE TO ONE OF THEM, TO FIND THE REMAINING SIDE AND ANGLES.

Fig. 7.



In the triangle A B C, *fig.* 7. given the angle C = 33° 20′ the side A B = 98, and the side B C = 95, 17, to find the angles at A and B, and the side A C.

Here, agreeably to the observation under rule 1. the angle A is acute, and not ambiguous; but had the side A B been less than the side B C, the arc *a a* would evidently have cut the side A C in two points on the same side of B C.

BY RULE 1.

To find the angles A and B.		To find the side A C.	
A B = 98	1.991226	Sine C = 33° 20′	9.739975
: Sine C 33° 20′	9.739975	: Side A B = 98	1.991226
:: B C 95.17	1.978479	:: Sine B 114° 25′ or 65° 35′	9.959310
	11.718454		11.950536
: Sine A 32° 15′	9.727228	: Side A C = 162.39	2.210561

The sum of the angles A and C, subtracted from 180° leaves the obtuse angle B = 114° 25′.

BY GUNTER'S SCALE.

1. Extend the compasses from 98 to 95, on the line of numbers, and that extent will reach from 33° 20′ to 32° 15′, on the line of sines.

2. Add the angles A = 32° 15′ and C = 33° 20′ together, and the sum will be 65° 35′; then extend the compasses from 33° 20′ to 65° 35′ on the line of sines, and that extent will reach from 98 to 162.4 on the line of numbers.

2. Given, { The angle C = 33° 20′, The side B C = 95. 17, The side A B = 60 } Required the angles A and B, and the side A C.

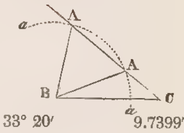
The geometrical construction of this triangle, *fig.* 8, is exactly the same as in the preceding example; only A B being shorter than B C, cuts A C in two points on the same side of B C, wherefore the angle A may be either acute or obtuse. We may work

BY RULE 1.

To find the angles A and B.	
Side A B = 60	1.778151
: Sine C 33° 20′	9.739975
:: Side B C 95.17	1.978479
	11.718454
: Sine A { 60° 39′ acute, 119° 21′ obtuse }	9.940303

The sum of the angles C and A subtracted from 180°, leaves the angle B = 86° 1′, if it be obtuse; or 27° 1′ if it be acute. It is evident that the two sides B A, and B A, are exactly equal, because they are radii of the same arc *a a*.

Fig. 8.



To find the side A C.	
Sine C = 33° 20′	9.739975
: Side A B 60	1.778151
:: Sine B 86° 1′	9.998950
	11.777101
: Side A C 108.92	2.037127
OR	
Sine C = 33° 20′	9.739975
: Side A B 60	1.778151
:: Sine B 27° 19′	9.661726
	11.439877
: Side A C = 50.11	1.699902

PROBLEM III.

GIVEN TWO SIDES AND THEIR INCLUDED ANGLE TO FIND THE OTHER SIDE AND REMAINING ANGLES.

In the plain triangle A B C, *fig.* 6, given the side A B = 98, the side B C = 95.17, and the inclined angle B = 114° 25′, to find the rest.

First, find the sum and difference of the two given sides.

The side A B = 98	The side A B – 98
B C 95.17	B C 95.17
Sum of the sides, 193.17	Difference of the sides, 2.83

Next, subtract the given angle B = 114° 25′, from 180, and it leaves 65° 35′ for the *sum of the other two angles*. Half of 65° 35′ is 32° 47′ 30″, Then,

By Rule 2.—As the sum of the two sides	193.17	2.285940
Is to their difference	2.83	0.451786
So is tangent of half the unknown angles	32° 47′ 30″	9.809055
		10.260841
To tangent of half their difference	0° 32′ 30″	7.974901
Add this to ½ the sum of the unknown angles	32 47 30	
Gives the greater angle at C =	33 20 00	
Subtracted, gives the less at A =	32 15 00	
OR		
As the sum of the two sides	193.17	2.285940
Is to their difference	2.83	0.451786
So is cotangent of ½ the given angle 57° 12′ 30″		9.809055
		10.260841
To tangent of ½ the difference of unknown angles.	0° 32′ 30″	7.974901

To find the side A C.		
As sine of C = 33° 20′		9.739975
Is to the side A B 98		1.991226
So is sine B = 114° 25′, or 65° 35′		9.959310
		11.950536
To side A C.	162.39	2.210561

BY GUNTER'S SCALE.

1. Extend the compasses from 193.17 to 2.83 on the line of numbers, and that extent will reach from 32° 47½′ to 0° 32½′ on the line of tangents. This is the method of working such examples as this; but so small an angle as 0° 32½′ cannot be taken from the scale.

2. Extend from 33° 20′ to 65° 35′ on the line of sines, and that extent will reach from 98 to 162.4 on the line of numbers.

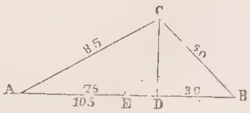
PROBLEM IV.

GIVEN THE THREE SIDES TO FIND THE ANGLES.

First, find the sum and difference of the two shorter sides.

Fig. 9.

Side A C	85	Side A C.	85
B C	50	B C.	50
Sum of the two sides	135	Difference	35



By *Rule* 3.—As the longest side A B =	105	2.021189
Is to the sum of the other two	135	2.130334
So is their difference	35	1.544068
		3.674402
To the difference of the segments E D	45	1.653213
Half of the said difference, is	22.5	
Added to half the base	52.5	
Gives the greater segment A D	75	
Subtracted, gives the less	30	

Thus, the triangle is divided into two right-angled triangles, A D C and B D C; in each of which the hypothenuse and one side are given, to find the angles as already taught in right-angled trigonometry.

To find the angle D C A.		To find the angle D C B.	
As hyp. A C 95	1.929419	As hyp. B C. 50	1.698970
Is to rad. 90°	10.000000	Is to rad.	10.000000
So is seg. A D 75	1.875061	So is seg. B D 30	1.477121
	11.875061		11.477121
To sine D C A 61° 56′	9.945642	To sine D C B, 36° 52′	9.778151

The angle D C A 61° 56′ subtracted from 90°, leaves the angle at A 28° 4′.

The angle D C B 36° 52′ subtracted from 90°, leaves the angle at B 53° 8′.

The angle D C A 61° 56′ added to the angle D C B 36° 52′ gives 98° 48′ for the obtuse angle at C, which was required.

The preceding solutions are all effected by means of the tabular sines, tangents and secants. But when any two sides of a right-angled triangle are given, the third side may be found without the aid of trigonometrical tables, by the proposition, that the square of the hypothenuse is equal to the sum of the squares of the two perpendicular sides; (see Geometry, Theorem 17.)

If the legs be given, extracting the square root of the sum of their squares, will give the hypothenuse. Or if the hypothenuse and one leg be given, extracting the square root of the difference of their squares, will give the other leg. It is generally most convenient to find the difference of the squares by *logarithms*. But this is not to be done by *subtraction*. For subtraction, in logarithms, performs the office of *division*. If we subtract the logarithm of the square of the base, from the logarithm of the square of the hypothenuse, we shall have the logarithm, not of the *difference* of the squares, but of their quotient.

To obtain the difference of the squares of two quantities, add the logarithm of the sum of the quantities, to the logarithm of their difference. After the logarithm of the difference of the squares is found, the *square root* of this difference is obtained by dividing the logarithm by 2.

Example 1.—If the base be 60 inches, and the perpendicular 45, what is the length of the hypothenuse?

1. By extracting the square root.	2. By Logarithms.	
60×60 = 3600		Logarithm.
45×45 = 2025	Sq. of the given sides = 5625	2)3.750123
56.25(75 inches.	Side required = 75	1.875061
49		
145 (725		
725		

Example 2.—If the hypothenuse be 75 inches, and the base 60, what is the length of the perpendicular?

1. By extracting the square root.	2. By Logarithms.	
75×75 = 5625		Logarithm.
60×60 = 3600	Sum of the given sides = 135	2.130334
20.25(45 inches.	Difference of do. = 15	1.176091
16		2)3.306425
85) 425	Side required 45	1.653212
425		

Example 3.—If the hypothenuse be 75 inches, and the perpendicular 45, what is the length of the base?

1. By extracting the square root.	2. By Logarithms.	
75×75 = 5625		Logarithm.
45×45 − 2025	Sum of the given sides = 120	2.079181
36.00(60 inches.	Difference of do. = 30	1.477121
36		2)3.556302
00	Side required 60	1.778151

There is a roof whose span or width is 40 feet, and height 10 feet, standing upon a rectangle plan, hipped at each end. What is the length of the common rafters, and likewise the hip rafters? (See *fig.* 64, *pl.* 28.)

Example.—One half of the span A D is 20 feet; and the perpendicular height C D is 10 feet, to find the length of the common rafters A C or B C. The square of 20 and 10 is 400, the square root of which is 22.36 feet, for the length of the common rafters A C or B C.

Now the hip rafters are the hypothenuses of right-angled triangles; having a common rafter for one of the legs, and the other leg being equal to half the width of the roof. Now we will suppose A C or B C to be the length of the common rafter, which is 22.36 feet, and A D half the width of the roof, (which is 20 feet,) to find the length of the hip rafter E G. The square of 22.36 and 20 is 899.9696, the square root of which is 29.99 feet, being a trifle less than 30 feet.

HEIGHTS AND DISTANCES.

PROBLEM I.

TO FIND THE PERPENDICULAR HEIGHT OF AN ACCESSIBLE OBJECT STANDING ON A HORIZONTAL PLANE.

Rule 1.—Measure from the object to a convenient station, on a base line, and there take the angle of the elevation, subtended by the object.

2. Then say: As radius is to the base :: so is the tangent of the angle of elevation, to the perpendicular height.

3. Or more briefly, thus—From the logarithm of the distance of the station from the object, increased by 10 in the index, subtract the tangent of the elevation; the remainder will be the logarithm of the perpendicular height, in the same denomination of measure, as the *distance* was taken in.

Example 1.—Wanting to know if a particular tree was of sufficient height to make a sill of a required length. I measured off 40 feet from the foot of the tree, and there found the angle, subtended by the tree, to be 56° 30′. Required the height of the tree. (See *fig*. 14, *pl*. 5.)

In this example, we have a plane triangle, right-angled at B, with the base and other angles given, to find the perpendicular. Making the base A B *radius*, because it is the side which is *given*, (see case 2, Trig.) the perpendicular becomes the *tangent* of the angle at A, and the proportion is stated thus:

As radius, or sine 90°	10.000000
Is to the base A B=40	1.602060
So is tangent A=56° 30′	10.179217
To the perp. BC 60.43	1.781277

Example 2.—Wanting to ascertain the elevation of a church and steeple, (see *fig*. 13, *pl*. 5,) similar to one I had undertaken to build, and not being able, otherwise, to obtain the measurements, I measured off 275 feet from the base of the porch, and by means of a hemistant, *fig*. 11, took the following angles:

1. To the top of the ridge, the angle A D E was 6° 14′.
2. To the top of the belfry, the angle A D D was 10° 18′.
3. To the commencement of the spire A D C was 13° 18′.
4. To the top of the spire A D B was 20° 0′.

Required the height of each part.

Here the same things are given, and required as in the first example, and the operations being precisely the same, may be abridged by simply adding together the first and second terms, and rejecting 10 from the sum of their indices: thus:

1. Base D A=275	2.439333	3. Base D A 275=	2.439333
Tang. A D E=6° 14′	9.038216	Tang. A D C 13° 18′=	9.373629
Perp. A E 30.03=	1.477649	Perp. A C=65	1.812961
2. Base D A=275	2.439333	4. Base D A 275=	2.439333
Tang. A D D 10° 18′	9.259428	Tang. A D B 20°=	9.561066
Perp. A D 49.97	1.698761	Perp. A B 100.1=	2.000399

Example 3.—Three places, A, B and C, (see *fig*. 66, *pl*. 28, are so situated, that A is directly south of B, and C directly east of A, at the distance of four miles. The bearing of C from B is 25° east. Required the distance of B, from A and C.

Here, making the hypothenuse B C radius, the proportion will be

As sine B= 25°	9.625948
Is to perp. A C 4	0.681241
So is rad. 90°	10.000000
To dist. B C=11.36	1.055293

In like manner, the distance A B will be found to be 10.29.

PROBLEM II.

TO FIND THE PERPENDICULAR HEIGHT OF AN OBJECT STANDING ON AN EMINENCE.

Rule 1.—Measure from the foot of the object to a convenient station on the plane beneath, and there take the angle of elevation, both of the top and bottom of the object.

2. Then say—as the cosine of the larger angle is to the difference of the two angles; so is the distance of the station from the foot of the object, to the height of the object.

Example.—Wanting to know the height of a tower standing on a hill, see *fig*. 65, *pl*. 28, I measured the distance from the base of the object at B, to the foot of the hill at A, and found it 136 feet. At A, I took the angle to the bottom of the tower, 48° 30′ and to the top, 67°. Required the height of the building, and the elevation of the hill. Thus:

As cosine 67°	9.591878
Is to the diff. of the ∠s. 18° 30′	9.501476 sine.
So is dist. A B=136	2.133339
	11.635015
To height B C=110.44	2.043137

2. To find the height of the hill say

As radius	10.000000
Is to 110.44	2.043137
So is sine 48° 30′	9.874456
To height of hill=82.71	1.917593

PROBLEM III.

TO FIND THE DISTANCE BETWEEN TWO INACCESSIBLE OBJECTS.

Rule.—Measure a base line between two stations, and the angles between this base and lines drawn from each of the stations to each of the objects.

Example.—Let B and C, *fig*. 67, *pl*. 28, be two buildings on the opposite banks of a river, and A D, two stations, lying in the same plane, 113 rods apart. Let the angle B A D, made by the first building and the second station, be 100°; the angle C A D, made by the second building and the second station, 36° 30′; let the angle C D A, made by the second building and the first station, be 121°, and the angle B D A, made by the first building and the first station, 49°. Required the distance between the two buildings B and C.

180° less the sum of B D A and B A D=A B D=31°
180 less the sum C A D and C D A=A C D=22° 30′.

1. In the oblique angled triangle A B D, find D B, thus:

As the sine of A B D=31°	9.711839
Is to the distance A D=113	2.053078
So is the sine of B A D=100°, (or 80°)	9.993352
	12.046430
To the distance D B 216.	2.334591

2. In the triangle A D C, find D C, thus:

As the sine of A C D=22° 30′	9.582840
Is to the distance A D=113	2.053078
So is the sine of C A D=36° 30′	9.774388
	11.827466
To the distance D C=175.64	2.244626

3. In the triangle B D C, we find the angle B D C, by subtracting the angle A D B=49° from A D C=121°: which gives 72°, with which, and the sides D B and D C, (as already found,) we determine the length of the required side B C, by case 2 of oblique angled trigonometry, wherein two sides, and their contained angle are given to find the remaining sides and angles. Thus:

	D B+D C		
As the sum of	216+175.64=391.64		2.592887
Is to their diff.	216—175.64	40.36	1.605951
So is tang. of $\frac{1}{2}$ supp. of B D C=54°			10.138739
			11.744690
To tang. of half the diff. of rem. angles.=8° 4′			9.151803

4. Having thus found half the difference of the required angles, the required side is soonest found by the following proportion, viz.

As the sine of half the diff.=8° 4′	9.147136
Is to the sine of half their sum=54°	9.907958
So is the diff. of the given sides 40.36	1.605951
	11.513909
To the distance BC=232.6	2.366773

The principles of trigonometry are no less essential to the measurement of lines and angles on water, than they are on land: hence their application to

NAVIGATION.

In applying the principles of trigonometry to the art of navigation, the *distance* which a ship sails, is represented by the hypothenuse of a right-angled triangle; the *course* or direction of the ship, is the angle at the perpendicular. The base represents the *departure*, which is always opposite to the course; and the perpendicular, which is always opposite to the complement of the course, represents the *difference of latitude*. To illustrate these observations by an example, see *fig*. 68. *pl*. 28.

Suppose a ship sails from the point A, on a course of 45° S. E. 84 miles to C. Required her departure A B, and difference of latitude C B.

Making the distance, that is, the hypothenuse, radius, the perpendicular will be,

As radius or 90°	10.000000
Is to the distance 84	1.924279
So is the sine of the course 45°	9.849485
To the departure A B 59.40	1.773764

To find the difference of latitude, use the *cosine* of the course, and proceed as above, exactly.

Note.—The line N S, that is drawn in the diagram of the circle, represents the meridian. (See *fig*. 15, *pl*. 5.)

GRECIAN ORDERS OF ARCHITECTURE.

When Dorus, the son of Helenus, and the nymph Optice, reigned over Achaia and all Peloponesus, he built in the ancient city of Argos, a Temple to Juno, which was formed by chance of this Order, and was afterwards used in the other cities of Achaia, while yet the ratio of its symmetries were not discovered.

Afterwards the Athenians, according to the responses of the Delphian Apollo, by the common consent of all Greece, sent out thirteen colonies at one time, into Asia, and appointing a leader to each colony, they gave the chief command to Ion, the son of Xenthus and Creusa, whom also Apollo, of Delphos, acknowledged as his son. These colonies he led into Asia, seized upon the country of Caria, and built the cities of Ephesus, Miletus, Myunta, (afterwards swallowed up by water, and its sacred rights and privileges given by Ion to the Milesians) Priene, Samos, Jeos, Colophana, Chios, Erethro, Phocis, Clazomeno, Lebedos and Melite.

This latter, on account of the arrogance of its inhabitants, was destroyed in the war declared against it by the unanimous determination of the other states, and, instead of it, by the beneficence of king Attalus and Arsinoe, the city of Smyrna was received by the Ionians. These states, when they had driven out the Carians and Lelegæ, called their country Ionia, after their leader Ion.

There they began to erect and dedicate temples to the immortal gods; and first they built a temple to Apollo Panionios, in the manner they had seen in Achaia, and which they called Doric, because they had first seen it in the Doric states. In this temple they wished to use columns, but not knowing their symmetries and proportions, to sustain the weight and present a graceful appearance, they measured the length of the human foot, and finding it to be a sixth part of the height of a man, they made use of this proportion for their columns, making the thickness or diameter of the shaft at the bottom, the sixth part of the height, including the capital. Thus the Doric column, having the proportions of the human body, began to be used with solidity and beauty in buildings.

Afterwards, when they were desirous of building a temple to Diana, they conceived a new species of order from a similar principle, making use of the proportions of a woman. They made the diameter of the column the eighth part of its height, and that it might appear more graceful, they put mouldings around the base to represent the shoe, and volutes in the capitals resembling the twisted braids of hair dropping to the right and left, and the cymatium and encarpi for the locks disposed on the forehead; they also made flutings on the shafts from top to bottom, like the folds in the garments worn by matrons.

Thus the two species of columns were composed, one imitating the strength and simplicity of man, the other the elegance and fine proportions of woman; but posterity, improving in judgment and knowledge, and aiming at still more graceful proportions, made the height of the Doric column seven diameters, and that of the Ionic eight and a half. This species was called Ionic, because it was invented by the Ionians.

The third, which is called Corinthian, imitates the delicacy of virgins; for in that tender age, the limbs are formed more slender, and admit of more graceful ornaments. The invention or origin of its capital is thus related :—

A Corinthian virgin, just marriageable, being seized with a disorder, died. After her interment, her nurse collected some vases, which pleased her when living, and putting them into a basket, carried them to her tomb, and placed them on its top; and that they might endure longer in the open air, she covered the basket with a tile. The basket happened to be placed upon the root of an achanthus, which being depressed in the middle, the leaves and stalks grew up in the spring, around the sides of the basket, but being resisted by the angles of the tile on the basket, were obliged to convolve at the extremities in the form of volutes. At that time, Callimachus, who, on account of his taste and skill in sculpture, was called by the Athenians, Catatechnos, happening to pass by this monument, observed the basket and the delicate foliage growing around it, and being pleased with the novelty of its form, he made some columns from this model, near Corinth, and composed the symmetry, and distributed the proportions of the Corinthian Order in the most exquisite manner.

DEFINITION OF THE ORDERS.

1. If any number of frustums of cones, or frustums of conoids of similar solids, and equal magnitudes with each other, be so arranged that their bases, which are the thickest ends of the frustums, may stand upon or in the same horizontal plane, and their axes in the same plane with each other, and perpendicular

to the horizon, and if on the tops of these frustums be laid a continued beam, and if over this beam be laid the ends of a number of equidistant joists, the other ends being either supported in the same manner, or by a wall, or any piece of building whatever, so that the upper and under surfaces may be in the same horizontal planes, and if over the ends of these beams be laid another beam parallel to the former, which lays upon the frustums, but projecting farther out from the axis of the columns than the vertical face of the lower beam which is over the frustums, and if this beam support the ends of rafters, whose upper surfaces lay in the same inclined plane, so as to support a covering or roof; the whole of this mass, together with the frustums supporting it, is called an order.

2. If the bottom or lower end of the frustum, finish with an assemblage of mouldings, projecting equally all around beyond the bottom of the frustum, then this assemblage is called a base.

3. If the upper end of the frustum finish with mouldings, or any kind of ornaments, and if these ornaments or mouldings be covered with a solid, whose upper and lower sides are squares, and the vertical or perpendicular sides rectangles; then this solid, together with the ornaments or mouldings under it, is called a capital.

4. If the frustum has no base, then the capital and the frustum together, is called a frustum column; but if the frustum has a base, then the base, frustum, and capital, taken together, are simply called a column.

5. The mass supported by the columns, is called an entablature.

6. The under beam of the entablature is called an architrave, or epistylium.

7. The space comprehended between the upper side of the epistylium, or architrave, and the under edge of the beam over the joists, is called the frieze, or zophorus.

8. The edge, or profile, of the inclined roof, supported by the joists, or cross-beams, jetting out beyond the face of the zophorus, or frieze, is called a cornice.

9. The lowest, or thickest part of the columns, is called the diameter of the columns.

10. Half of the diameter of the columns, is called a module.

11. If a module be divided into thirty, or any other number of equal parts, then each of these parts is called a minute.

12. The shortest distance from the bottom of the frustum of one column, to the bottom of the frustum of the next column, is called the intercolumniation.

13. When the intercolumniation is one diameter and a half of a column, it is called pycnostyle, or columns thick-set.

14. When the intercolumniation has two diameters of the columns, then it is called systile.

15. When the space between the columns is two diameters and a quarter, then the intercolumniation is called eustyle.

16. When the intercolumniation is three diameters of the columns, then it is called decastyle.

17. When the distance between the columns has four diameters of the columns, then that intercolumniation is called aræostyle, or columns thin-set.

18. When there are four columns in one row, then that number is called tetrastyle.

19. When there are six columns in one row, then it is called hexastyle.

20. When there are eight columns in one row, then it is called octastyle.

PLATE 29.

ELEVATION OF THE DORIC ORDER ON THE TEMPLE OF MINERVA AT ATHENS, CALLED THE PARTHENON.

This example is one of the most magnificent of Grecian Architecture now remaining. It was constructed of the finest marble, of the order periptere octastyle, viz., having eight columns on each front, and seventeen on each side including those at the angles, each column measuring six feet one inch in diameter, and including capital, thirty-four feet two inches in height. Its ground dimensions were two hundred and twenty-five feet in length, and one hundred feet in breadth; and the entrance on each front was twelve feet six inches in width, and twenty-eight feet eight inches high.

A. Elevation, showing the return of each flank at the angle of the building.

B. Elevation through the pediment.

C. Plan of the soffit at the angles.

E. Section through the cornice, and the manner of capping the triglyphs.

F. Plan of the triglyphs, tænia, regula and drops at the angles of the building.

G. Elevation of the same.

PLATE 30.

A. Elevation of the great hexastyle Temple at Pæstum.

B. Section through the entablature.

C.. Plan of a mutule.

D. The echinus of the capital.

E. Profile of the annulets.

F. The detail of astragal.

Fig. 3, *plate* 32, is the antæ capital of this example.

Fig. 1. Elevation of a Grecian Doric Order, the height expressed in diameters and minutes.

Fig. 2. Elevation of a Doric column and entablature,—its proportions from Fig. 1.

Fig. 3. Section through the entablature of Fig. 2.

Fig. 4. Profile through the front of the cornice.

Fig. 5. Section of capital.

The projections of these two examples are counted from a vertical line passing through the centre of the column and entablature.

PLATE 31.

I have here given an example of a Doric entablature and capital of a column, with all their details expressed in minutes.

PLATE 32.

Examples of antæ.

This is a species of square columns attached to a wall or building, either in a line with the columns, or behind them; in Grecian architecture, the capitals of the antæ differ from those of the columns, but in Roman they are the same. The following examples, taken from Grecian buildings, will show how they differ from the columns of the same building.

A. Elevation of the entablature and antæ, from the charagic monument of Thrasyllus. Fig. 6, is the antæ capital drawn to a

large size. The projections of the mouldings in this example are counted from a vertical line, passing through the centre of the antæ and entablature.

B. From the inside of the portico of the Temple of Minerva at Athens, called the Parthenon. Fig. 8, is a section of the cornice in B.

C. From the inside of the Doric portico at Athens.

D. and E. Two examples for antæs, which can be applied for frontispieces, inside finishing, and in many cases for terraces, towers, &c.

D. Is divided into seven parts, one of which gives the diameter. Fig. 7, is the capital to D.

E. Is divided into eight parts, one of which is the diameter. The antæ capital from the Erectheon, or from the temple of Minerva Polias can be applied to this example.

Fig. 1. From the Parthenon at Athens.

Fig. 2. Capital and base, from the Temple of Theseus, at Athens.

Fig. 3. From the hexastyle Temple at Pæstum.

Fig. 4. From the Doric portico at Athens.

Fig. 5. From the Propylea at Athens.

IONIC ORDER.

It has already been observed, in the general definitions of the orders, that every order consists of a column and an entablature.

Every column consists of a base, shaft, and capital, except in the Doric where the base is omitted.

Every entablature consists of an architrave, frieze and cornice.

The base, shaft, capital, architrave, frieze and cornice, are the principal members of an Order, and the peculiar mode or form of the members determines the particular name of the Order. But, as many of the mouldings are common to all the Orders, and are generated in a similar manner, what has been said in the general definition, and also on the Doric Order, will render it unnecessary to repeat the same things here, as such mouldings cannot form any particular feature of any particular Order. I shall therefore show, in the following definitions, how these members ought to be modified, so that they may constitute that Order invented by the Ionians, and called from their name, the Ionic Order.

DEFINITIONS.

1. If, from the under side of the abacus of an Order there project two or more spirals on each end of the front, in a plane parallel to the frieze, so that the extremety of each shall be at the same distance from the axis of the column; and also two others on the opposite side of the abacus, parallel to the former and projecting the same distance from the axis of the column, so that each of the spirals shall have the same number of revolutions, and equal and similar to each other; the projecting part contained between any two spirals is called a volute.

2. An order which has volutes and mouldings in the capital, of the annular kind, and the ichnography of the abacus square, as in the Doric Order, the architrave finishing of plain faciæ and mouldings, either plain or enriched, the frieze, a plain surface, the cornice consisting of a simarecta, then a fillet and an echinus only; and if to the underside of the corona are hung a row of equal and similar parallelopipeds equidistant from each other, whose fronts are in a plane, parallel to the plane of the frieze, then each of these parallelopipeds is called a dentil.

3. An Order so constructed is similar to that invented by the Ionians, and consequently is the Ionic Order.

PLATE 33.

ELEVATION OF THE IONIC TEMPLE ON THE RIVER ILLYSSUS, NEAR ATHENS.

On the southern bank of the Illyssus, not far from the fountain Enneacrunas, (which at present, having recovered its ancient name, is called Callirrha) are the remains of a little Ionic Temple. The mouldings are but few in number, and differ much from all other examples of that Order. Their forms being very simple, but withal so elegant, and thé whole so well proportioned, that it may well be ranked among those works of antiquity which are most entitled to our attention. This little temple was built of marble, from the quarries of Mount Pentilicus, and was of the order of amphiprostlyeas tetrastyle.

To the left hand of the plate is the order of the Temple.

To the right is the plan of one half of the capital, half the size of the original.

In the centre is a section and elevation of the capital half the size of the original.

PLATE 34.

Fig. 1. The cornice from the Temple on the Illyssus, half the size of the original, and a method of drawing the raking mouldings to mitre with the horizontal. To describe which, observe the following

RULE.—Draw a vertical line through the centre of the height and projection A B C D, and divide it into four equal parts; then draw another line at right angles, through the centre at D. Extend the compasses the distance of three of these divisions, and with one foot in D, set off E and F. In E, draw from D down to the first division, and in F, draw up to the first division; then draw the diagonal lines from E down, and from F up, and intersecting the curve and division lines at their juncture, and G H are the centres to complete it.

Then draw the line A C at right angles with the rake of the moulding, and make the raking projection A B, equal to A B the level projection; the remainder is drawn in the same manner as the level or horizontal moulding.

Fig. 2 and 3. The base and capital of the antæ, half the size of the original.

Fig. 4. Base of column, half the size of the original.

Fig. 5. Architrave band, of the full size.

Fig. 6. Bedmould, of the full size.

PLATE 35.

SPIRALS OF THE VOLUTE FROM THE CAPITALS OF THE TEMPLE ON THE ILLYSSUS, SHOWING A SECTION THROUGH THE FACE, SEVEN-EIGHTHS OF THE SIZE OF THE ORIGINAL.

To find the centres for drawing the spiral to this volute.

RULE.—Having described the circumference of the eye, whose diameter is six and a half minutes—and drawn the horizontal line $m\,n$ through the centre at o, divide $m\,o$, into two parts at 3, and produce the vertical line 3, 2, equal to one of the two parts; from 2, draw a line parallel to $m\,n$ and terminating perpendicu-

lar from n; divide the vertical line 3, 2, into four equal parts, and set two of these parts on each side to the right and left of o and join 4 5—6 7—8 9—10 11; then from the centre of the line 4, n, raise a perpendicular up to the outer circumference of the eye at 1, and join 1, 2, when 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, will be the centres for drawing the outside spiral to the volute. Then to draw it,

Set the dividers in 1, and extending them up to the top of the volute at a, describe the arc $a\,b$ until it intersects the continued line 1, 2; then set in 2, b, and draw $b\,c$, then 3 c, and draw $c\,d$, and so forth from all the centres.

Or you may reverse the process of drawing, by setting first in 11, l and drawing $l\,k$, and in 10 k, draw $k\,j$, and so forth.

To the left of the plate against the hem to the top of the volute, A B C D E F G H I J K L M shows a method of diminishing the hem of the volute.

PLATE 36.

Erectheon at Athens.

"To the north of the Parthenon, at the distance of about 150 feet, are the remains of three contiguous temples. That towards the east was called the Erectheon; to the westward, but under the same roof, was the Temple of Minerva, with the title Polias, as protectress of the city; adjoining to which, on the south side, is the Pandrosium, so named, from its being dedicated to the nymph Pandrossus, one of the daughters of the Cecrops. To the left hand of the plate, is seen the portico of the Pandrosium; the entablature and roof supported by caryatides. To the right is the hexastyle portico of the Erectheon, and in the centre is the tetrastyle portico of the Minerva Polias. The whole edifice was called by Pausanius, the Erectheum, after an appellation of Neptune, and because it contained the salt spring called Erectheis; and not because within it was the tomb of Erecthonius, fourth king of Athens. This celebrated edifice was erected in the age of the illustrious Pericles, when taste and invention were in their meridian among the Athenians, and when they were anxiously engaged in restoring the temples which had been destroyed by the Persians. In this beautiful specimen of the Ionic Order, they seem to have been ambitious of excelling their Asiatic brethren, in their own peculiar Order of architecture, by the addition of new and elaborate ornaments, imagined with the utmost ingenuity and elegance of taste, and executed with the sharpness and perfection of a cameo, which it could hardly have been supposed that marble was capable of receiving.

PLATE 37.

Fig. 1. The Order of the portico of the Erectheon.

Fig. 2. Profile of the upper mouldings of the cornice, about one-fourth of the size of the original.

Fig. 3. The bedmould; full size of the original.

Fig. 4. The architrave band; ditto.

Fig. 5. Base of columns; one-fourth original size.

Fig. 6. Ovolo of the cymatium; drawn to be carved.

Fig. 7. Architrave moulding; ditto.

Fig. 8 and 9. Centres to draw the spirals. Either of these methods will draw a spiral to the height of about thirty-six minutes, though the width and form differs a trifle.

PLATE 38.

A. Half the elevation of the capital of the Erectheon, showing a section through the face; one-fourth size.

B. Flank elevation; one-fourth original size.

C. Plan of capital; ditto.

D. Half the elevation of the antæ capital; ditto.

E. Base of the antæ; ditto.

F. Section of the eye of the volute; full size.

PLATE 39.

Fig. 1. The order of the portico of the Temple of Minerva Polias.

Fig. 2. Half the elevation of the capital of Minerva Polias, with horizontal and vertical sections through the front; half the original size.

Fig. 3. Stretch-out of the echinus, and plaited torus of the capital; full size of the original.

Fig. 4. Base of the columns; one-fourth size.

Fig. 5. The eye to the volute: full size of the original, showing more clearly how to find the centres in drawing the spirals.

TO FIND THE CENTRES IN DRAWING THE SPIRAL TO THIS VOLUTE.

Rule.—Having described the circumference of the eye, whose diameter is five and two-tenth minutes, and drawn vertical and horizontal lines through the centre at o, as in *fig.* 5, divide the vertical line from o up to the circumference of the eye, into nine equal parts;—continue the vertical line from the circumference up, the distance of three of these parts, and divide this continued line into three equal parts; subdivide the upper part or division into four equal divisions, the third of which from the top down at 1, will be the first centre. From the fifth division of the vertical line, up from o, draw a horizontal line to 2, equal in length to seven of the nine parts; from 2 draw a vertical line down, equal in length to six of these parts; thence draw a horizontal line to the left, the distance of nine parts. A continued diagonal line from 1, cutting the angle at 2, will show the termination of the first arc of the spiral, and a similar line from 2, cutting the line 3, 4, at the distance of half a part from the angle near 3, will show the termination of the second arc. The remaining centres, and the terminations of the arcs, may readily be found by an inspection of the diagram in the circumference of the eye. The centres being found, proceed to draw the outside spiral as described in plate 35. The middle hem is to be drawn in the centre of the two outside spirals, as represented in the plate.

PLATE 40.

One quarter of the plan of the capital of Minerva Polias; half the size of the original.

PLATE 41.

One quarter of the flank elevation of the capital of Minerva Polias; half the original size.

PLATE 42.

Fig. 1. Elevation of one half the antæ capital of Minerva Polias; half of the original size.

Fig. 2. The base of the antæ; ditto.

Fig. 3. Flank elevation of the cornice from the portico of Minerva Polias; one eighth the size of the original.

PLATE 43.

FROM THE TEMPLE OF MINERVA POLIAS, AT PRIENE, IN IONIA.

Fig. 1. The order from the temple, showing the elevation of the entablature on the flanks of the building.

Fig. 2. Elevation of the entablature on the front of the pediment.

Fig. 3. Section through the cornice of the pediment. It is remarkable that the enrichment of the upper mouldings differs from that of the lateral cornice.

Fig. 4. Plan of the dentils, showing on the angles of the building.

Fig. 5. Section through the cornice on the pediment.

PLATE 44.

A. Represents half the elevation of the capital of the Temple of Minerva, at Priene, in Ionia, with a section through the front.

B. One quarter of the flank elevation, with a section of the same.

C. One quarter of the plan.

D. A moulding, which can be applied for the abacus, ornamented at the angles.

E. A plan of one half of the capital from the remains of the Temple of Apollo Didymaus, near Miletus, in Ionia. Supposing that many architects might prefer this capital to one before described, I have thought proper to give a plan of it; but the form of the hem, and the spirals to the volutes, being so similar to the other examples of the order, I have deemed it unnecessary to make a full drawing of it.

F. The eye to the volute, drawn on a large scale.

TO FIND THE CENTRES, IN DRAWING THE SPIRAL TO THIS VOLUTE.

Rule.—Having described the circumference of the eye, whose diameter is $2\frac{2}{3}$ minutes, and drawn a vertical line through the centre; inscribe within this circumference, a hexagon, having three of its sides upon each side of the vertical line. Divide the two upper and two lower sides into three equal parts each, and draw a line from 1, through the centre of the eye down to 3, and another from 2, down to 4, when 1, 2, 3, 4, will be the first centres. Then divide these two diagonal lines into six equal parts each, three above and three below the centre of the eye, and 5, 6, 7, 8, 9, 10, 11, 12, are the second centres. Then divide three of these lines, viz. centre, 9—centre, 10—centre, 11, into two equal parts each, and 13, 14, 15, are the centres complete for drawing the outside spiral of four revolutions. The spiral is then drawn from the same rules as the spirals of three revolutions, which have already been described.

The diameter, or scale, upon which this capital is drawn, being $12\frac{1}{4}$ inches, renders it suitable in size for practice, in small porticos. This example, with the exception of the base, which is somewhat objectionable, is not inferior to any other example of the order. In drawing this capital I have taken the liberty of altering the proportions a trifle where I thought necessary; which makes it, in my opinion, more perfect and beautiful.

PLATE 45.

FROM THE TEMPLE OF BACCHUS, AT TEOS, IN IONIA.

This Temple was first began in the Doric Order, by Hermongenus, but he afterwards changed it into the Ionic, and dedicated it to Bacchus.

This example is drawn from an accurate measurement of that celebrated building; and by diminishing the columns to fifty minutes at their upper diameters, may be reckoned among those works of antiquity which most deserve our attention. The architrave is well proportioned to itself, and also the cornice; the dentils of the cornice add greatly to the character of the order.

The capital, and the spirals of the volute hem are elegant and beautiful, and the base I think not inferior to any other of the order.

The base of the columns, it is thought, from the little difference between the shaft at the base here exhibited, did not belong to the capital shown in *Fig.* 2, but to some of the interior columns; for the ancients always made the interior ranges of columns less in diameter than the exterior, as is to be found in the celebrated Athenian building, the Temple of Minerva and the Propylea.

Fig. 1. The order from the Temple.

Fig. 2. One half of the capital, with horizontal and vertical sections through the spirals of the volutes.

Fig. 3. A section through the front of the capital.

Fig. 4. The base of the column.

Fig. 7. The architrave band.

Fig. 8. The cornice, drawn on a large scale. The projections of the mouldings are counted from a vertical line passing through the cornice perpendicular to the frieze; the remaining parts are counted from the centre of the column and architrave.

Fig. 5. The eye of the volute, on a large scale.

TO FIND THE CENTRES IN DRAWING THE SPIRAL TO THIS VOLUTE.

Rule.—Having described the circumference of the eye, whose diameter is $3\frac{5}{8}$ minutes, draw a horizontal line through the centre of the eye at *o*, and divide it into eight equal parts; set off two of these parts from *o* down, and one part from *o* up; draw a line from 1 to 2, equal in length to six parts of the eight; from 2 to 3, equal in length to three parts of the eight; from 3 to 4, equal to five of those parts; and from 4 to 5, equal to two and a half parts; then find the other centres as the figures direct. The whole will appear more clear by inspection, than it can be made by description.

Fig. 6. A method of diminishing the hem of the spirals.

CORINTHIAN ORDER.

DEFINITIONS.

1st. An order which has two annular rows of leaves capital, each leaf of the upper row growing between those of the lower row in such a manner, that a leaf of the upper row may be in the middle of each side of the face of the capital; and if between each space of the upper leaves there spring stalks with volutes, two of which meet at the angles of the abacus, and two in the middle of the capital, either touching or interwoven with each other; a capital so constructed is called Corinthian.

2d. An order which has a Corinthian capital and an Ionic, or any other entablature, is called the Corinthian order.

PLATE 46.

CORINTHIAN ORDER, FROM THE MONUMENT OF LYSICRATES AT ATHENS, COMMONLY CALLED THE LANTHORN OF DEMOSTHENES.

Fig. 1. The order from the monument.
Fig, 2. Base of the columns, half the size of the originals.
Fig. 3. Plan of the capital.
Fig. 4. Profile of the capital.
Fig. 5. Section of the cornice—half the size of the original.
Fig. 6. Plan of the dentils.

DEFINITIONS OF ORNAMENTS.

1. An artificial arrangement or disposition of leaves is called foliage.

2. The subdivisions of single leaves are called raffles. The leaves which are chiefly used in architecture, are the acanthus, olive, parsley, laurel and lotus.

3. An artificial arrangement of leaves, branches, fruit, flowers, drapery, &c., either singly or combined in any manner with each other, are called ornaments in architecture.

4. A string, consisting of flowers, fruit, leaves, and branches, either singly, by themselves, or intermixed with each other, and supported at the two extremes, the middle part forming itself into a curve by its gravity; this figure, so suspended, is called a festoon.

5. A curve line, which is continually changing its course in contrary directions on the same side of it; that is, first concave and then convex, concave again, and then convex again, and so on alternately in this manner, to any number of curves of contrary flexure, is called a serpentine line.

6. If from a stalk, in the form of a serpentine line, a number of branches issue out, twisting themselves in the form of spiral lines on each side of the serpentine line, in all the concave parts on the alternate sides of it, and if these spirals and the stalk be decorated with foliage, a composition so formed is called winding foliage.

TO DRAW ORNAMENTS.

The learner should, in the first place, draw a great variety of curve and spiral lines of different descriptions, and compare these figures with each other, by which means he will be able, by sight only, to distinguish one particular species of curve from another; then he ought to imitate, with precision, the same things by hand, in all the varieties of positions which he can suggest to himself; and thus he will acquire a freedom of hand in every direction. When he proceeds to copying leaves, a general outline ought to be drawn, circumscribing the whole leaf; he should then form outlines of all the veins, and round every compartment, circumscribing all the different sets of points or raffles; and afterwards proceed to draw the raffles themselves. The learner having, after sufficient practice in copying, acquired a freedom of hand, I would then advise to draw from nature a variety of such things as will be most suitable to the purposes to which they are to be applied. By so doing, the parts of his compositions will always appear rich and natural; and hence he will obtain a greater facility of invention. Having had sufficient practice in drawing from nature, he may then apply himself to the designing of ornaments, for which purpose he will find the first part of the problem, viz. that of drawing curve and spiral lines by hand, to be of the utmost utility in forming the general outlines of his designs; and for finishing the smaller parts, such as raffles, flowers, fruits, &c. he must apply the knowledge he has acquired in drawing from nature, which will complete his composition.

ELEMENTS OF FOLIAGE.—LEAVES.

Of the acanthus, or bear's-breech, or branhursinæ, there are several different species.

1. The mollis, or common bear's-breech, a native of Italy.

2. The spinosus, or prickly bear's-breech, the leaves of which are deeply jagged in every regular order, and each segment is terminated with a sharp spire, as is also the compalement of the flower, which renders it troublesome to handle them.

3. The ilisifolias, or shrubby bear's-breech, grows in both the Indies. It is an evergreen shrub which rises about four feet high, and is divided into many branches, garnished with leaves like those of the common holly, and armed with spires in the same manner; the flowers are white, and shaped like those of the common acanthus, but smaller.

4. Nigra, or Portugal bear's-breech, with smooth sinuated leaves, of a livid green color.

5. The middle bear's-breech, with entire leaves, having spires on their border.

ROMAN ARCHITECTURE.

TUSCAN ORDER.

PLATE 47.

This order is very similar to the Doric, and is evidently derived from it. It was first executed by the inhabitants of Tuscany, from which it derives its name.

There are two orders of Italian origin, called Latin orders, which are distinguished by the names of Tuscan and Roman. They were probably invented with a view of extending the characteristic bounds on one side, still farther towards strength and similarity; and on the other towards elegance and profusion of enrichment. At what period these orders were invented, and by whom improved to such perfection, remains doubtful. Vitruvius has attempted to give their origin and history, but his relation has been justly questioned, and is probably not much to be depended upon.

There have been a number of examples taken from the drawings of Paladio, Scamozzi, and other modern authors, and they all differ in their proportions, especially in the general character of their mouldings. I have selected the Trajan column, at Rome, for this example. This column is considered one of the proudest monuments of Roman splendor, and consists of a base, shaft and capital of the Tuscan order. It was erected by the Senate and people of Rome, in acknowledgment of the services of Trajan, and has contributed more to immortalize that emperor, than the united efforts of all historians.

De Cambria notices the Antonine column erected at Rome, in honor of Antoninus Pius; and another similar one at Constantinople, raised in honor of the emperor Theodorus, after his victory over the Scythians; both of which prove, by their resemblance to the Trajan column, that this sort of appropriation, recommended by him, had passed into a rule among the ancient masters of the art. Though much has been written against this order, on account of its plainness, I shall not here dispute either the accuracy, justice or fitness of the remarks of other authors; but shall venture to affirm, that not only the Tuscan column, but the entire order, as exhibited in this work, may justly be considered elegant specimens of architecture, and in numerous instances, usefully and tastefully applied in practice. Besides, as an order, it is a necessary gradation in the arts, although not recognized by the Grecian architects.

Combining the idea of strength and simplicity, for rural purposes, it is not surpassed by any of the ancient orders, being peculiarly applicable to farm houses, coach houses, green houses, grottos, fountains, barns, sheds, &c.; to park and garden gates, and in short, wherever magnificence is not required and expense is to be avoided.

Sebastian Serlio recommends the use of it in prisons, arsenals, public granaries, seaports and gates of fortified places. Le Clerc observes, that although the Tuscan order is treated with contempt by Vitruvius, Paladio and others, as unworthy of being identified, yet, according to the composition of Vignola, there is a beauty in its simplicity which entitles it to notice, and recommends it to a place both in private and public buildings, as in porticoes and colonnades surrounding squares; even in royal palaces, if suitably introduced to adorn the inferior apartments, offices, &c. where strength and simplicity are required, and where richer and more delicate orders would be extremely improper.

TO DRAW THIS ORDER TO ANY GIVEN HEIGHT.

Rule.—Divide A, B, Fig. 1, into nine equal parts, as shown in the outside division in the margin of the plate, and give one of these parts to the diameter of the column just above the base at *c, d;* then divide *c d* into sixty equal parts, (or first divide into six and subdivide into ten, as represented in the scale *c d*, Fig. 1,) and these divisions are called minutes, or sixtieths of the diameter of the column. I have drawn a scale of one module or thirty minutes in Fig. 4, and drawn all the details of this order large. The heights of the mouldings, and the aggregate heights are expressed in minutes by figures placed in the margin divisions. The projections of the mouldings are expressed in minutes by figures placed at the extreme projection of each member, and are counted from perpendiculars, raised at the extremities of the superior and inferior diameters of the shaft, or from the vertical line that is dotted, passing through the base, capital, architrave and cornice.

Fig. 1. Elevation of the order.

Fig. 2. Cornice, on a large scale.

Fig. 3. Capital, on a large scale.

Fig. 4. Base of columns, large scale.

Fig. 5. Architrave band, large scale.

Fig. 6. Capital after the Grecian style, which may be prefered to Fig. 3.

PLATE 48.

Fig. 1. Elevation of a Doric order, as approved by Sir William Chambers. On this plate are given three profiles of the Doric order.

Fig. 1, 2. Are copied by Ligorio* from various fragments of antiquity in and near Rome.

Fig. 3. Entablature of Paladio, as executed in the Basilica, at Vicenza.

Fig. 4. Design for a Doric base, by Le Clerc.

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* Peter Ligorio, a Neapolitan, distinguished as a painter and architect. His designs composed thirty volumes. He died A. D. 1580.

PLATE 49.

ELEVATION OF A CORINTHIAN ORDER, AND ITS DETAILS, FROM SIR WILLIAM CHAMBERS.

PLATE 50.

A and B. Two examples for volutes.

TO DRAW A SPIRAL TO THE HEIGHT OF ABOUT THIRTY-FOUR MINUTES AS IN A.

Rule.—Having described the circumference of the eye, whose diameter is six minutes, draw a vertical line through the centre of the eye at *o*, as in D, and form on the left of the vertical line a square equal to two minutes, by setting one minute from *o* down and one up, and two minutes to the left of *o*, and drawing the lines 9, 10, 11, 12. Then draw the diagonal lines *o*, 10, and *o*, 11, and divide them into three equal parts each; likewise divide the vertical lines *o*, 9, and *o*, 12, into three equal parts each, and join 1, 2—3, 4—5, 6—and 7, 8, which gives all the centres for drawing the outside spiral. Then,

Set your compasses in 1, the first division below the centre, and, extending them up to the circumference at *a*, describe the arc *a*, *b*. Set again in 2, *b*, and produce *b*, *c*; and proceed in this manner until the spiral is completed.

Should it be required to draw the spiral for a middle hem, like those of the Temples of Erectheus, and Minerva Polias; for the four last centres, set one half of one of these divisions from 9 down, and produce a line parallel to 9, 10. Set the same distance to the left of 10, and produce a line parallel to 10, 11; the angles of these parallel lines at 9, 10, 11, are three of the required centres, and the middle of the vertical line *a*, 12, is the fourth and last centre, from which the spiral is drawn, as represented by the dotted line in A.

TO DRAW A SPIRAL TO THE HEIGHT OF THIRTY AND A HALF MINUTES, AS IN B.

Rule.—Having described the circumference of the eye, whose diameter is six minutes, set one minute from the centre of the eye at *o*, as in C, perpendicularly up, and divide it into three equal parts; set one minute upon each side from the third point perpendicular from *o*, and produce the lines 11, 12, and the diagonal lines 11, *o*—12, *o*; from 11, drop a vertical line, in length one minute and five-sixths of a minute, down to 10, and from 10 produce a horizontal line of one minute and two thirds in length to 9; the other centres may be readily found by an inspection of the Figure. The spirals are drawn as in A.

E, F and G. Ornaments designed for the eyes of the volutes, and are of the same size as C D.

Fig. 1, 2. Plan and elevation of one quarter of a Corinthian and Composite pilaster capital, from Sir William Chambers.

Fig. 3. Elevation of a leaf, from the capitals of the columns on the baths of Dioclesian, at Rome.

Fig. 4. Profile of Fig. 3.

Fig. 5, 6. Plan and elevation of a leaf taken from the Temple of Jupiter Stator, at Rome.

SCHOOL-HOUSE ARCHITECTURE.

COMMON ERRORS IN SCHOOL-HOUSES.

They are, almost universally, badly located, exposed to the noise, dust and danger of the highway, unattractive, if not positively repulsive in their external and internal appearance, and built at the least possible expense of material and labor.

They are too small. There is no separate entry for boys and girls appropriately fitted up; no sufficient space for the convenient seating and necessary movements of the scholars; no platform, desk, or recitation room for the teacher.

They are badly lighted. The windows are inserted on three or four sides of the room, without blinds or curtains to prevent the inconvenience and danger from cross-lights, and the excess of light falling directly on the eyes or reflected from the book, and the distracting influence of passing objects and events out of doors.

They are not properly ventilated. The purity of the atmosphere is not preserved by providing for the escape of such portions of the air as have become offensive and poisonous by the process of breathing, and by the matter which is constantly escaping from the lungs in vapor, and from the surface of the body in insensible perspiration.

They are imperfectly warmed. The rush of cold air through cracks and defects in the doors, windows, floor and plastering, is not guarded against. The air which is heated is already impure from having been breathed, and made more so by noxious gases arising from the burning of floating particles of vegetable and animal matter coming in contact with the hot iron. The heat is not equally diffused, so that one portion of a school-room is frequently overheated, while another portion, especially the floor, is too cold.

They are not furnished with seats and desks, properly made and adjusted to each other, and arranged in such a manner as to promote the comfort and convenience of the scholars, and the easy supervision on the part of the teacher. The seats are too high and too long, with no suitable support for the back, and especially for the younger children. The desks are too high for the seats, and are either attached to the wall on three sides of the room, so that the faces of the scholars are turned from the teacher, and a portion of them at least are tempted constantly to look out at the windows,—or the seats are attached to the wall on opposite sides, and the scholars sit facing each other. The aisles are not so arranged that each scholar can go to and from his seat, change his position, have access to his books, attend to his own business, be seen and approached by the teacher, without incommoding any other.

They are not provided with black-boards, maps, clock, thermometer, and other apparatus and fixtures which are indispensable to a well regulated and instructed school.

They are deficient in all of those in and out-door arrangements which help to promote habits of order, and neatness, and cultivate delicacy of manners and refinement of feeling. There are no verdure, trees, shrubbery and flowers for the eye, no scrapers and mats for the feet, no hooks and shelves for cloaks and hats, no well, no sink, basin and towels to secure cleanliness, and no places of retirement for children of either sex.

GENERAL PRINCIPLES OF SCHOOL ARCHITECTURE.

LOCATION—STYLE—CONSTRUCTION—YARD.

The location should be dry, quiet, pleasant, and in every respect healthy. To secure these points and avoid the evils which must inevitably result from a low and damp, or a bleak and unsheltered site, noisy and dirty thoroughfares, or the vicinity of places of idle and dissipated resort, it will sometimes be necessary to select a location a little removed from the territorial centre of the district. If possible, it should overlook a delightful country, present a choice of sunshine and shade, of trees and flowers, and be sheltered from the prevailing winds of winter by a hill-top, or a barrier of evergreens. As many of the pleasant influences of nature as possible should be gathered in and around that spot, where the earliest, most lasting, and most controlling associations of a child's mind are formed.

The style of the exterior should exhibit good, architectural proportion, and be calculated to inspire children and the community generally with respect for the object to which it is devoted. It should bear a favorable comparison, in respect to attractiveness, convenience and durability, with other public edifices, instead of standing in repulsive and disgraceful contrast with them.

The school-house should be constructed throughout in a workmanlike manner. No public edifice more deserves, or will better repay, the skill, labor and expense, which may be necessary to attain this object, for here the health, tastes, manners, minds, and morals of each successive generation of children will be, in a great measure, determined for time and eternity.

The building should be surrounded by a yard, of never less than half an acre, protected by a neat and substantial inclosure. This yard should be large enough in front, for all to occupy in common for recreation and sport, and planted with oaks, elms, maples, and other shady trees, tastefully arranged in groups, and around the sides. In the rear of the building, it should be divided by a high, and close fence, and one portion, appropriately fitted up, should be assigned exclusively for the use of the boys, and the other, for the girls. Over this entire arrangement, the most perfect neatness, seclusion, order and propriety should be enforced, and every thing calculated to defile the mind, or wound the delicacy or the modesty of the most sensitive, should receive attention in private, and be made a matter of parental advice and co-operation.

In cities and populous districts, particular attention should be paid to play-ground, as connected with the physical education of children. In the best conducted schools, the play-ground is now regarded as the *uncovered* school-room, where the real dispositions, and habits of the pupils are more palpably developed, and can be more wisely trained, than under the restraint of an ordinary school-room. These grounds are provided with circular swings, and are large enough for various athletic games. To protect the children in their sports in inclement weather, in some places, the school-house is built on piers; in others, the basement story is properly fitted up, and thrown open as a play-ground; and in others, the wood, or coal-shed is built large for that purpose. Under any circumstances the school-room should not be used for any other, than purposes of study and conversation. The following views exhibit improvements under this division of the subject, in school-houses recently erected.

Fig. 1.—Providence High School.

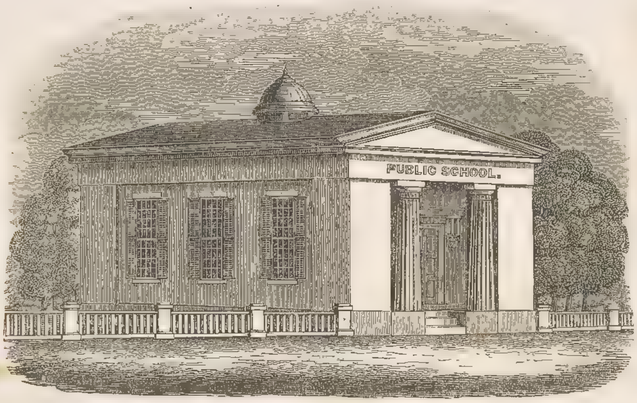
Fig. 2.

OCTAGONAL SCHOOL-HOUSE, See Fig. 33.

Fig. 3.

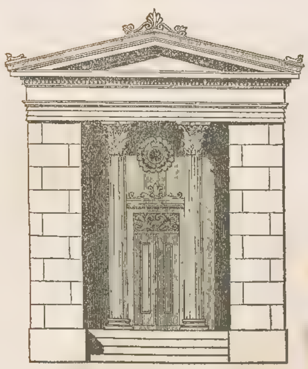
See Fig. 33.

Fig. 4.



WHITING STREET SCHOOL, NEW HAVEN.

Fig. 5.

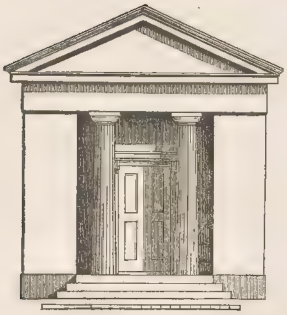


DISTRICT SCHOOL, HARTFORD.

Fig. 6.

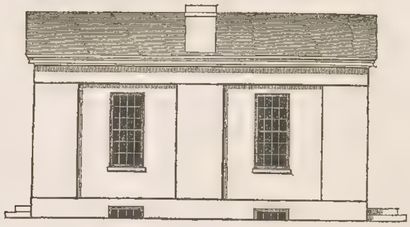
DISTRICT NO. 6, WINDSOR, CONN.
See Fig. 31.

Fig. 7.



DISTRICT SCHOOL, HARTFORD.
See Fig. 30.

Fig. 8.



PRIMARY SCHOOL, HARTFORD.

Fig. 9.

GRAMMAR SCHOOL, PROVIDENCE.

Fig. 10.

PRIMARY SCHOOL, PROVIDENCE.

Fig. 11.

MIDDLETOWN HIGH SCHOOL.
See Fig. 36.

SIZE.

In determining the size of a school-house, due regard must be had to the following particulars:

First.—A separate entry, or lobby, for each sex, furnished with scraper, mat, hooks or shelves, sink, basin and towels. A separate entry thus furnished, will prevent much confusion, rudeness, and impropriety, and promote the health, refinement, and orderly habits of children.

Second.—A room, or rooms, large enough to allow, 1st, each occupant a suitable quantity of pure air, i. e. at least 150 cubic feet; 2d, to go to and from his seat without disturbing any one else; 3d, to sit comfortably in his seat, and engage in his various studies with unrestricted freedom of motion; and 4th, to enable the teacher to approach each scholar in his seat, pass conveniently to any part of the room, supervise the whole school, and conduces the readings and recitation of the several classes properly arranged.

Third.—One or more rooms for recitation, apparatus, library, and other purposes.

LIGHT.

The arrangements for light should be such as to admit an abundance to every part of the room, and prevent the inconvenience and danger of any excess, glare, or reflection, or of cross-light. A dome, or sky-light,* or windows set high, admit and distribute the light most steadily and equally, and with the least interruption from shadows. Light from the north is less variable, but imparts less of cheerfulness and warmth than from other directions. Windows should be inserted only on two sides of the room, at least three and a half or four feet from the floor, and should be higher and larger, and fewer in number than is now common. There should be no windows directly back of the teacher, or on the side towards which the scholars face, unless the light is modified by curtains or by ground glass. Every window should be suspended with weights, and furnished with blinds and curtains; and if in a much frequented street, the lower sash should be glazed with ground glass.

In the plan of an octagonal, see p. 90, fig. 2.

VENTILATION.

Every school-room should be provided with means of ventilation, or of renewing the vital portions of the atmosphere which are constantly absorbed, and of removing impurities which at the same time are generated, by the breathing and insensible perspiration of teacher and pupils, and by burning fires and lights.

The atmosphere which surrounds our earth to the height of forty-five miles, and in which we live, and move, and have our being, is composed mainly of two ingredients, oxygen and nitrogen, with a slight admixture of carbonic acid. The first is called the vital principle, the breath of life, because by forming and purifying the blood it alone sustains life, and supports combustion. But to sustain these processes, there is a constant consumption of this ingredient going on, and, as will be seen by the facts in the case, the formation and accumulation of another ingredient, carbonic acid, which is deadly hostile to animal life and combustion. This gas is sometimes found in wells, and will there extinguish a lighted candle if lowered into it, (and which should always be lowered into a well before any person ventures down,) and is not an uncommon cause of death in such places. It is almost always present in deep mines and at the bottom of caverns.

The air which we breathe, if pure, when taken into the mouth and nostrils, is composed in every one hundred parts, of twenty-one oxygen, seventy-eight nitrogen, and one of carbonic acid. After traversing the innumerable cells into which the lungs are divided and subdivided, and there coming into close contact with the blood, these proportions are essentially changed, and when breathed out, the same quantity of air contains eight per cent. less of oxygen, and eight per cent. more of carbonic acid. If in this condition (without being renewed,) it is breathed again, it is deprived of another quantity of oxygen, and loaded with the same amount of carbonic acid. Each successive act of breathing reduces in this way, and in this proportion, the vital principle of the air, and increases in the same proportion that which destroys life. But in the mean time what has been going on in the lungs with regard to the blood? This fluid, after traversing the whole frame, from the heart to the extremities, parting all along with its heat, and ministering its nourishing particles to the growth and preservation of the body, returns to the heart changed in color, deprived somewhat of its vitality, and loaded with impurities. In this condition, for the purpose of renewing its color, its vitality and its purity, it makes the circuit of the lungs, where by means of innumerable little vessels, inclosing like a delicate net work each individual air cell, every one of its finest particles comes into close contact with the air which has been breathed. If this air has its due proportion of oxygen, the color of the blood changes from a dark purple to a bright scarlet; its vital warmth is restored, and its impurities, by the union of the oxygen of the air with the carbon of blood, of which these impurities are made up, are thrown off in the form of carbonic acid. Thus vitalized and purified, it enters the heart to be sent out again through the system on its errand of life and beneficence, to build up and repair the solid frame work of the body, give tone and vigor to its muscles, and re-string all its nerves to vibrate in unison with the glorious sights and thrilling sounds of nature, and the still sad music of humanity.

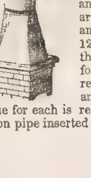
*See plan of Octagonal School-house, Fig. 2.

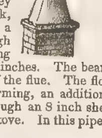
But in case the air with which the blood comes in contact, through the thin membranes that constitute the cells of the lungs, does not contain its due proportion of oxygen, viz. twenty or twenty-one per cent. as when it has once been breathed, then the blood returns to the heart unendued with newness of life, and loaded with carbon and other impurities, which unfit it for the purposes of nourishment, the repair, and maintenance of the vigorous actions of all the parts, and especially of the brain, and spinal column, the great fountains of nervous power. If this process is long continued, even though the air be but slightly deteriorated, the effects will be evident in the languid and feeble action of the muscles, the sunken eye, the squalid hue of the skin, the unnatural irritability of the nervous system, a disinclination to all mental and bodily exertion, and a tendency to stupor, headache and fainting. If the air is very impure, i. e. has but little or no oxygen, and much carbonic acid, then the imperfect and poisoned blood will act with a peculiar and malignant energy on the whole system, and especially on the brain; and convulsions, apoplexy, and death must ensue.

The necessity of renewing the atmosphere, does not arise solely from the consumption of the oxygen, and the constant generation of carbonic acid, but from the presence of other destructive agents, and impurities. There is carburetted hydrogen, which Dr. Dunglinson in his Physiology, characterizes, "as very depressing to the vital functions. Even when largely diluted with atmospheric air, it occasions vertigo, sickness, diminution of the force and velocity of the pulse, reduction of muscular vigor and every symptom of diminishing power." There is also sulphuretted hydrogen, which the same author says, in its pure state, kills instantly, and in its diluted state, produces powerful sedative effects on the pulse, muscles, and whole nervous system. There are also offensive and destructive impurities arising from the decomposition of animal and vegetable matter in contact with the stove, or dissolved in the evaporating dish.

The objects to be attained are—the removal of such impurities, as have been referred to, and which are constantly generated, wherever there is animal life and burning fires, and the due supply of that vital principle, which is constantly consumed by breathing and combustion. The first can be in no other way effectually secured, but by making provision for its escape into the open air, both at the top and the bottom of the room; and the second, but by introducing a current of pure air from the outside of the building, warmed in winter by a furnace, or in some other mode, before entering the room. The two processes should go on together—i. e. the escape of the vitiated air from within, and the introduction of the pure air from without. The common fire-place and chimney secures the first object very effectually, for there is always a strong current of air near the floor, towards the fire, to support combustion, and supply the partial vacuum in the chimney occasioned by the ascending column of smoke and rarified air, and in this current the carbonic acid and other impurities will be drawn into the fire and up the chimney. But there is such an enormous waste of heat in these fire-places, and such a constant influx of cold air through every crevice in the imperfect fittings of the doors and windows, to supply the current always ascending the chimney, that this mode of ventilation should not be relied on. The common mode of ventilating, by opening a window or door, although better than none, is also imperfect and objectionable; as the cold air falls directly on the head, neck, and other exposed parts of the body, when every pore is open, and thus causes discomfort, catarrh, and other more serious evils, to those sitting near, besides reducing the temperature of the whole room too suddenly and too low. This mode, however, should be resorted to at recess.

There should be one or more openings, expressly for ventilation, both at the top and the bottom of the room, of not less than twelve inches square, capable of being wholly or partially closed by a slide of wood or metal, and, if possible, these openings, or the receptacle into which they discharge, should be connected with the chimney or smoke-flue, in which there is already a column of heated air. By an opening in or near the ceiling, the warmer impurities (and air when heated, and especially when over-heated, will retain noxious gases longer) will pass off. By an opening *near the floor*, into the smoke flue, the colder impurities (and carbonic acid, and the other noxious gases, which at first rise, soon diffuse themselves through the atmosphere, cool, and subside towards the floor) will be drawn in to supply the current of heated air and smoke ascending the chimney. These openings, however, may let cold air in, and will not always secure the proper ventilation of a school-room, unless there is a current of pure warm air flowing in at the same time. Whenever there is such a current there will be a greater economy, as well as a more rapid and uniform diffusion of the heat, by inserting the outlet for the vitiated air near the floor, and at the greatest distance from the inlet of warm air.





NOTE.—The use of ***J. L. Mott's Patent Cowls*** have been found highly useful in the ventilation of School-rooms as well as of other buildings, and is thus described in a letter from Joseph Curtis, Esq., of New York City.

"A school-house of one story, say 30 by 45 to 65 feet, requires on each side, two flues; if more than two stories, an additional pair of flues is required for each story. They are constructed without any additional cost for mason work, and in the following manner; by a recess of 4 inches (in a 12 inch wall,) by 20 inches, and continue it to, and through the stone copeing, on which the cowl is placed. The furring for the lath being one inch, leaves a flue of 100 square inches. The beams resting in the wall, will be, (at this end,) on each side of the flue. The floor and ceiling complete the flue. If stoves are used for warming, an additional flue for each is required, and should be located so as to lead the air through an 8 inch sheet iron pipe inserted through the floor within 4 inches of the bottom of the stove. In this pipe is

a valve, which when opened admits a current of air from the top of the building. The horizontal air passages formed by the beams, floor and ceiling, should be so located as to divide the building into about equal parts, as to give as near as may be equal division. The two air passages nearest the centre of the house are for ingress of air from the ceiling, (in winter,) through perforated sheet zinc, 12 inches broad, extending across the room and flush with the plastering. The other two air passages, situated outside of the two former, are for the agress, (in winter,) through the floor, in which are 144 brass tubes, (the size of the hole is that of No. 16 wire,) to every square foot.

"From this it will be perceived that the cowls, being shifted to the opposite sides as the season changes from summer to winter, you have a pure air, with more economy of fuel than by any other mode now in use; inasmuch as in winter, the cold air ingress is directed to the most heated part of the room, and that which is most distant from the children, while the exhausting cowl, (the power of which is greater than that of the receiving one,) takes the cold and foul air through the floor, and discharges it at the top of the house; in summer the ingress is from the floor, in small jets, and the egress from the ceiling."

TEMPERATURE.

Fuel of the right kind, in the right condition, in suitable quantity and in due season must be provided. The best modes of consuming it so as to extract its heat, and diffuse it equally through all parts of the room, and retain it as long as is safe, must be resorted to. The means of regulating it, so as to keep up a uniform temperature in different parts of the room, and to graduate it to the varying circumstances of a school at different periods of the day, and in different states of the weather, must not be overlooked.

The open stove with large pipe, not bending till the horizontal part is carried ten or twelve feet above the heads of the children, affords as effectual, economical and unobjectionable a mode of consuming the fuel and disseminating the heat as any stove of this kind. It is far superior in point of economy to the open fire-place, as ordinarily constructed, in which near seven-eighths of the heat evolved ascends the chimney, and only one-eighth, or according to Rumford and Franklin, only one-fifteenth is radiated from the front of the fire into the room. It has to some extent the cheerful light of the open fire, to which habit and association have attached us, and the advantages of the latter, in opening broadly near the floor, and thus drawing in the colder air with the carbonic acid in the current which goes to sustain the combustion and ascend the large pipe of the stove.

Various plans have been proposed and adopted, to make the common stove, whether close or open, serviceable in warming pure air before it is thrown into the room. Mr. Woodbridge in his essay on school-houses, describes one as follows:—the stove is inclosed on three sides in a case of sheet iron, leaving a space of two or three inches beneath and around the stove, and as it rises around it becomes warmed before it enters the room at the top of the case. The case is moveable so as to allow of the cleaning out of any dust which might collect between it and the stove. Mr. Palmer, in his Manual for Teachers, secures the same object by conducting the air from without, into a passage which traverses the bottom of the stove five or six times before it enters the room, and thus becomes warm.

In Millar's *Patent Ventilating School-house Stove*, the air is conducted from without, into a chamber below the fire-plate, and after circulating through pipes around the fire, escapes into the room.

The same thing can be secured by a similar arrangement connected with stoves for burning anthracite coal. In the Olmsted stove, for instance, the pure air from without can be made to pass in contact with the exterior, as well as the interior surface of the radiators and thus be warmed before entering the room. This stove has an advantage, in admitting of the slow combustion of billets of wood in connection with nut or pea coal, and thus maintaining a fire which will keep up a uniform temperature of the proper degree at the cheapest rate. The large radiating surface, which is nothing more than prolonged pipe, conveniently arranged, imbibes and diffuses all the heat evolved by the combustion of the fuel, so that at the point where it enters the chimney, the heat of the pipe is scarcely perceptible.

The best mode, at the same time, of warming and ventilating a school-room, especially if it is large, is by pure air heated in a stove or furnace placed in the cellar or a room lower than the one to be warmed. No portion of the room, or the movements of the scholars, or the supervision of the teacher, are encumbered or interrupted by stove or pipe. The fire in such places can be maintained without noise and without throwing dust or smoke into the room. The offensive odors and impurities of burnt air, or rather of particles of vegetable or animal matter floating in the air, are not experienced. The heat can be conducted into the room at different points, and is thus diffused so as to secure a uniform summer temperature in every part of it. A room thus heated, even without any special arrangements for this object, will be tolerably well ventilated, for the constant influx of warm pure air into the room will force that which is already in it out at every crack and crevice, and thus reverse the process which is ordinarily going on in every school-room. By an opening or rather several small openings into the ceiling, or a flue, which in either case should connect with the outer air, the escape of the impure air will be more effectually secured.

In our arrangement for artificial warmth, especially in all stoves for burning anthracite coal, where intense heat is liable to be communicated to the iron surface, if we would preserve the purity of the atmosphere at all degrees of temperature, it is necessary to secure the presence of a certain quantity of moisture, by an evaporating dish supplied with pure water. The water should be frequently changed. The gathering and settling of dirt and other impurities in the vessel containing the water, can be guarded against by closing the top except to admit a suspended linen or cotton cloth, which will absorb the water and give it out again from its exposed surface.

SEATS AND DESKS FOR SCHOLARS AND TEACHER.

In the construction and arrangement of the seats and desks of a school-room, due regard should be had to the convenience, comfort and health of those who are to occupy them. To secure these objects, they should be made for the young and not for grown persons, and of varying heights, for children of different ages, from four years and under, to sixteen and upwards. They should be adapted to each other and the purposes for which they will be used, such as writing and ciphering, so as to prevent any awkward, inconvenient or unhealthy positions of the limbs, chest or spine. They should be easy of access, so that every scholar can go to and from his seat and change his position, and the teacher can approach each scholar and give the required attention and instruction, without disturbing any other person than the one concerned. They should be so arranged as to facilitate habits of attention, take away all temptations and encouragement to violate the rules of the school on the part of any scholar, and admit of the constant and complete supervision of the whole school by the teacher.

Each scholar should be furnished with a seat and desk, properly adapted to each other, as to height and distance, and of varying heights, (the seats from nine inches and a half, to fifteen and a half, with desks to correspond,) for children of different age or size. The seat should be so made, that the feet of every child when properly seated, can *rest* on the floor, and the upper and lower part of the leg form a right angle at the knee; and the back, whether separated from, or forming part of the adjoining desk behind, should recline to correspond with the natural curves of the spine and the shoulders. The seat should be made, as far as possible, like a convenient chair.

The desk should not be removed from the seat either in distance or height, so far as to require the body, the neck or the chest to be bent forward in a constrained manner, or the elbow or shoulder blades to be painfully elevated whenever the scholar is writing or ciphering. These last positions, to which so many children are forced by the badly constructed seats and desks of our ordinary school-houses, have led not unfrequently to distortions of the form, and particularly to spinal affections of the most distressing character.

The arrangements for the teacher should be such, that he can survey the whole school at a glance, address his instruction, when necessary, to the whole school, approach each scholar in his seat without incommoding any other, and conduct the recitations most conveniently to himself, and with the least interference with the study of the school.

With this view, his seat and desk should be placed in front of the school on a raised platform; the aisles should be so arranged as to separate each range of the scholars' seats; and an open space, or appropriate seats, should be provided for the reciting classes, in front or the side of his desk; or what would be better, a recitation room opening from the platform, or else a special platform in the rear of the school.

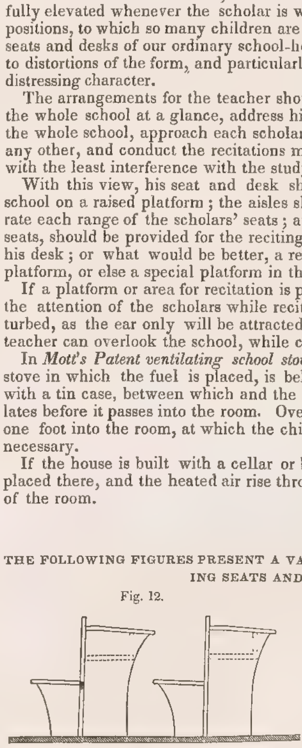
If a platform or area for recitation is provided in the rear of the school, the attention of the scholars while reciting will be less likely to be disturbed, as the ear only will be attracted by what is going on, and the teacher can overlook the school, while conducting the recitations.

In *Mott's Patent ventilating school stove*, the heater, or portion of the stove in which the fuel is placed, is below the floor, and is surrounded with a tin case, between which and the heater the air from without circulates before it passes into the room. Over the fire is a sheet iron box rising one foot into the room, at which the children can warm their feet when necessary.

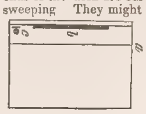
If the house is built with a cellar or basement room, the stove can be placed there, and the heated air rise through openings into different parts of the room.

THE FOLLOWING FIGURES PRESENT A VARIETY OF MODES OF CONSTRUCTING SEATS AND DESKS.

Fig. 12.

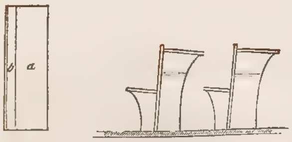


Represents a section of seat and desk for one pupil—the front of the desk slopes $2\frac{1}{2}$ inches in 16, and constitutes the back of the preceding seat. The seat inclines a little from the edge. The desk is two feet long by eighteen inches wide, three inches of which is level, and the remaining part is inclined one inch to the foot. The edge of the desk and the seat is in the same perpendicular line. There is a shelf for books one foot wide. The ends of the desk are curved so as to be convenient for getting out and in the seat, and for sweeping They might be still more curved, and iron supporters would be still better, as occupying less space. The level portion of the desk has a groove (*a*) running along the line of the slope, to prevent pencils and pens from rolling off; an opening on the back side (*b*) to receive a slate, with which every desk should be furnished as a part of the furniture of the school-room; and an opening (*c*) to receive an inkstand, which is covered by a metallic lid.



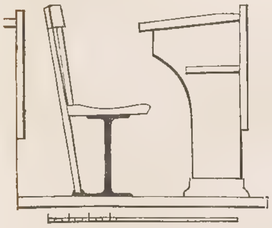
The seats and desks should vary in height, the former from 9 or 10 inches to 16 or 17, and the latter, from 23 to 29 inches. The youngest pupils being seated nearest to the teacher's desk.

Fig, 13.



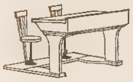
Represents a seat and desk for two pupils, but nearly similar to Fig. 12. The Primary and Intermediate Schools, Providence, R. I, are furnished with such.

Fig. 14.



Represents a section of chair and desk used in the Providence Grammar Schools. The chair is on an iron pedestal, and is attached to the floor by four screws. This chair can be furnished by the makers in Providence, in quantities to fit up a school-room, from $1 to $1.25.

Fig. 15.



Represents a modification of the above, as used in the High Schools. The desk and seat are attached to a platform which is movable.

Fig. 16.

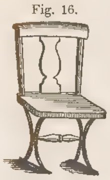
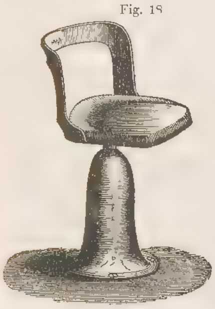


Fig. 17.

Figure 16, represents one of "Kimball's Improved School Chair, used in the High Schools of Salem, Mass., and in many of the district schools in the neighborhood. The supporters are of cast iron, and when screwed to the floor are perfectly firm.

Figure 17, represents a modification of Fig. 16, for Primary Schools.

Fig. 18



Mott's Patent School Chair.

This chair is made of cast iron, except the seat, and is so constructed, that the seat and back may be turned round, while the bottom, being screwed fast to the floor, remains stationary. Several school-rooms belonging to the Public School Society of the City of New York are furnished with this chair

Fig. 19.

Fig. 20.

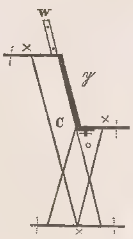
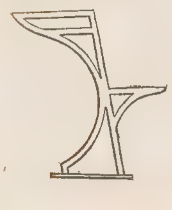


Fig. 19 View of School Desk and Chair, used in the Girl's High School, Newburyport.

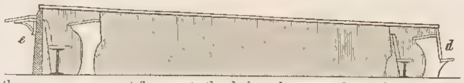
Fig. 20. One side of cast iron frame of do. W, part of frame to which the back of the chair, and ends of desk are attached. C, projection, into which a cast iron brace is screwed.

Fig. 21.

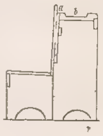


Represents a cast iron frame for the end of a desk. These castings weigh from ten to thirteen pounds, and are made from four sizes. They have been used for the end pieces in Fig. 22.

Fig. 22.

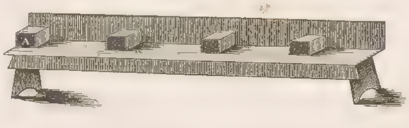


Represents a modification of the plan of a seat and desk for two, so as to economize the room, secure great firmness to the desk, and separate the pupils as effectually as an aisle of the ordinary width. Each range of desks is divided by a partition extending from the floor to four inches above the surface of the desk. The seat can be attached to the desk as in Figs 12 and 13, or a chair can be used as represented above.



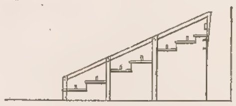
To accommodate two of the larger pupils in winter, a desk like a table leaf can be attached to the highest end of each range, (*e*) and to accommodate the same number of smaller children in summer, movable sand desks can be placed at the lowest end.(*d*) The sand desk has an opening (*a*) to receive a slate, and a groove (*b*) to receive a thin layer of sand, if it should be thought desirable to use sand, before using the slate, as is done in the New York primary Schools.

Fig. 23.



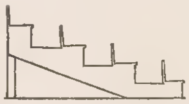
Represents a view of a bench used in some of the primary schools, Boston, on which the childred are separated by a little compartment (A,) for books, which also serves as a support for the arms.

Fig. 24.



Gives the end view of the gallery, in the Primary Schools of the New-York Public School Society. The one here represented contains seven seats, each thirty-one feet long, with backs seven inches high. The gallery is set two and a half feet from the wall, and is left entirely open underneath, and is used as a ward-robe for the younger children.

Fig. 25.



Presents a slight modification of the gallery.

APPARATUS.

One blackboard, at least, is indispensably necessary. This should be so placed, as to be easily accessible, and in full view of the whole school. The larger it is, the more useful it can be made. The board should be free from knots, or cracks, well seasoned, smoothly planed, and then rubbed with sand-paper, and painted black, without varnish. On the lower side should be placed a trough to receive the chalk or crayon, tin or brass holders, (called port-crayons) a rubber of cloth, wash-leather, or sponge. If the board is broad, or in two or more parts, it should de kept from warping or opening by cleates of iron or wood on the back side or ends.

If there is but one blackboard, it should be moveable, so as to be used in different parts of the room. For this purpose, it must be suspended on hooks, or rings inserted in the upper edge, or what is better, on a movable frame, like the painter's easel.

Fig. 26.

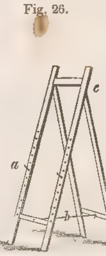


Fig. 27.

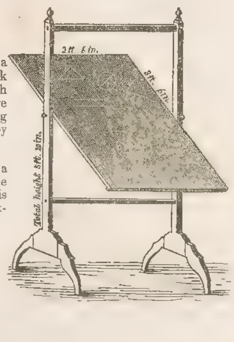


Fig. 26 represents a movable frame for a black board. *a* Pins on which the board rests. *c* Hinge or joint to the supporting legs which are braced by hook *b*.

Fig. 27. represents a movable blackboard, the construction of which is evident without any explanation.

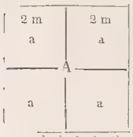
Fig. 28.

Represents a cheap movable black board for a Primary School

PLANS OF SCHOOL ROOMS, &c.

In determining the details of construction and arrangement for a school-house, due regard must, of course, be had to the varying circumstances of country and city, of a large and a small number of scholars, of schools of different grades, and of different systems of instruction.

1. In by far the largest number of country districts, as they are now situated, there will be but one school-room, with a smaller room for recitations and other purposes needed. This must be arranged and fitted up for scholars of all ages, for the varying circumstances of a summer and of a winter school, and for other purposes, religious and secular, than those of a school, and in every particular of construction and arrangement, the closest economy of material and labor must be studied. A union of two or more districts, for the purpose of maintaining in



each (a a a a) a school for the younger children, and in the center (A) of the associated districts a school for the older children of all, or, what would be better, a consolidation of two or more districts into one, for these and all other school purposes, would do away with the almost insuperable difficulties which now exist in country districts, in the way of comfortable and attractive school-houses, as well as of thoroughly governed and instructed schools.

2. In small villages, or populous country districts, at least two school-rooms should be provided, and as there will be other places for public meetings of various kinds, each room should be appropriated and fitted up exclusively for the use of the younger or the older pupils. It is better, on many accounts, to have two schools on the same floor, than one above the other.

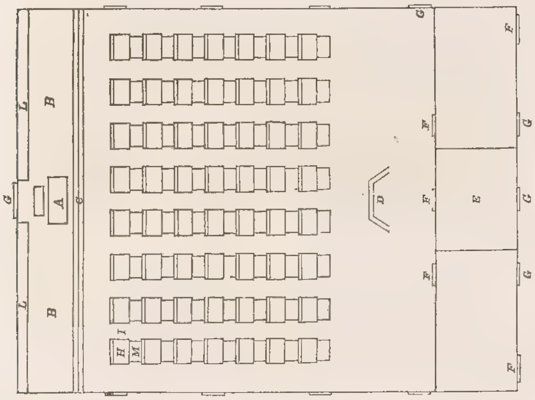
3. In large villages and cities, a better classification of the schools can be adopted, and, of course, more completeness can be given to the construction and arrangement of the buildings and rooms appropriated to each grade of schools. This classification should embrace at least three grades—viz. Primary, with an infant department; Secondary, or Grammar; Superior, or High Schools. In manufacturing villages, and in certain sections of large cities, regularly organized Infant Schools should be established and devoted mainly to the culture of the morals, manners, language and health of very young children.

4. The arrangement as to supervision, instruction and recitations, must have reference to the size of the school; the number of teachers and assistants; the general organization of the school, whether in one room for study, and separate class rooms for recitation, or the several classes in distinct rooms under appropriate teachers, each teacher having specified studies; and the method of instruction pursued, whether the mutual, simultaneous, or mixed.

Since the year 1830, and especially since 1838, much ingenuity has been expended by practical teachers and architects, in devising and perfecting plans of school-houses, with all the details of construction and fixtures, modified to suit the varied circumstances enumerated above, specimens of which, with explanations and descriptions, will be here given.

Fig. 29.

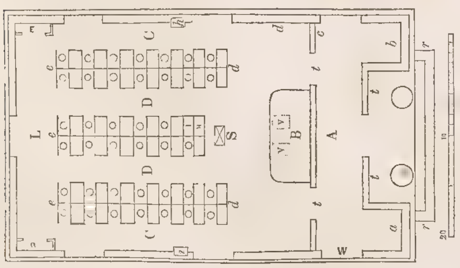
Plan recommended by Horace Mann, Secretary of Board of Education for Mass.



A. Represents the teacher's desk. *B B.* Teacher's platform, from 1 to 2 ft. in height. *C.* Step for ascending the platform, *L L.* Cases for books, apparatus, cabinet, &c. *H.* Pupil's single desks, 2 ft. by 18 inches. *M.* Pupil's seat, 1 ft. by 20 inches. *I.* Aisles, 1 ft. 6 inches in width. *D.* Place for stove, if one be used. *E.* Room for recitation, for retiring in case of sudden indisposition, for interview with parents, when necessary, &c. It may also be used for the library, &c. *F F F F F.* Doors into the boys' and girls' entries—from the entries into the school-room, and from the school-room into the recitation room. *G G G G.* Windows. The windows on the sides are not lettered. The seats and desks are substantially the same as represented in Fig. 12.

Fig. 30.

Plan of School-room in Washington District, Hartford, Conn. drawn by Henry Barnard.

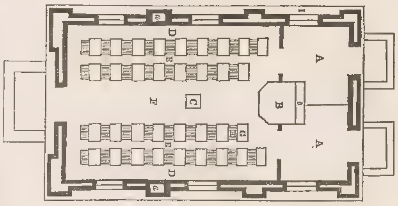


The exterior dimensions are 40 ft. by 26 ft., and the school-room, exclusive of the recess for the pillars, and the entry, is 30 ft. by 25.

A. Entry, on one-side of which (*a*) is fitted up for girls, and the other (*b*) for boys. B. Teacher's platform, 9 ft, long by 4 ft. 6 inches wide, and 9 inches high, with a blackboard occupying the wall behind. V V. Teacher's desk. C C. side aisles 3 ft. wide. L. Rear aisles 4 ft. wide. D D. Aisles each 2 ft. 7 inches. S. Stove. H. Desk. (See Fig. 22.) I. Chair. (See Fig. 14.) *d.* Sand desk. (See Fig. 22.) *e.* Leaf, &c. (See Fig. 22.) *i.* Smoke flue. *h.* Ventilating flue with opening at top and bottom. W W. Seven windows. *r r.* Scrapers for feet. *t t t t.* Mats. *c.* Sink for water pail, basin, &c. E. Closet for library of 600 vols. G. Closet for apparatus, &c.

Fig. 31.

Plan of School-room in District No. 6, Windsor, Conn. as drawn by Henry Barnard
See Fig. 6.



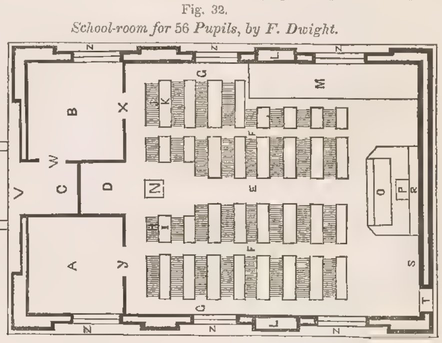
The building is 33 feet 6 inches long, 21 feet 8 inches wide.

The school-room is 24 feet 5 inches long, by 19 feet 4 inches wide, and 15 feet 6 inches high in the clear, and was intended for a school of 36 pupils, but will accommodate 8 or 10 more on movable seats.

A A. Entries 7 ft. 3 inches by 9 ft. 3 inches in the rear of the building; one for boys and the other for girls; each supplied with scraper, mats, shelves and hooks for hats and outer garments. B. Teacher's platform, 5 feet 2 inches wide, by 6 feet deep. *b.* Shelves for books, in front of which is a movable blackboard, 5 feet by 4, suspended on weights, and steadied by a groove on each side, so as to admit of being raised and lowered by the teacher. D D. Passages round the room 2 feet wide. E E. Aisles 15 inches wide. E. Aisles 5 feet 3 inches wide. C. Stove. *a a.* Flues—one for smoke, and the other for ventilation. G. Desk for one pupil 2 feet long and 18 inches wide. H Seat for one pupil, varying from 9½ inches to 17½ inches.

Fig. 32.

School-room for 56 Pupils, by F. Dwight.



The building is 36 ft. long by 26 wide, and 19 ft. high from the ground to the eaves, including 2 ft. base. V. Main entrance. C. Outer entry. W. Door leading into clothes entry B. X. Door into school-room 24 ft. by 24, and 15 ft. high in the clear. N. Stove. D. Recess for wood. Y. Door to recitation and library room A. M. Platform for recitation. O. Teacher's desk. P. His seat, and R. shelves for his books, &c. S. Map of the World, and on the opposite side of teacher, a blackboard. E. Center aisle 2 ft. wide. F F. Division aisle, 18 inches, and G G. side aisles, 20 inches. K. Desk for two pupils, 4 ft. long by 18 inches wide. J. Seat for two, 12 inches wide, and varying from 9½ inches to 16 high. H I. Seat and desk for one pupil. Z. Windows three on each side. L L. Ventilation and smoke flue.

Plan of Octagonal School-house, drawn by Messrs. Town & Davis, New-York
See Fig. 2

The octagonal shape will admit of any number of seats and desks, (according to the size of the room,) arranged parallel with the sides, constructed as described in specification, or on such principles as may be preferred. The master's seat may be in the centre of the room, and the seats be so construcred that the scholars may sit with their backs to the centre, by which their attention will not be diverted by facing other scholars on the opposite side, and yet so that at times they may all face the master, and the whole school be formed into one class. The lobby next to the front door is made large, (8 by 20,) so that it may serve for a recitation-room. This lobby is to finish eight feet high, the inside wall to show like a screen, not rising to the roof, and the space above be open to the school-room, and used to put away or station school apparatus. This screen-like wall may be hung with hats and clothes, or the triangular space next the window may be inclosed for this purpose. The face of the octagon opposite to the porch, has a wood-house attached to it, serving as a sheltered way to a double privy beyond. This wood-house is open on two sides, to admit of a cross draught of air, preventing the possibility of a nuisance. Other wing-rooms (A A) may be attached to the remaining sides of the octagon, if additional conveniences for closets, library, or recitation-rooms be desired.

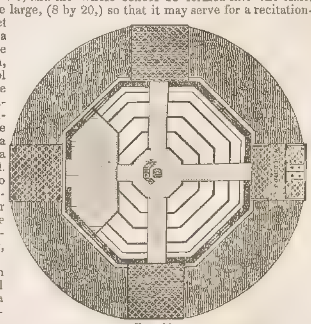


Fig. 33.

The mode here suggested, of a lantern in the centre of the roof for lighting all common school-houses, is so great a change from common usage in our country, that it requires full and clear explanations for its execution, and plain and satisfactory reasons for its general adoption, and of its great excellence in preference to the common mode. They are as follows, viz.

1. A sky-light is well known to be far better and stronger than light from the sides of the building in cloudy weather, and in morning and evening. The difference is of the greatest importance. In short days, (the most used for schools,) it is still more so.

2. The light is far better for all kinds of study than side light, from its quiet uniformity and equal distribution.

3. For smaller houses, the lantern may be square, a simple form easily constructed. The sides, whether square or octagonal, should incline like the drawing, but not so much as to allow water condensed on its inside to drop off, but run down on the inside to the bottom, which should be so formed as to conduct it out by a small aperture at each bottom pane of glass.

4 The glass required to light a school-room equally well with side lights would be double what would be required here, and the lantern would be secure from common accidents, by which a great part of the glass is every year broken.

5. The strong propensity which scholars have to look out by a side window would be mostly prevented, as the shutters to side apertures would only be opened when the warm weather would require it for air, but never in cool weather, and therefore no glass would be used. The shutters being made very tight, by calking, in winter, would make the school-room much warmer than has been common; and, being so well ventilated, and so high in the centre, it would be more healthy.

6. The stove, furnace, or open grate, being in the centre of the room, has great advantages, from diffusing the heat to all parts, and equally to all the scholars; it also admits the pipe to go perpendicularly up, without any inconvenience, and it greatly facilitates the ventilation, and the retention or escape of heat, by means of the sliding cap above.

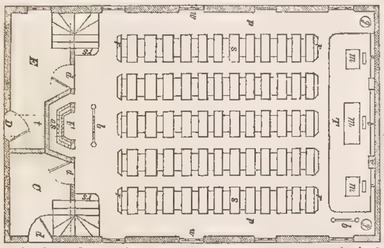
Construction.—Foundation of hard stone, laid with mortar; the superstructure framed and covered with 1¼ inch plank, tongued, grooved, and put on vertically, with a fillet, chamfered at the edges, over the joint, as here shown. In our view, a rustic character is given to the design by covering the sides with slabs; the curved side out, tongued and grooved, without a fillet over the joint; or formed of logs placed vertically, and lathed and plastered on the inside. The sides diminish slightly upward. A rustic portch is also shown, the columns of cedar poles, with vines trained upon them. The door is battened, with braces upon the outside, curved as shown, with a strip around the edge. It is four feet wide, seven high, in two folds, one half to be used in inclement weather. The cornice projects two feet six inches, better to defend the boarding; and may show the ends of the rafters. Roof covered with tin, slate or shingles. Dripping eaves are intended, without gutters. The roof of an octagonal building of ordinary dimensions may with ease and perfect safety be constructed without tie-beams or a garret floor, (which is, in all cases of school-houses, waste room, very much increasing the exposure to fire, as well as the expense.) The wall-plates, in this case, become ties, and must be well secured, so as to form one connected *hoop*, capable of counteracting the pressure outward of the angular rafters. The sides of the roof will abut at top against a similar timber octagonal frame, immediately at the foot of the lantern cupola. This frame must be sufficient to resist the pressure inward of the roof, (which is greater or less, as the roof is more or less inclined in its pitch,) in the same manner as the tie-plates must resist the pressure outward. This security is given in an easy and cheap manner; and may be given entirely by the roof boarding, if it is properly nailed to the angular rafters, and runs horizontally round the roof. By this kind of roof, great additional height is given to the room by *camp-ceiling;* that is, by planing the rafters and roof-boards, or by lathing and plastering on a thin half-inch board ceiling, immediately on the underside of the rafters, as may be most economically performed. This extra height in the centre will admit of low side-walls, from seven to ten feet in the clear, according to the size and importance of the building, and, at the same time, by the most simple principle of philosophy, conduct the heated foul air up to the central aperture, which should be left open quite round the pipe of the stove, or open grate standing in the centre of the room.

In the design given, the side-walls are ten feet high, and the lantern fifteen feet above the floor; eight feet in diameter, four feet high. The sashes may open for additional ventilation, if required, by turning on lateral pivots, regulated by cords attached to the edges above. The breadth of each desk is seventeen inches, with a shelf beneath for books, and an opening in the back to receive a slate. The highest desks are twenty-seven inches, inclined to thirty, and the front forms the back of the seat before it. The seat is ten to twelve inches wide, fifteen high, and each pupil is allowed a space of two feet, side to side.

For the sake of variety, we have given a design in the pointed style, (see Fig. 3.) Any rectangular plan will suit it. The principal light is from one large mullioned window in the rear end. The side openings are for air in summer—not glazed, but closed with tight shutters. The same ventilating cap is shown, and height is gained in the roof by framing with collar beams set up four or five feet above the eaves. The sides, if not of brick or stone, may be boarded vertically, as before described. The porch may be of any convenient size to shelter the door of a recitation-room, through which may be the passage to the school-room. One end of the recitation-room may be partitioned off for a book-room, and one opening on each side may be glazed for light.

Fig. 34.

School-room for 120 Pupils, by G. B. Emerson.

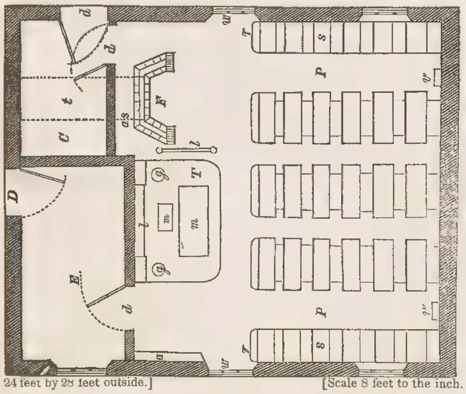


51 feet by 31 feet outside.] [Scale 16 feet to the inch

D. Entrance door. E. Entry. F. Fireplace. C. Wood closet. T. Teacher's platform. *a.* Apparatus shelves. *t.* Air tube beneath the floor. *d.* Doors. *g.* globes. *l.* Library shelves. *m.* Master's table and seat. *p.* Passages. *r.* Recitation seats. *s.* Scholar's desks and seats. *r s.* Stairs to recitation room in the attic. *v.* Ventilator. *w.* Windows. *b.* Movable blackboard. *a s.* A space behind the fire place.

Fig. 35.

School-room for forty-eight Pupils, by G. B. Emerson.



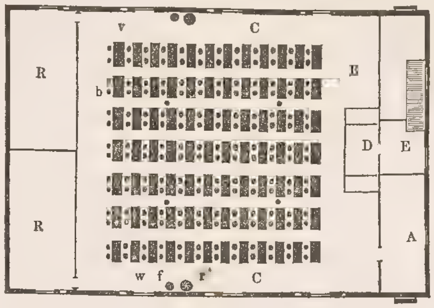
24 feet by 28 feet outside.] [Scale 8 feet to the inch.

D. Entrance door. E. Entry. F. Fireplace. C. Wood closet, or recitation room. T. Teacher's platform. *a.* Apparatus shelves. *t.* Air tube beneath the floor. *d.* Doors. *g.* Globes. *l.* Library shelves. *m.* Master's table and seat. *p.* Passages. *r.* Recitation seats. *s.* Scholars' desks and seats. *v.* Ventilator. *w.* Windows. *b.* Movable blackboard. *a s.* Air space behind the fireplace.

Fig. 36.

Plan of Male Department, Middletown High School.

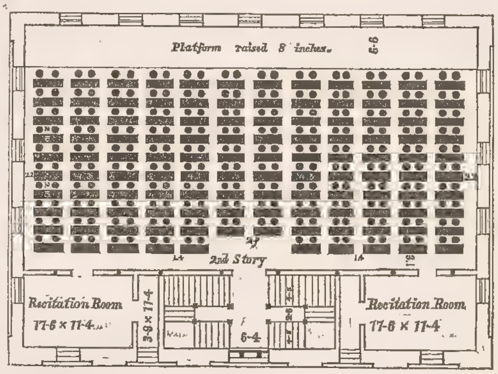
See Fig. 11,



The building is 72 ft. by 54. Male department 50 ft. by 47, with two recitation rooms R R each 25 ft. by 12. There are seven ranges of desks, each accommodating ten pupils, and all facing the teacher's platform D. Each range is separated from the other by an aisle 18 inches wide, and the whole surrounded by an open space C C six ft. wide. There are eight flues for ventilation, opening from the top and bottom of the room, and discharging into the attic above. The whole building is heated by the furnace in the cellar.

Fig. 37.

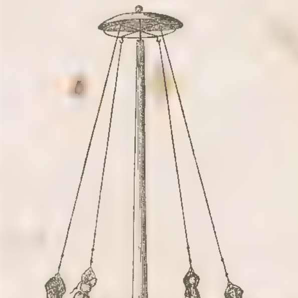
Plan of second floor, Brimmer School, Boston.



The building is 74 feet in length by 53 in depth, and three stories high.

The school-room on the second floor is 70 ft. by 37 ft. wide, and 15 ft. in the clear, and contains 118 desks and 236 chairs of four sizes. The scholars sit with their backs to the platform

Fig. 38.—Rotary Swing.



For the excellent and safe exercise afforded by the Rotary Swing, erect, at the distance of thirty feet from each other, two posts or masts, from sixteen to eighteen feet high above the ground; nine inches diameter at the foot, diminishing to seven and a half at top; of good well-seasoned hard timber; charred with fire about three feet under ground, fixed in sleepers, and bound at top with a strong iron hoop. In the middle of the top of the post is sunk perpendicularly a cylindrical hole, ten inches deep, and two inches in diameter, made strong by an iron ring two inches broad within the top, and by a piece of iron an inch thick to fill up the bottom, tightly fixed in. A strong pivot of iron, of diameter to turn easily in the socket described, but with as little lateral play as possible, is placed vertically in the hole, its upper end standing four inches above it. On this pivot, as an axle, and close to the top of the post, but so as to turn easily, is fixed a wheel of iron, twenty-four inches diameter, strengthened by four spokes, something like a common roasting-jack wheel, but a little larger. The rim should be flat, two inches broad, and half an inch thick. In this rim are six holes or eyes, in which rivet six strong iron hooks, made *to turn in the holes,* to prevent the rope from twisting. To these hooks are fixed six well-chosen ropes, an inch diameter, and each reaching down to within two feet of the ground, having half-a-dozen knots, or small wooden balls, fixed with nails, a foot from each other, beginning at the lower extremity, and ascending to six feet from the ground. A tin cap, like a lamp cover, is placed on the top of the whole machine, fixed to the prolongation of the pivot, and a little larger than the wheel, to protect it from the wet. To this, or to the wheel itself, a few waggoners' bells appended, would have a cheerful effect on the children. The operation of this swing must, from the annexed cut be obvious.

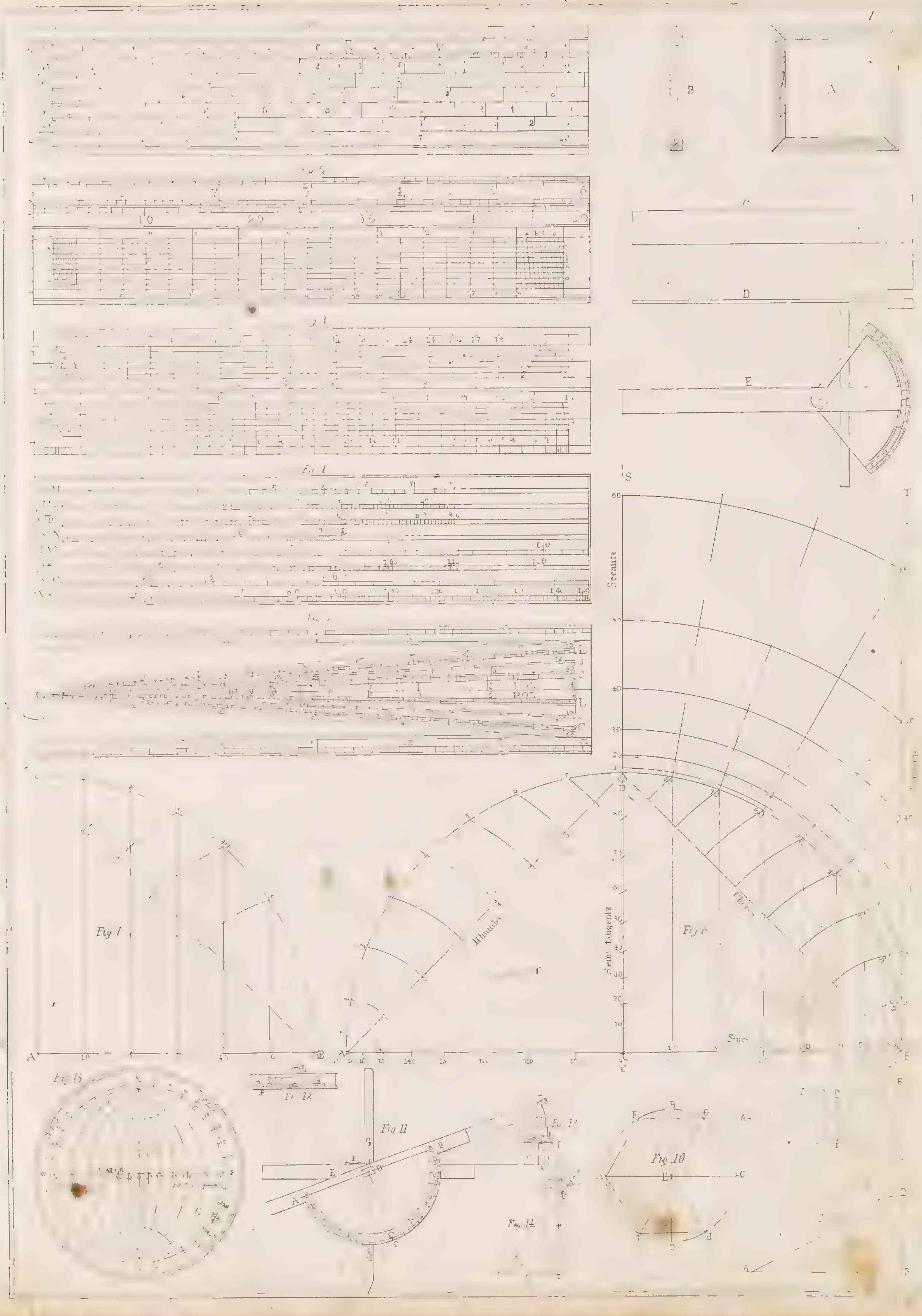
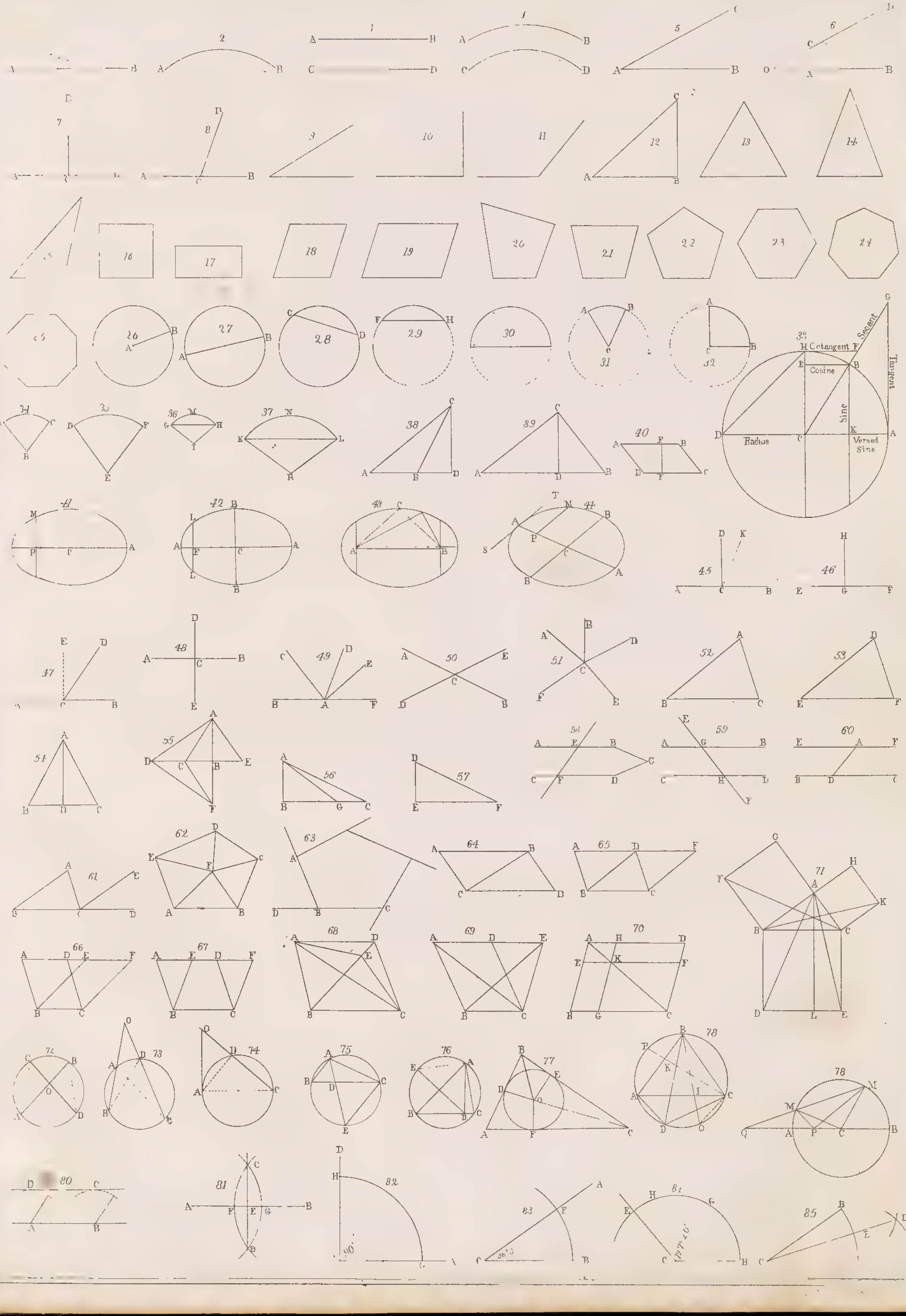
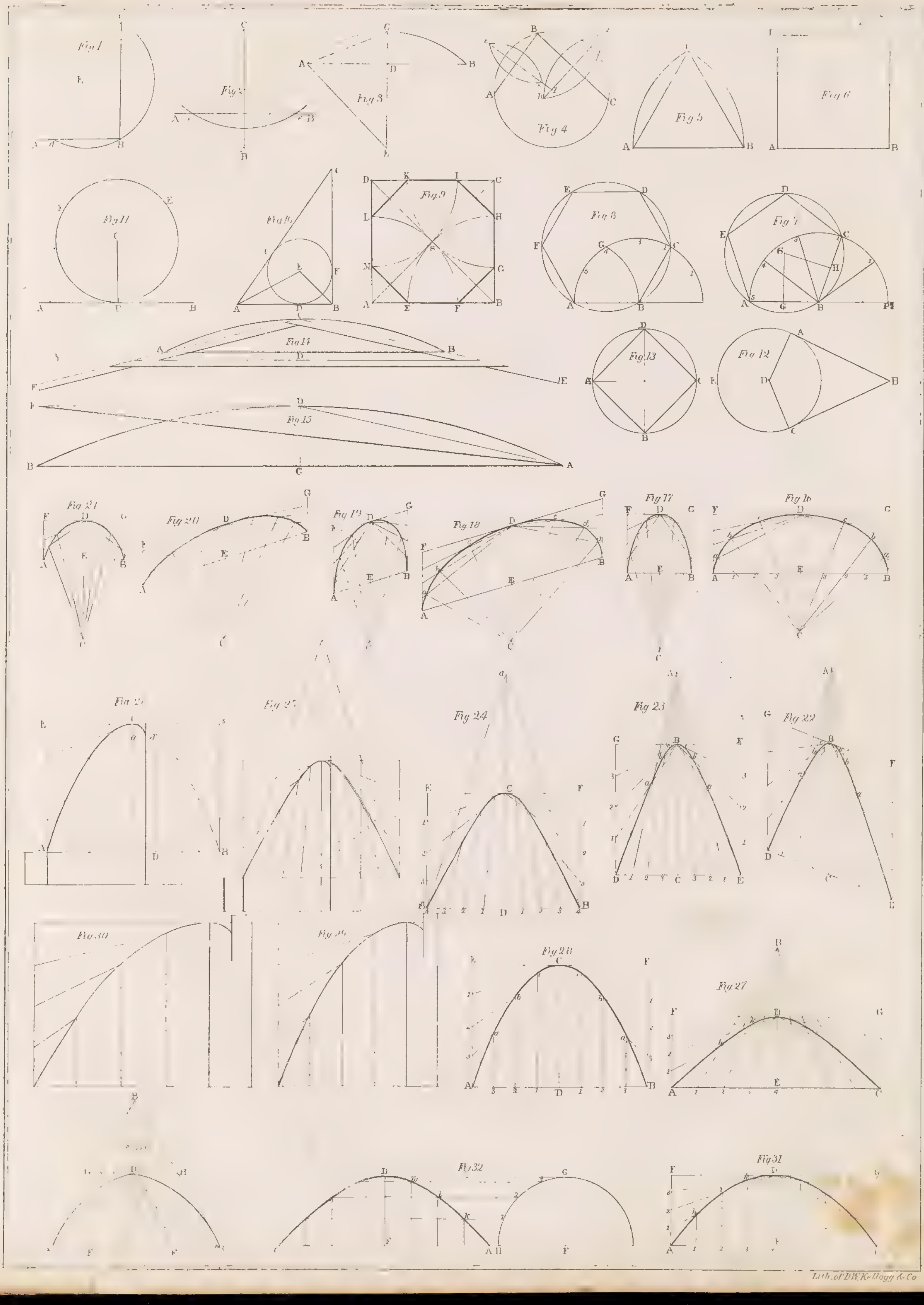


Fig 1
Rhumbs
Secants
Fig 10
Fig 11
Fig 14


Cotangent
Secant
Tangent
Cosine
Sine
Radius
Versed Sine



Lith. of D.W. Kellogg & Co

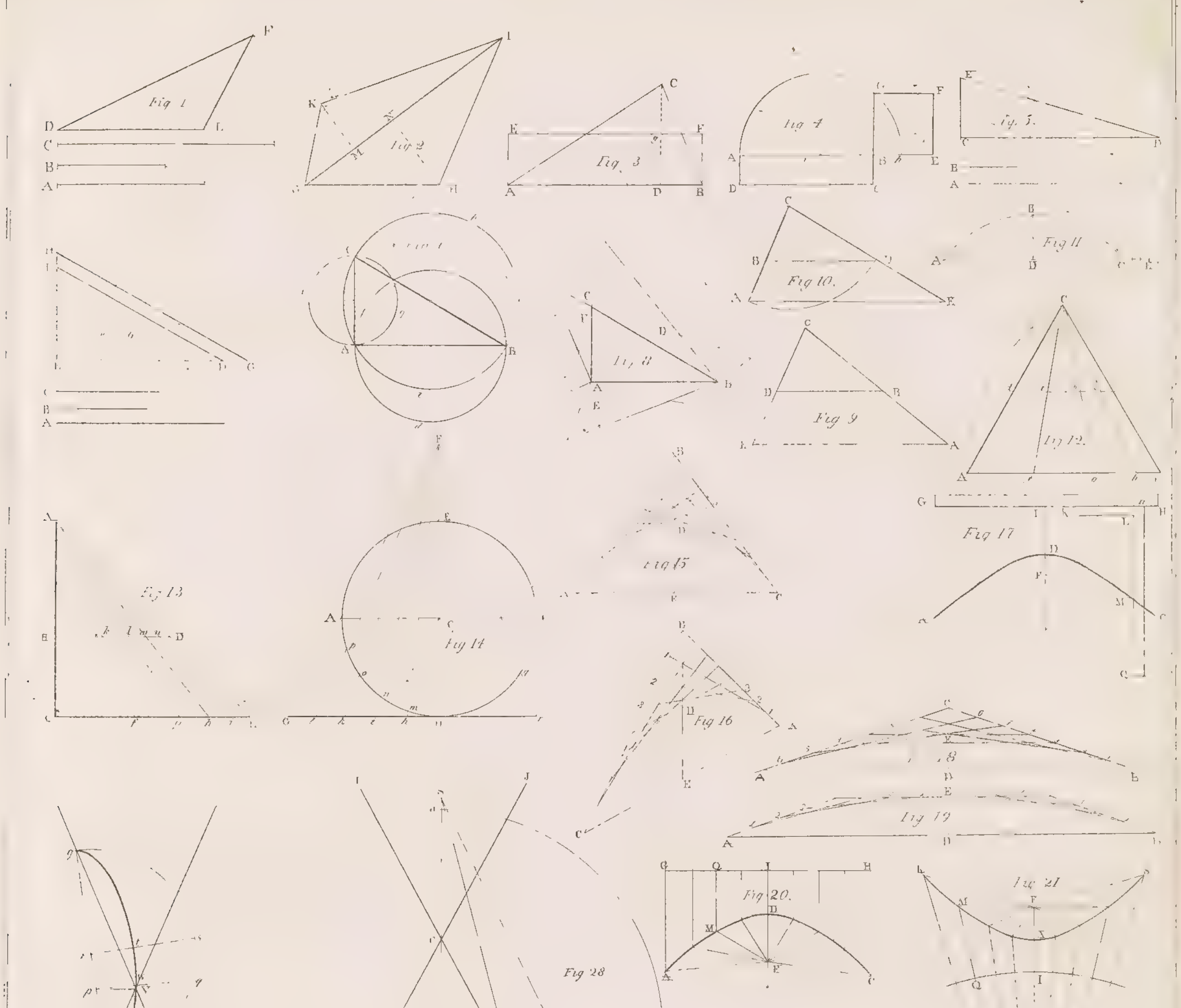


Fig 1
Fig 2
Fig 3
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Fig 28
Fig 29

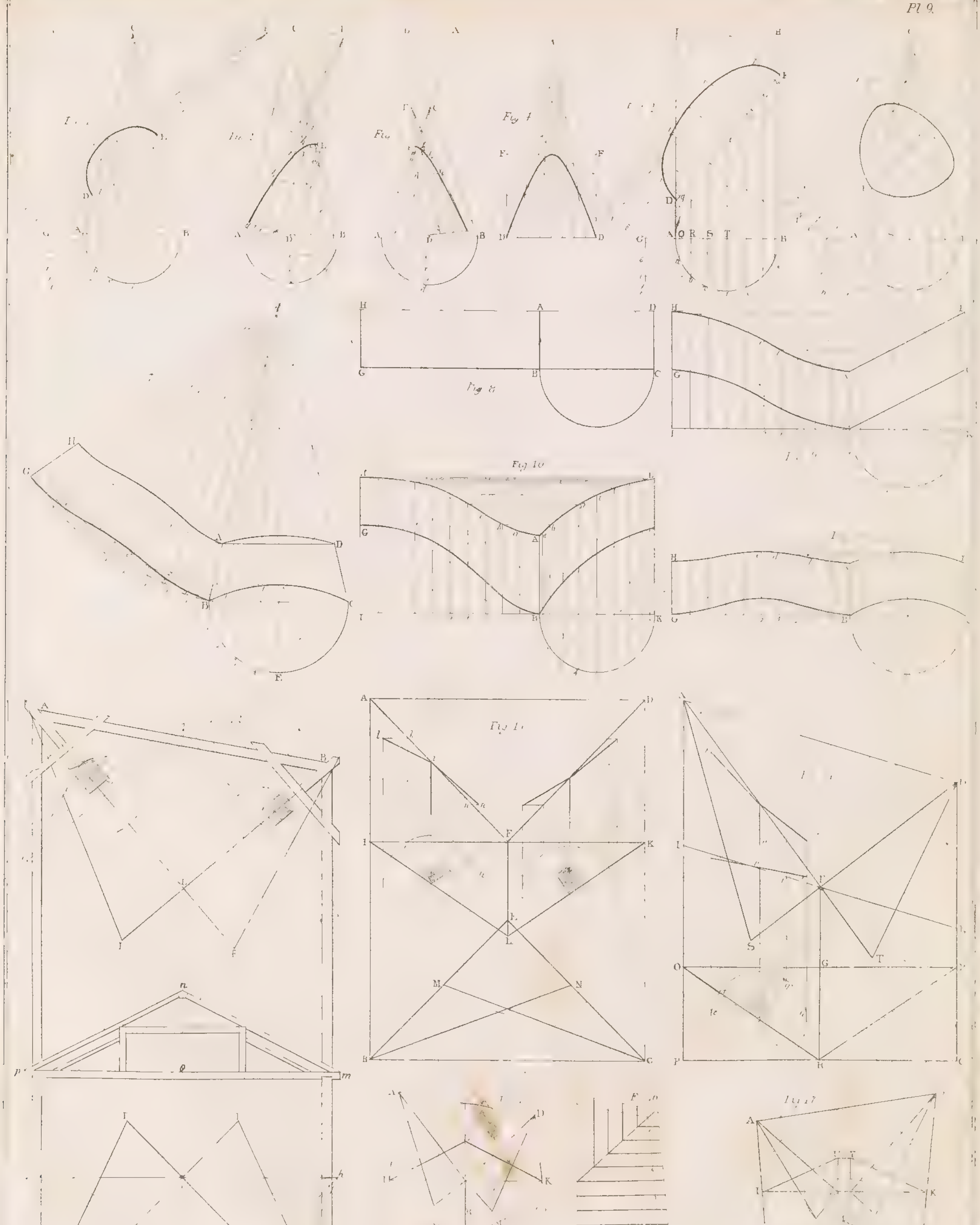


Fig 4
Fig 8
Fig 10

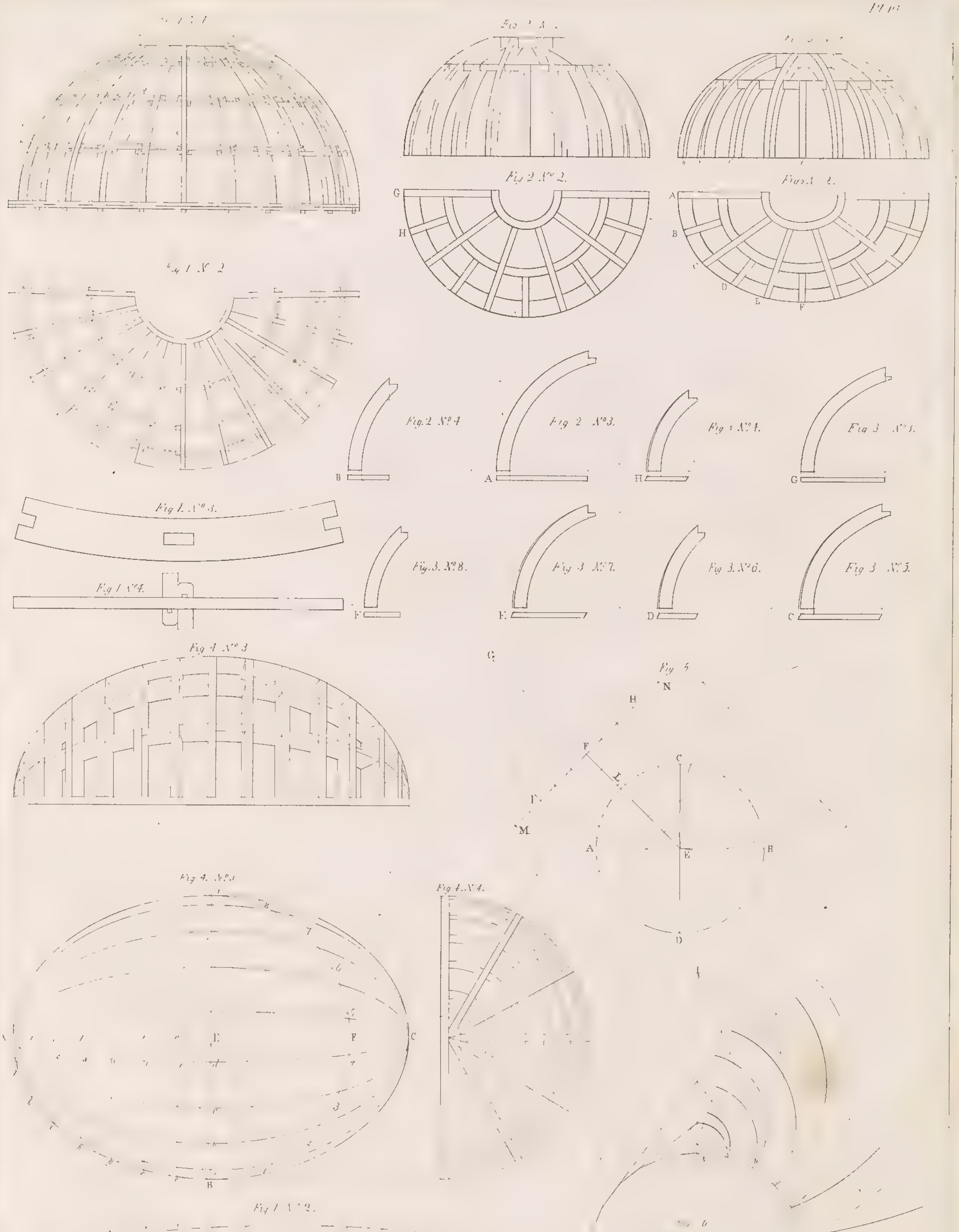
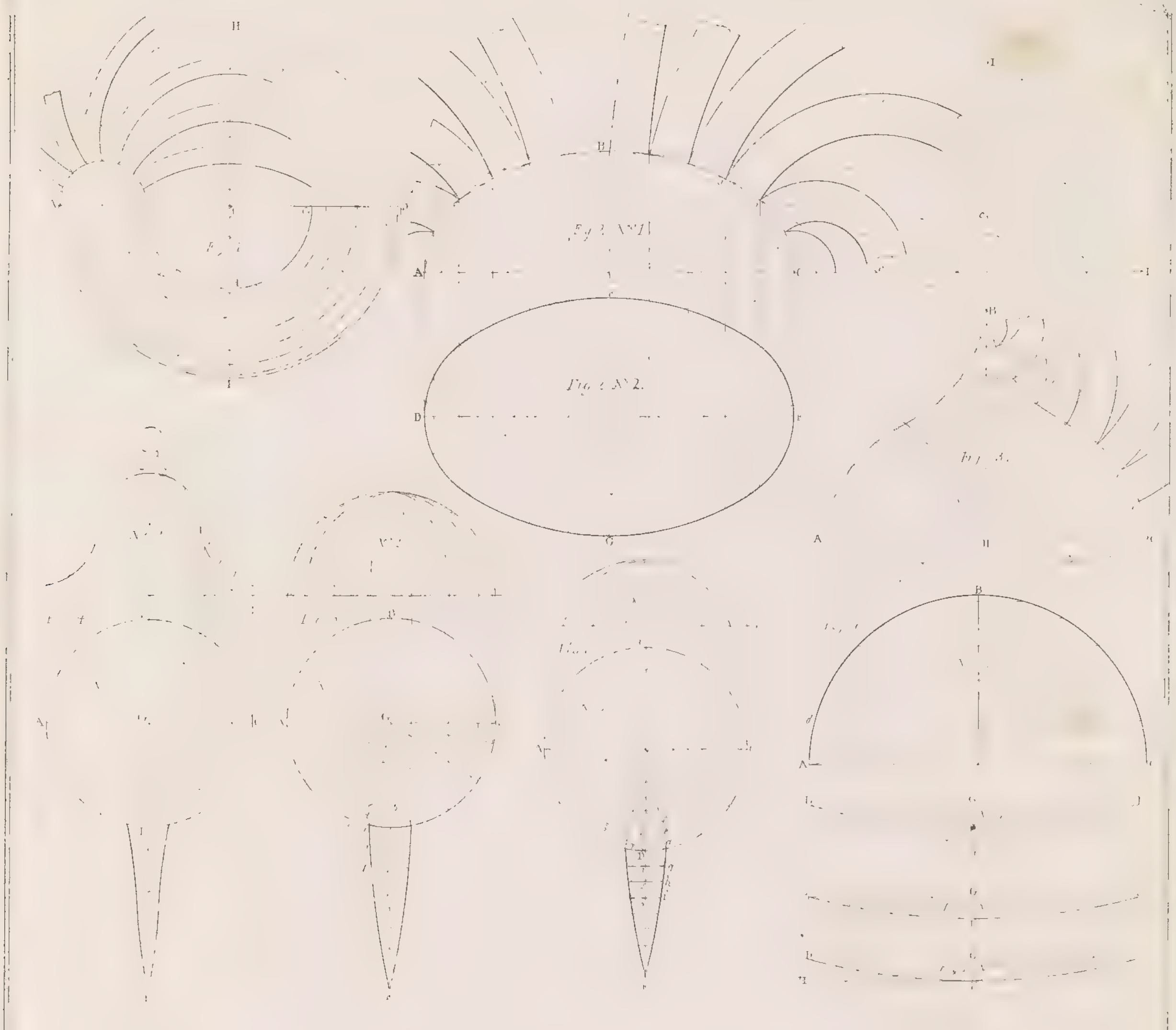
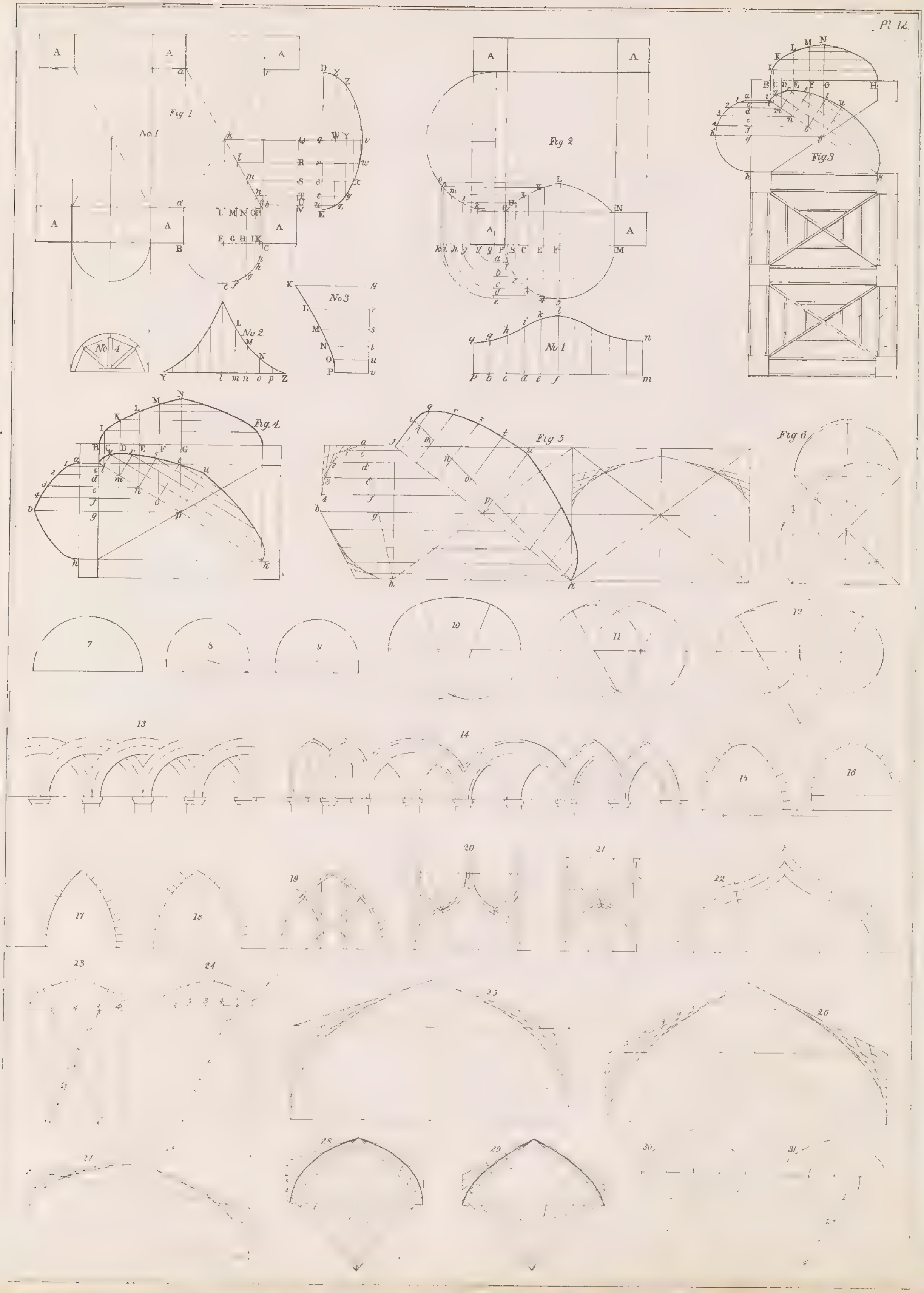


Fig. 2 N° 2.
Fig. 2 N° 4
Fig. 2 N° 3.
Fig. 3 N° 4.
Fig. 1 N° 3.
Fig. 1 N° 4.
Fig. 3. N° 8.
Fig. 3 N° 7.
Fig. 3. N° 6.
Fig. 3 N° 5.
Fig. 4 N° 3
Fig. 5
Fig. 4. N° 1
Fig. 4. N° 4.
Fig. 1 N° 2.



Pl. 12.



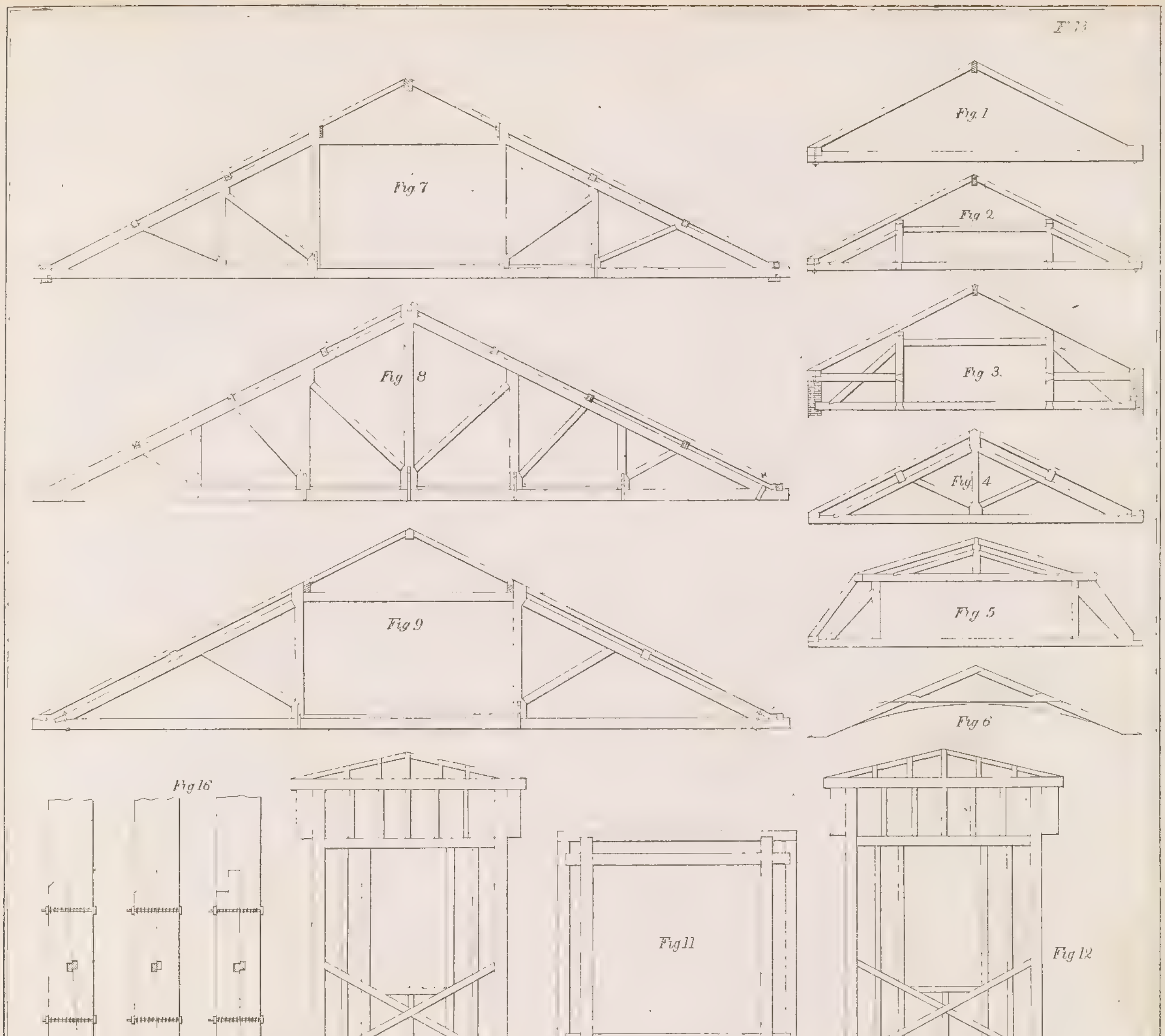


Fig. 1
Fig 2
Fig 3.
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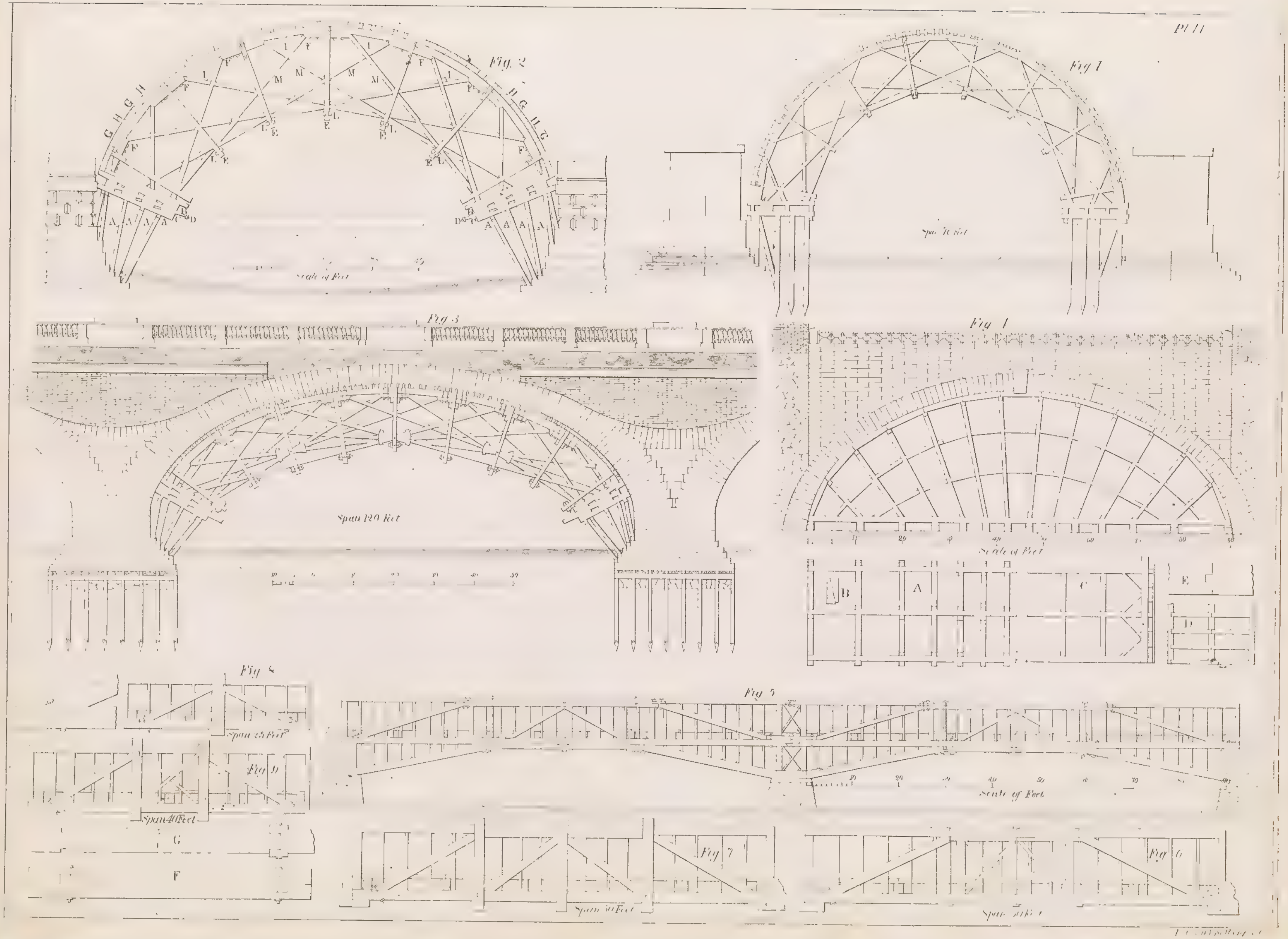
Fig 15
Fig 16


Pl. 11
Fig. 2
Scale of Feet
Fig 1
Span 76 Feet
Fig 3
Span 120 Feet
Fig 4
Scale of Feet
Fig 8
Span 25 Feet
Fig 9
Span 40 Feet
Fig 5
Scale of Feet
Fig 7
Span 50 Feet
Fig 6

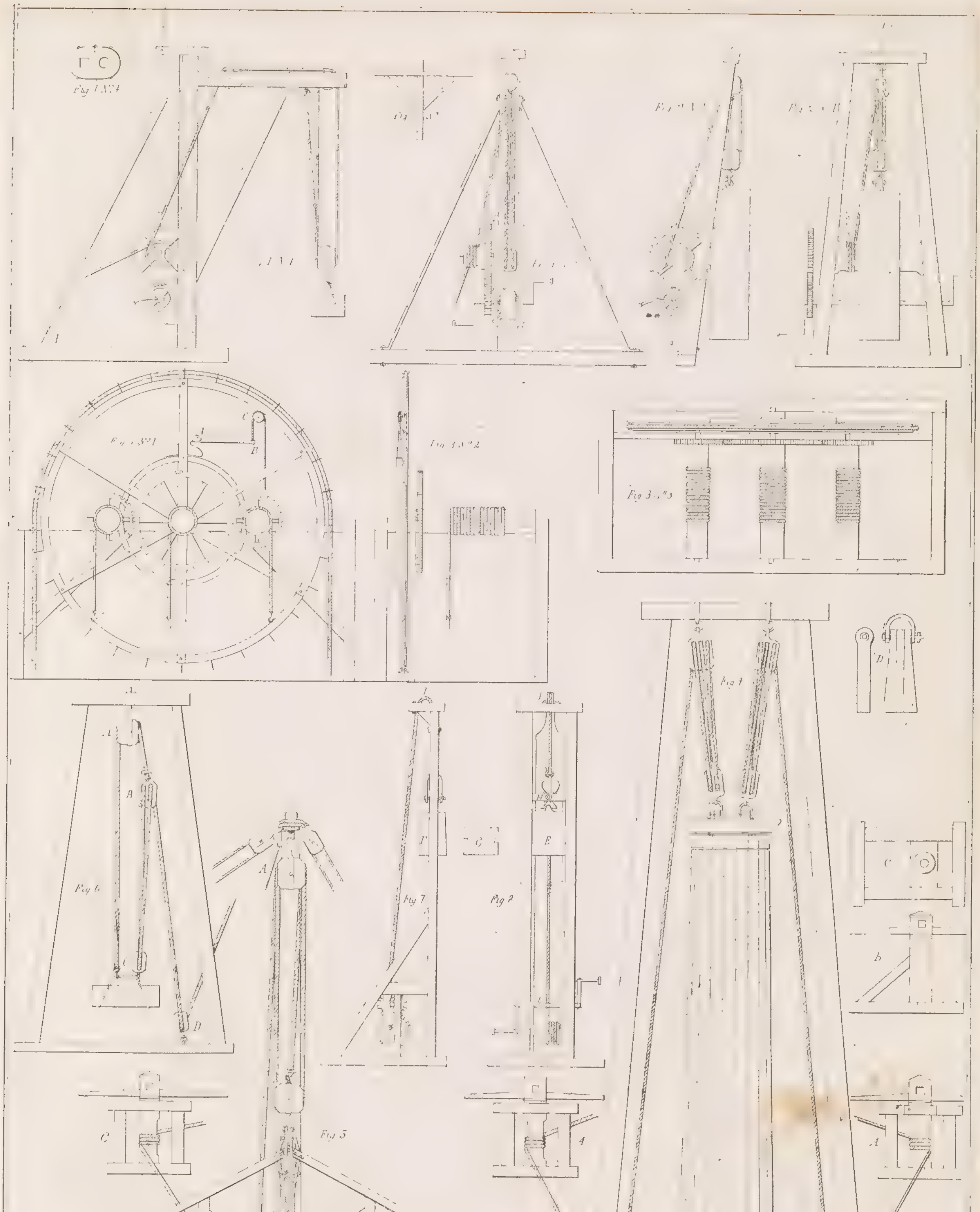


Fig. 6
Fig. 7
Fig. 8
Fig. 5
Fig. 4

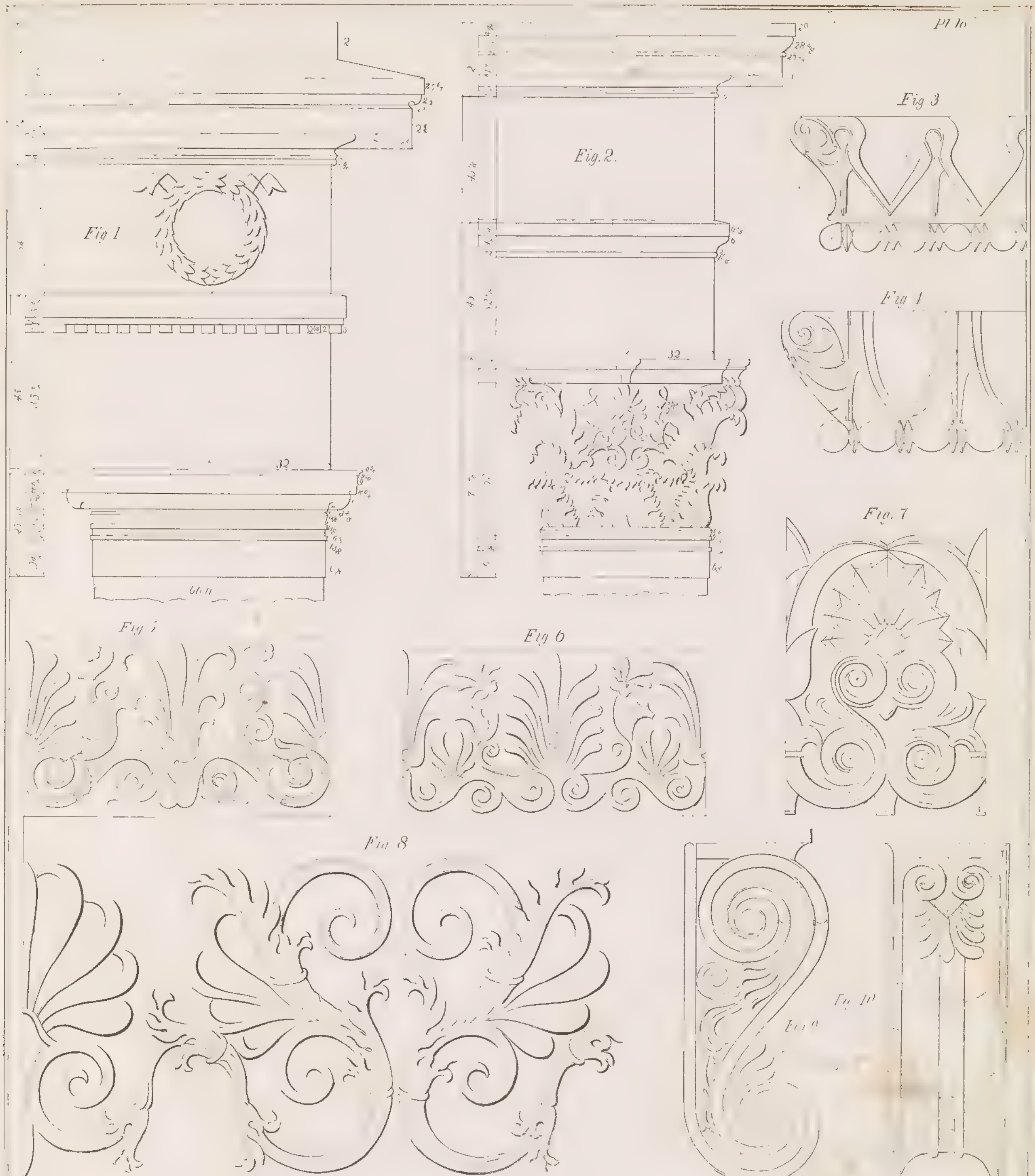


Fig 1
Fig. 2.
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Fig 1.

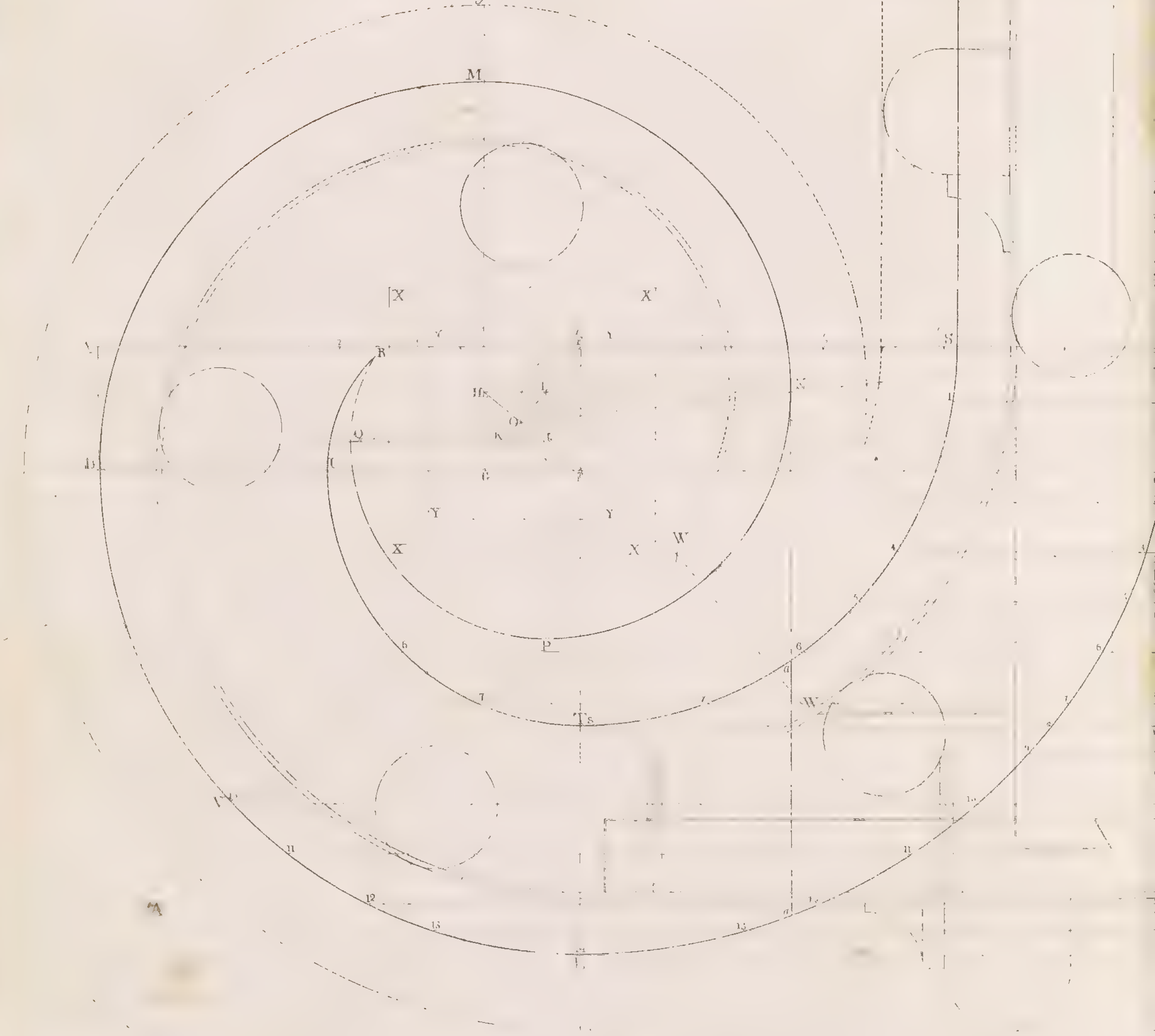


Fig 4.

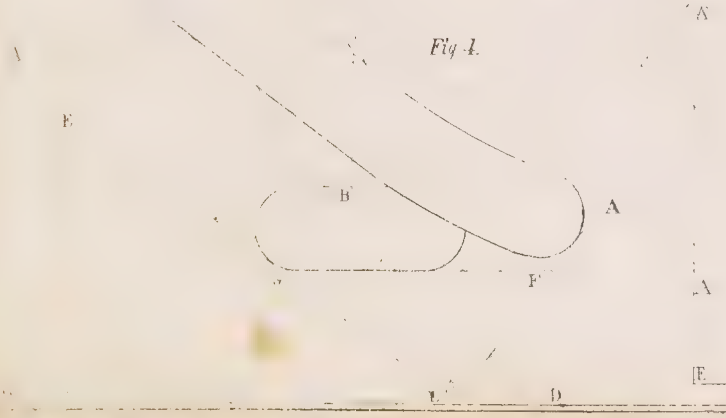


Fig 6

Fig 3

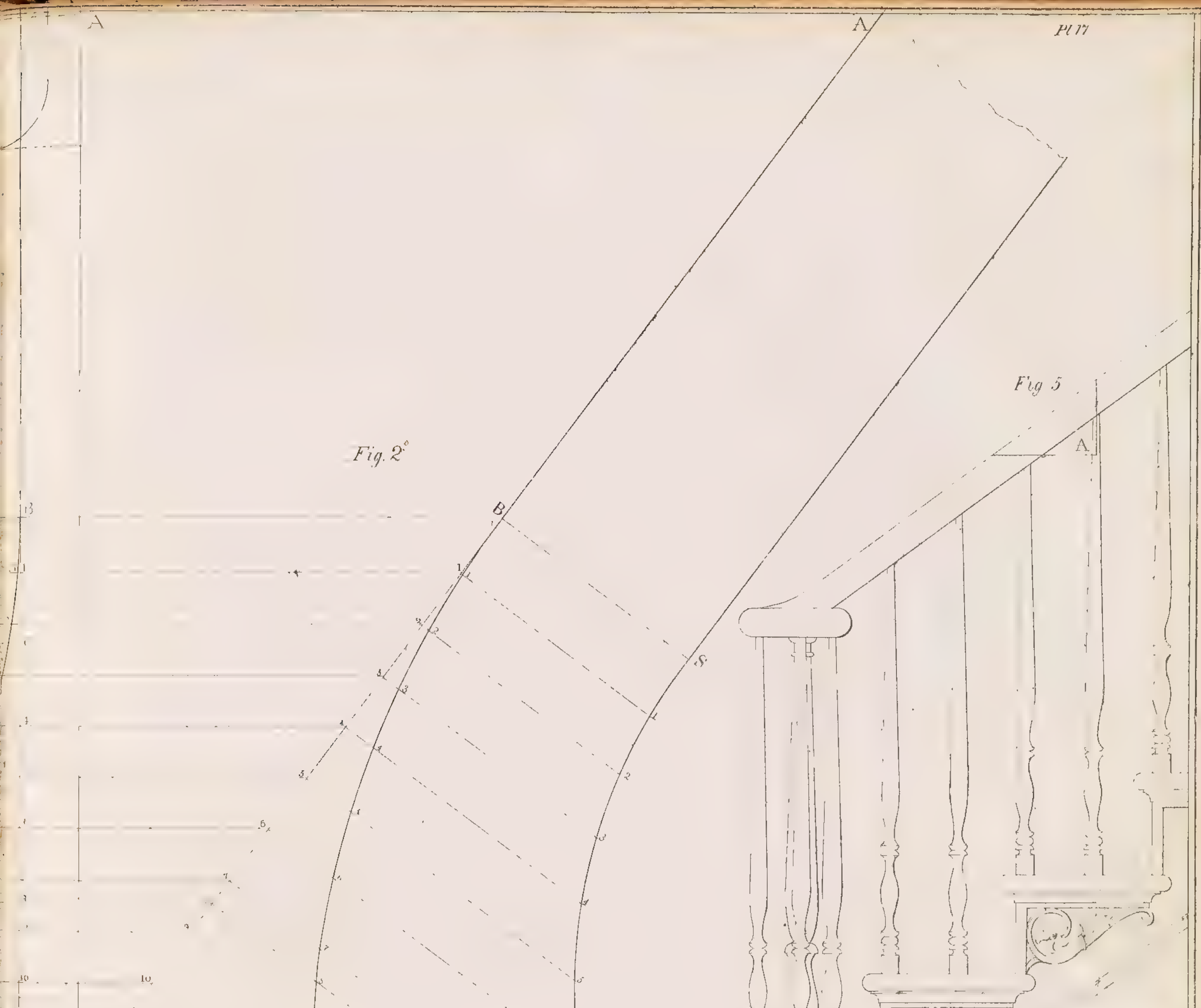


Fig. 2°

Fig 5

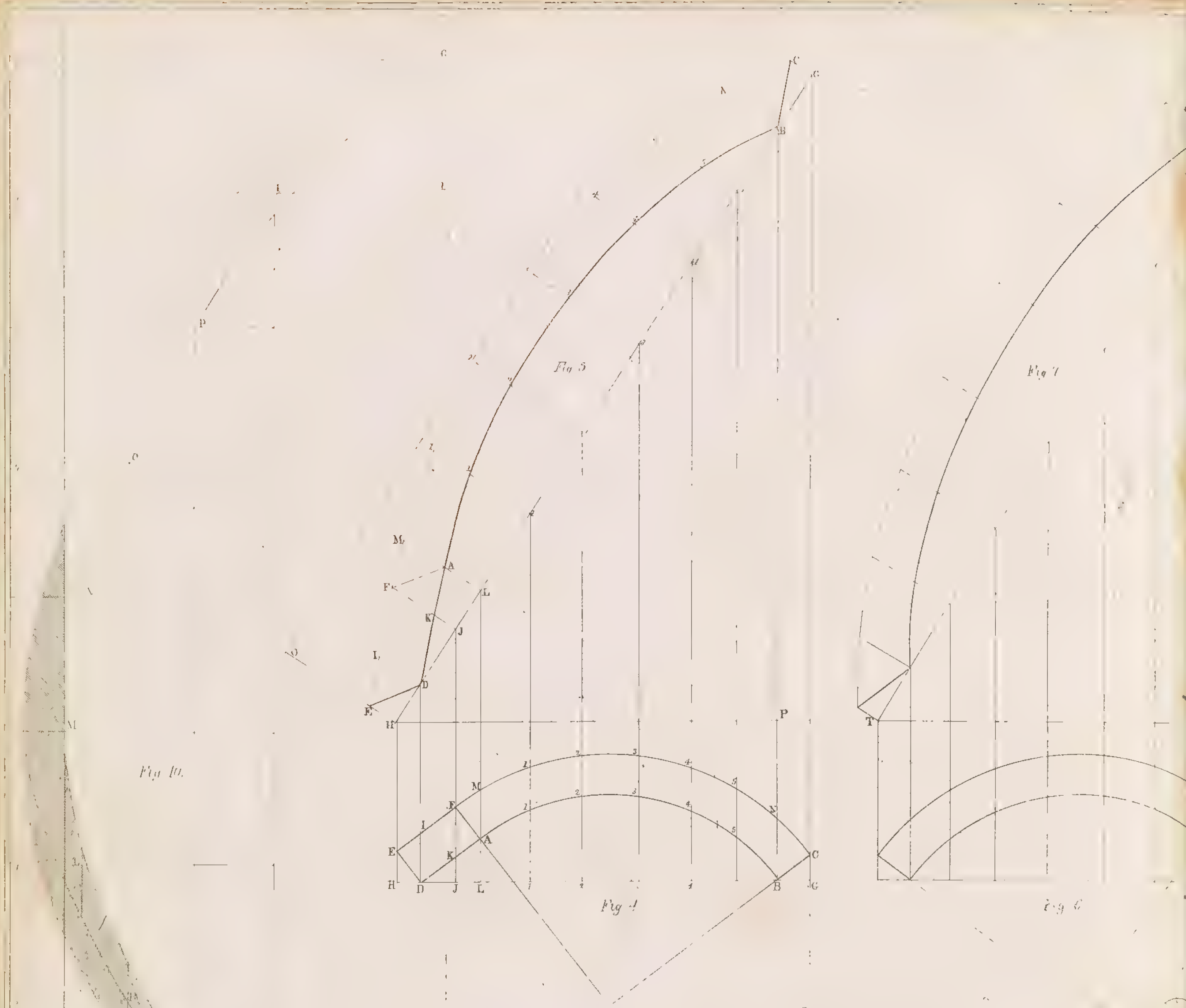


Fig 5
Fig 7
Fig 4
Fig 6
Fig 10.

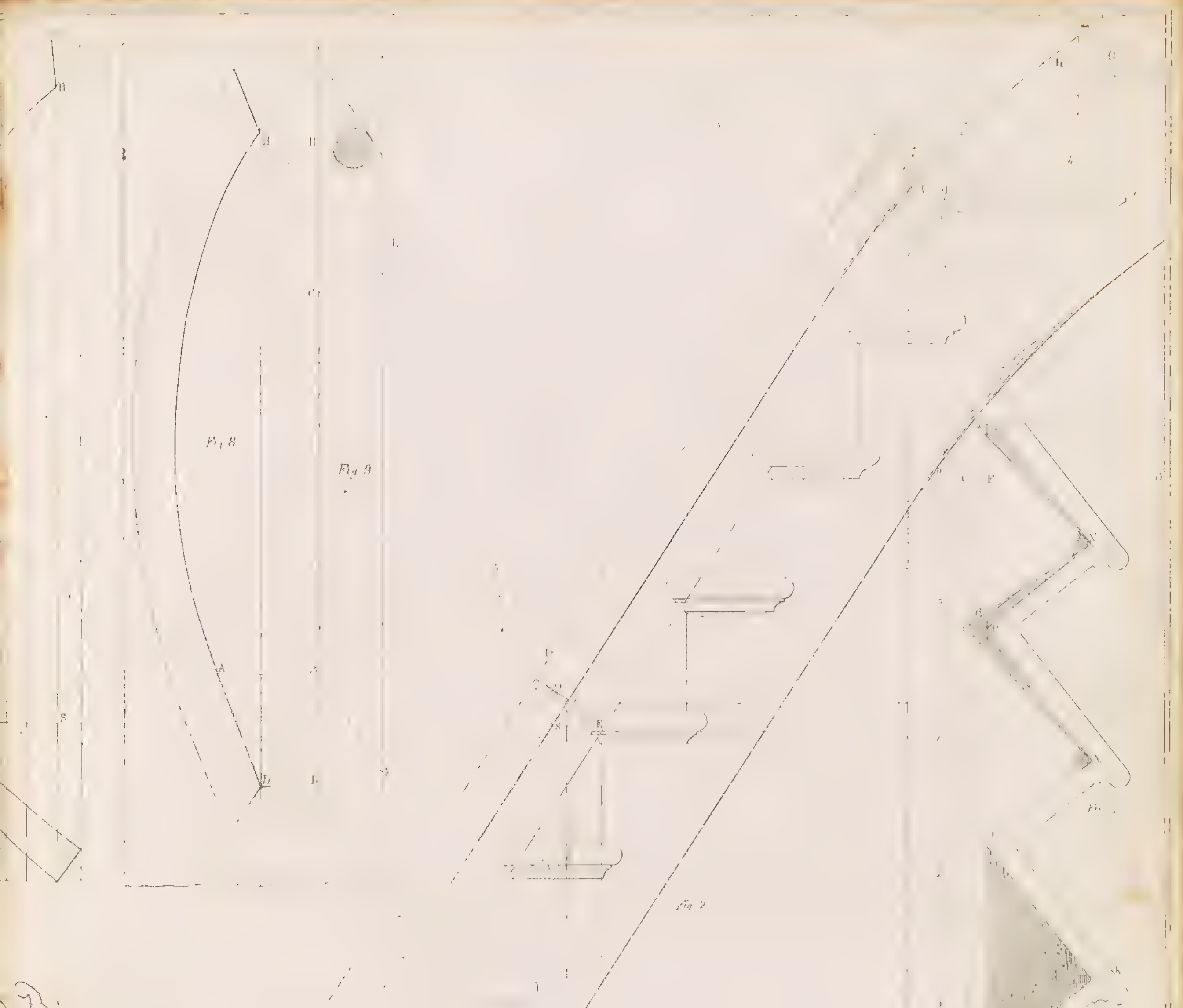


Fig 8
Fig 9
Fig 12
Fig 2

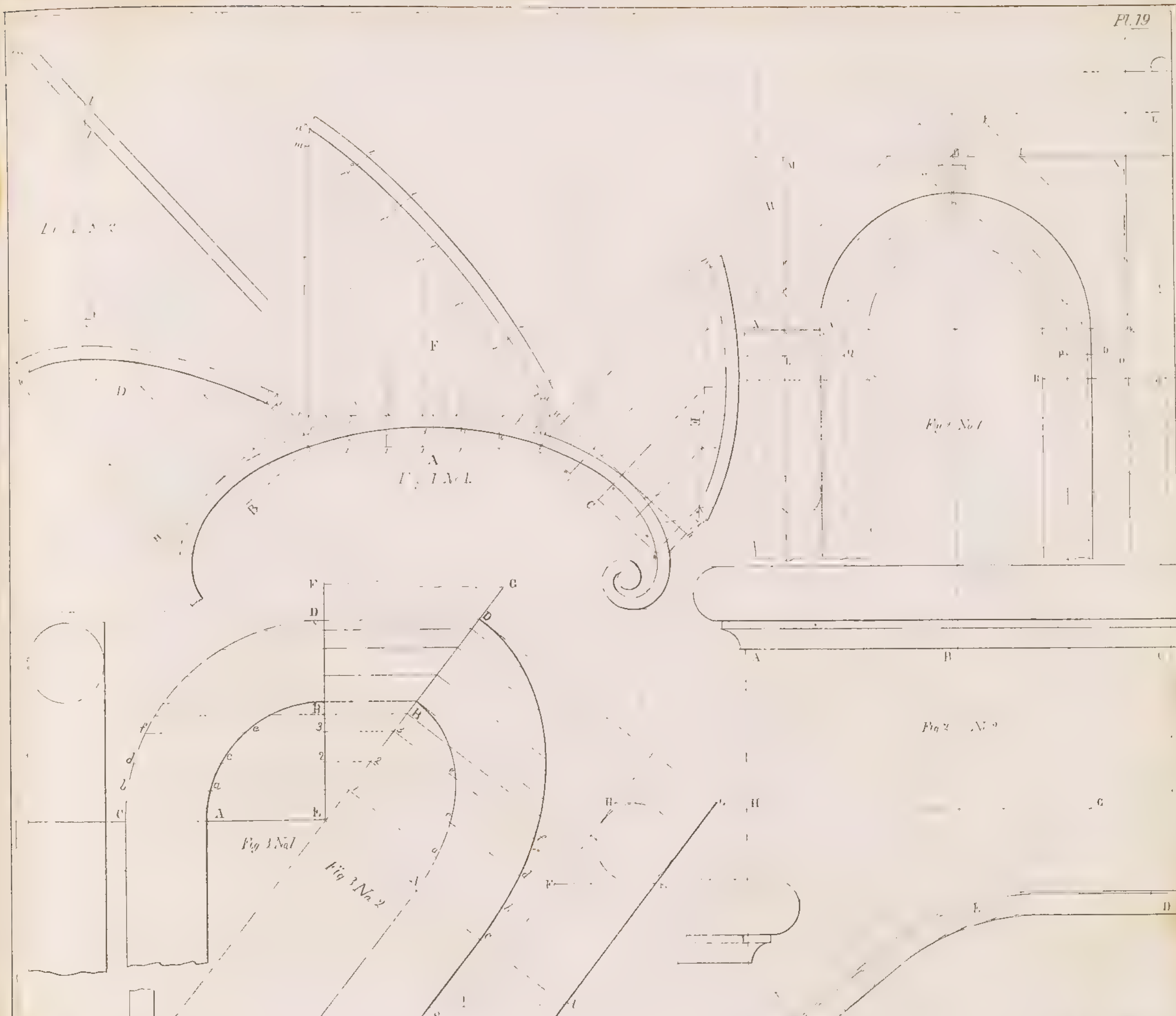


Fig. 1 No. 1
Fig. 2 No. 1
Fig. 2 No. 2
Fig. 3 No. 1
Fig. 3 No. 2
Fig. 4 No. 1
Fig. 5 No. 1
Fig. 5 No. 2
Fig. 6
Fig. 7

Pl. 7

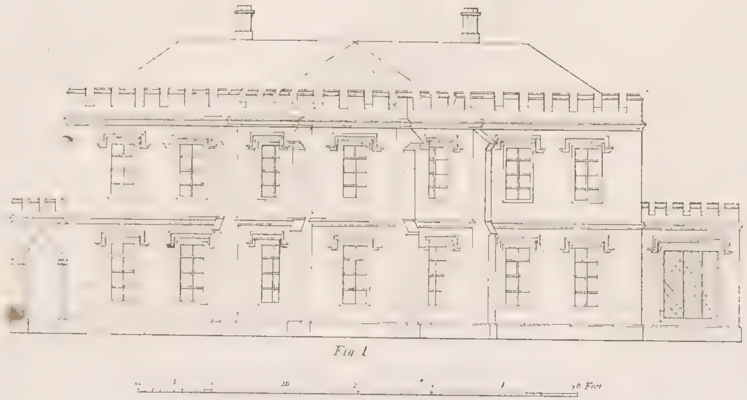


Fig. 1

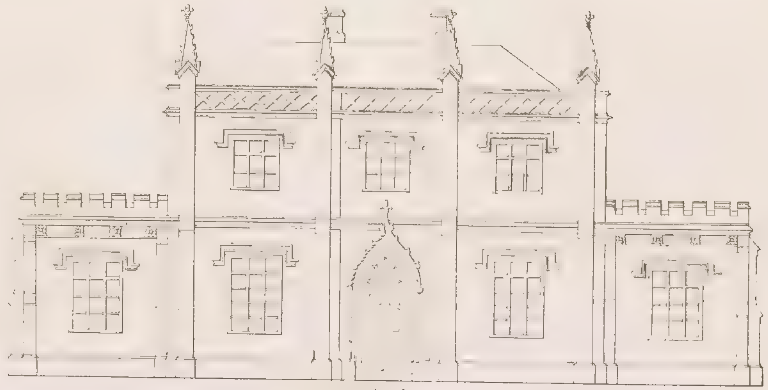


Fig. 3

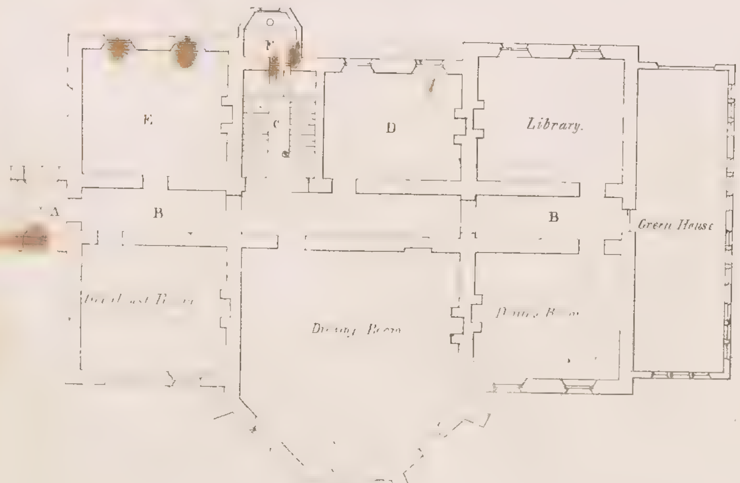


Fig. 2

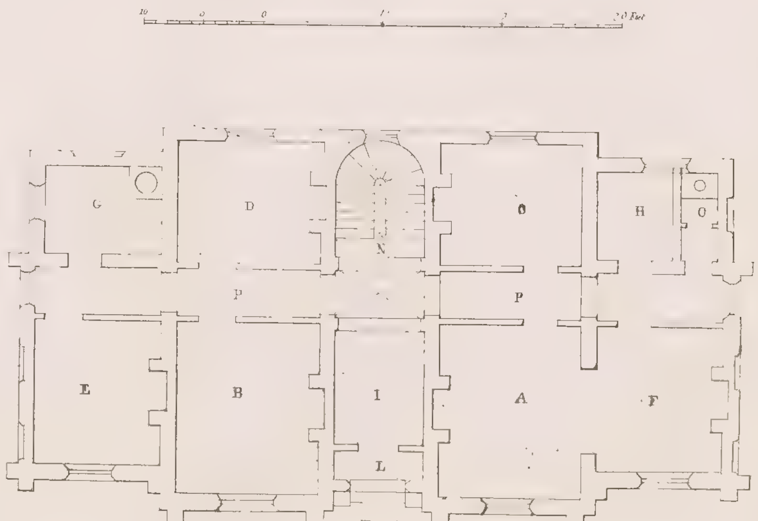
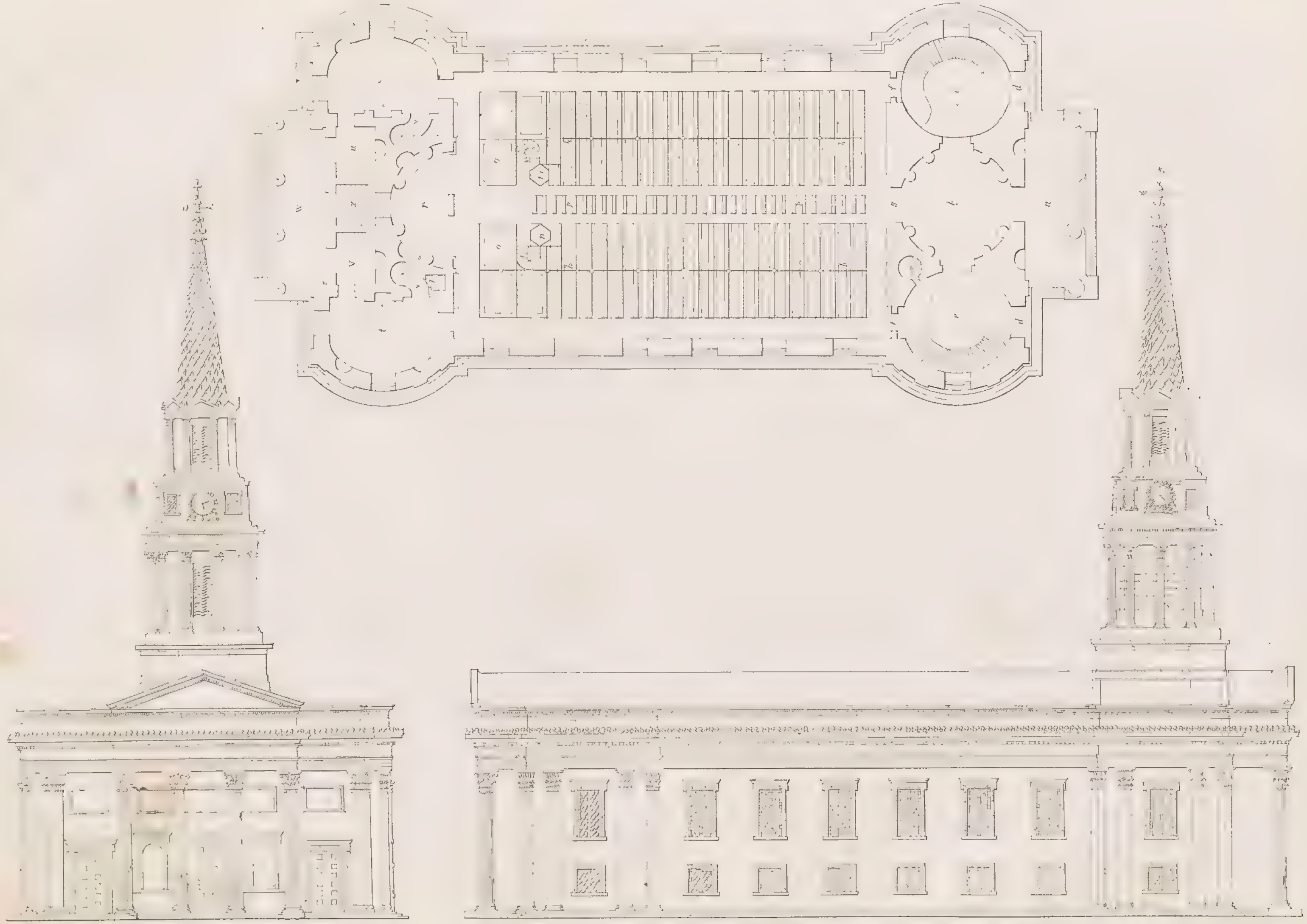
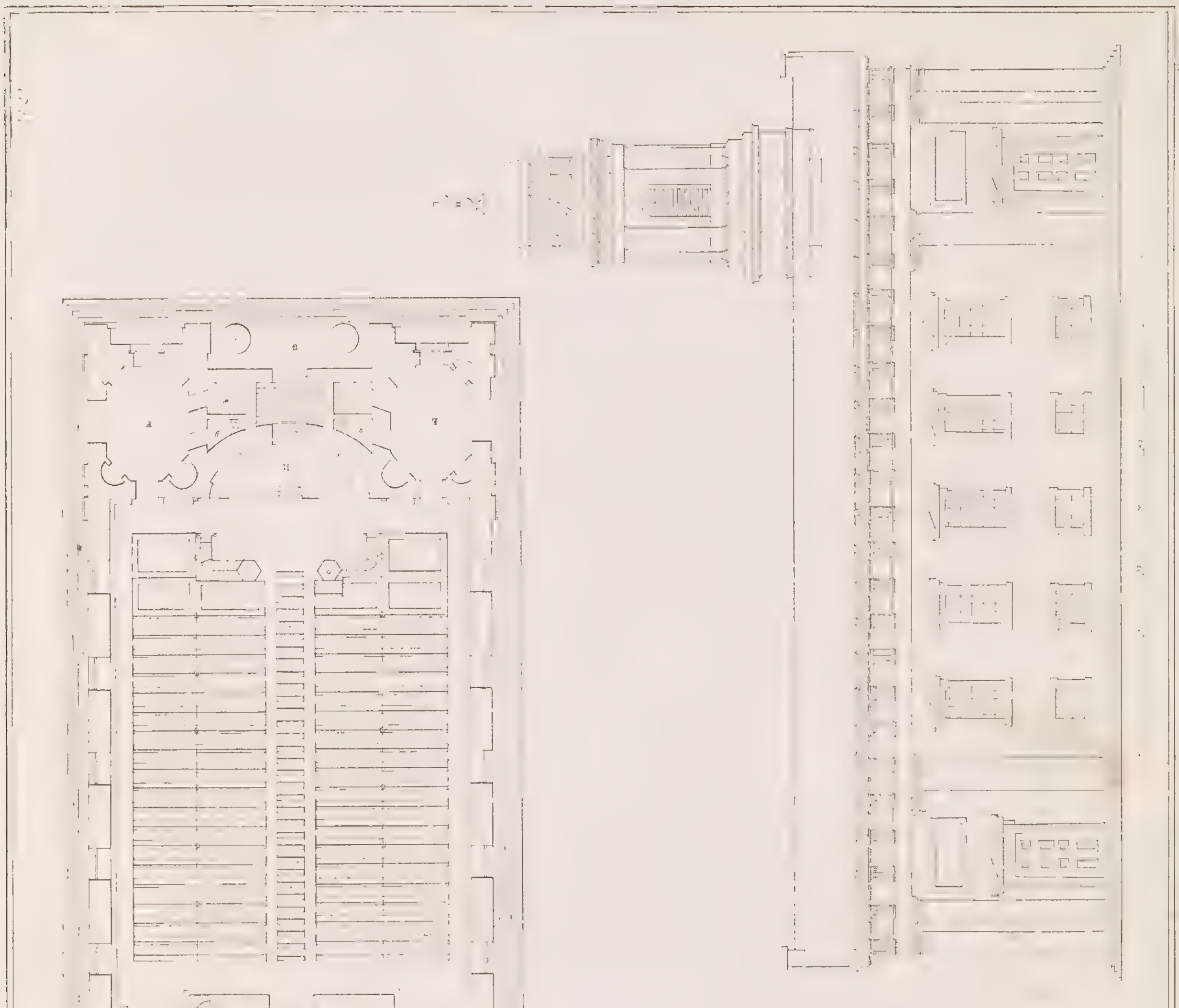


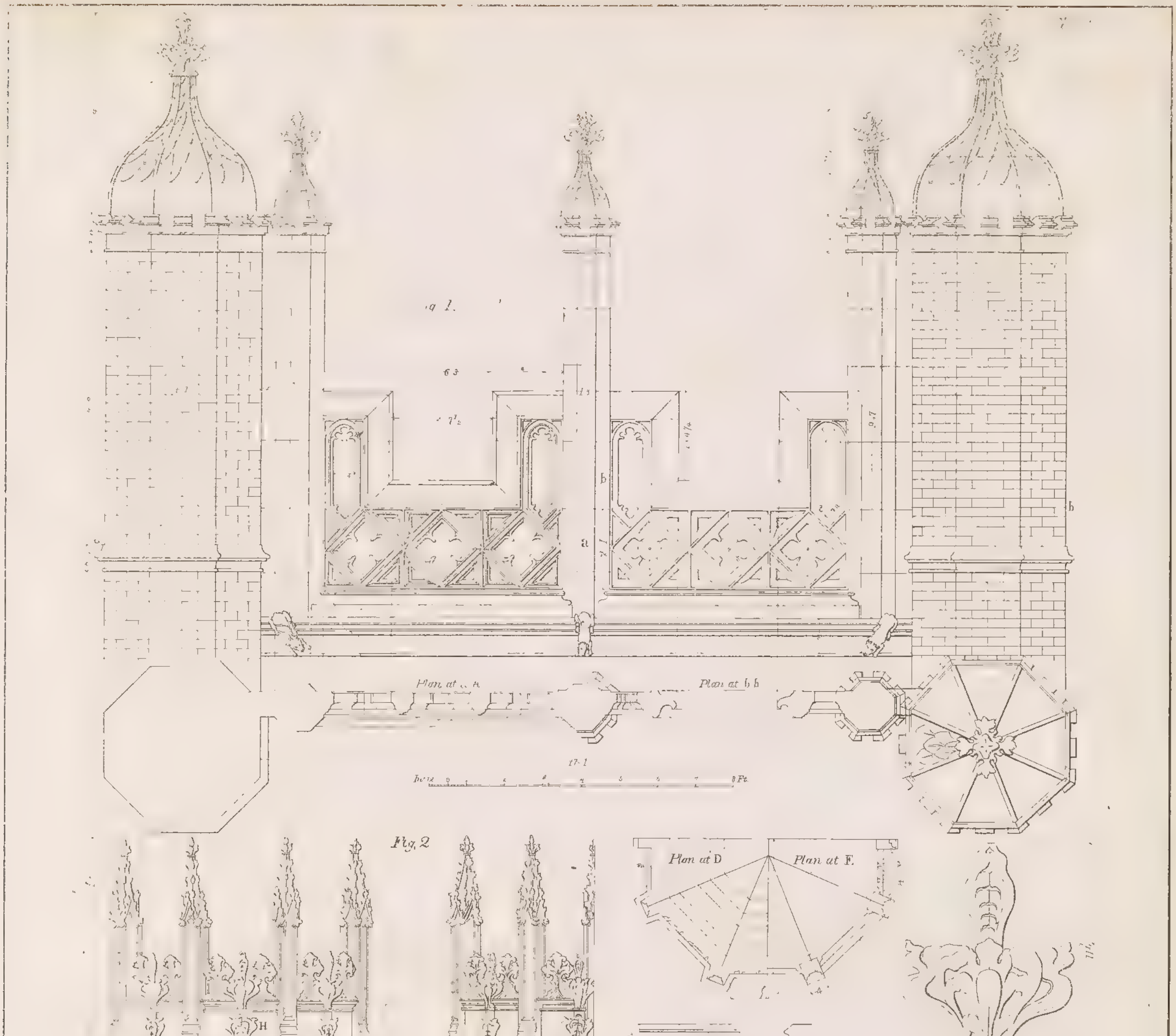
Fig. 4

Lith of D W Kellogg & Co

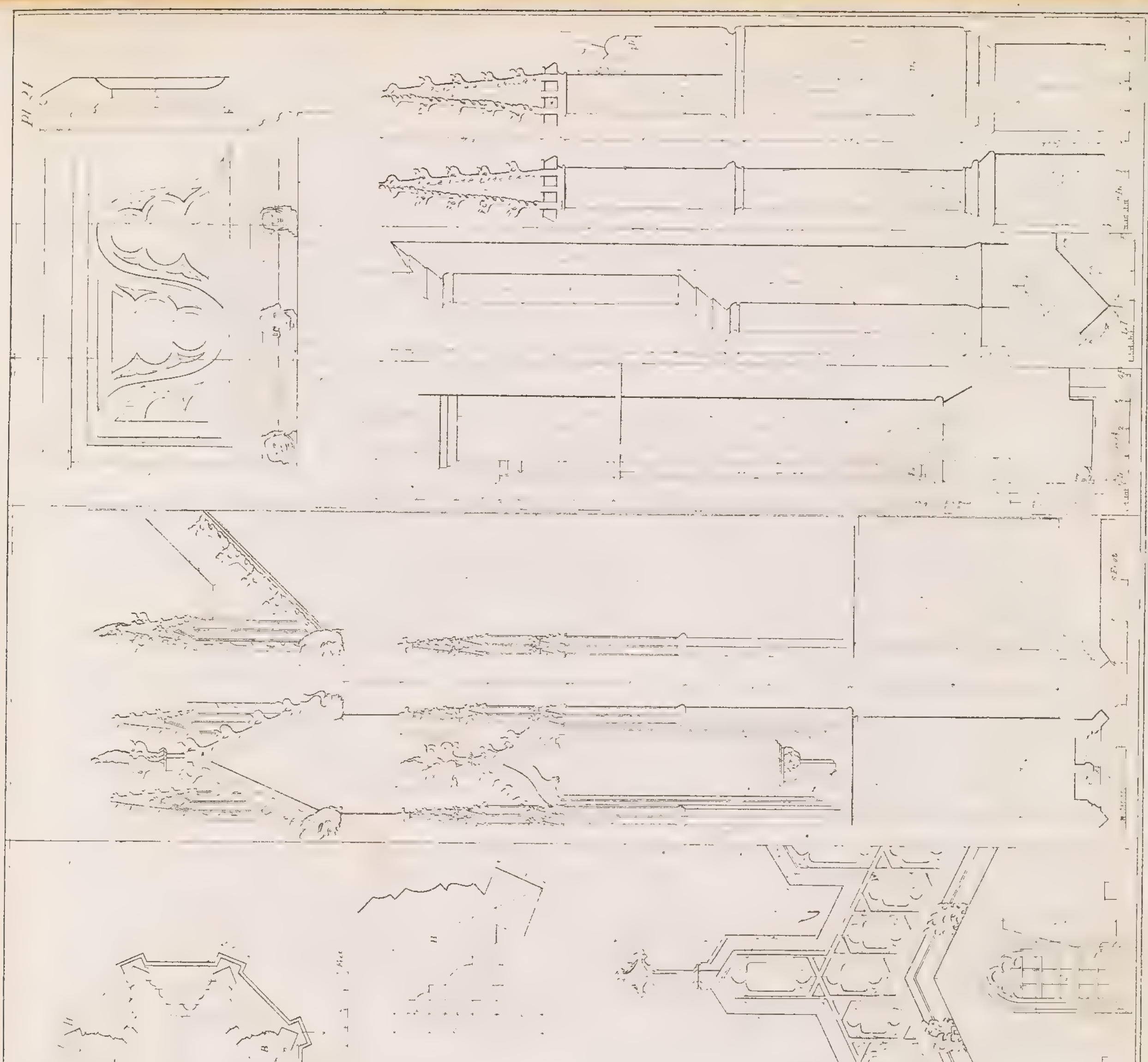
Pl. 21







Plan at b b
Fig. 2
Plan at D
Plan at E.
Section at A.
Section at A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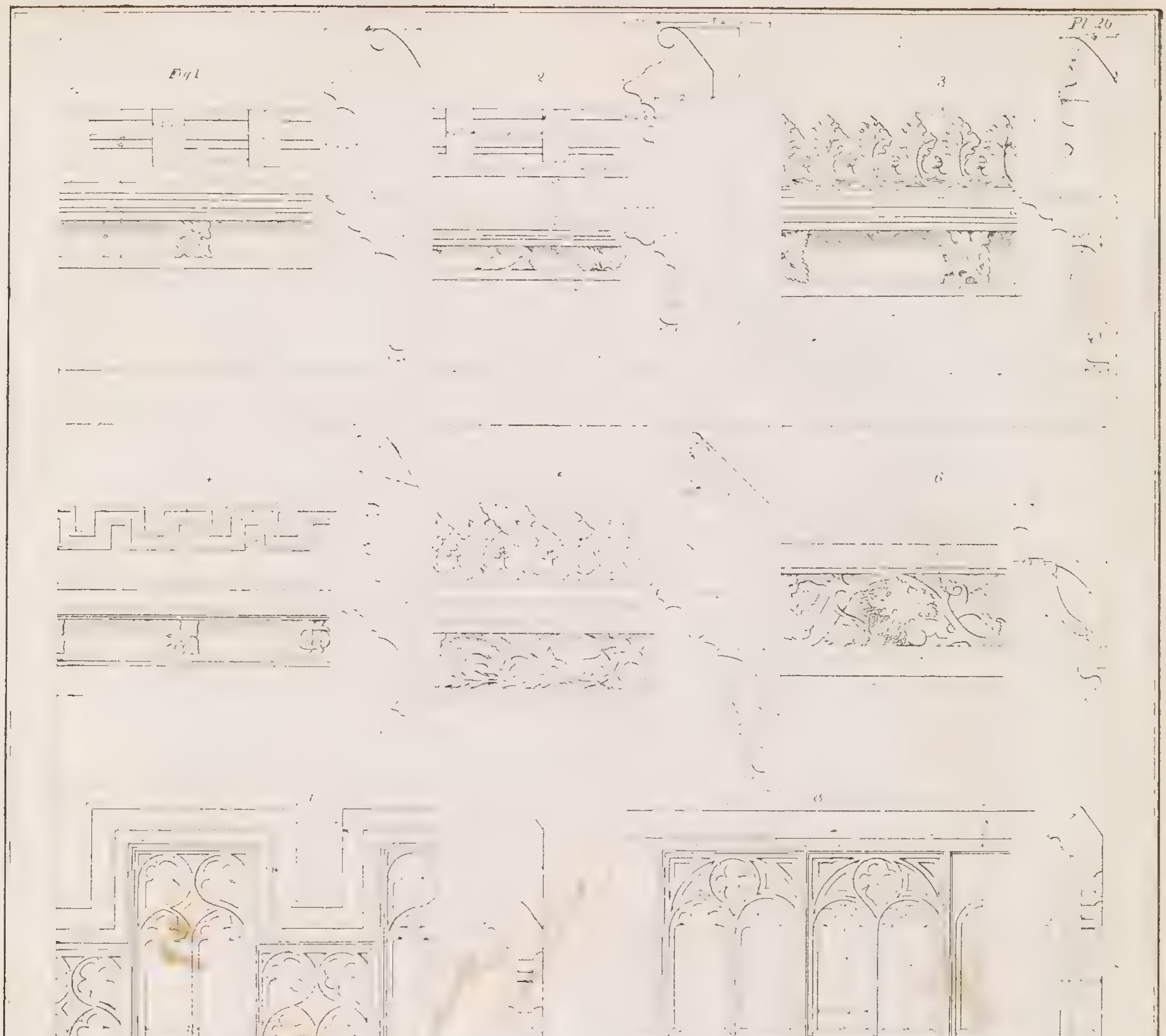

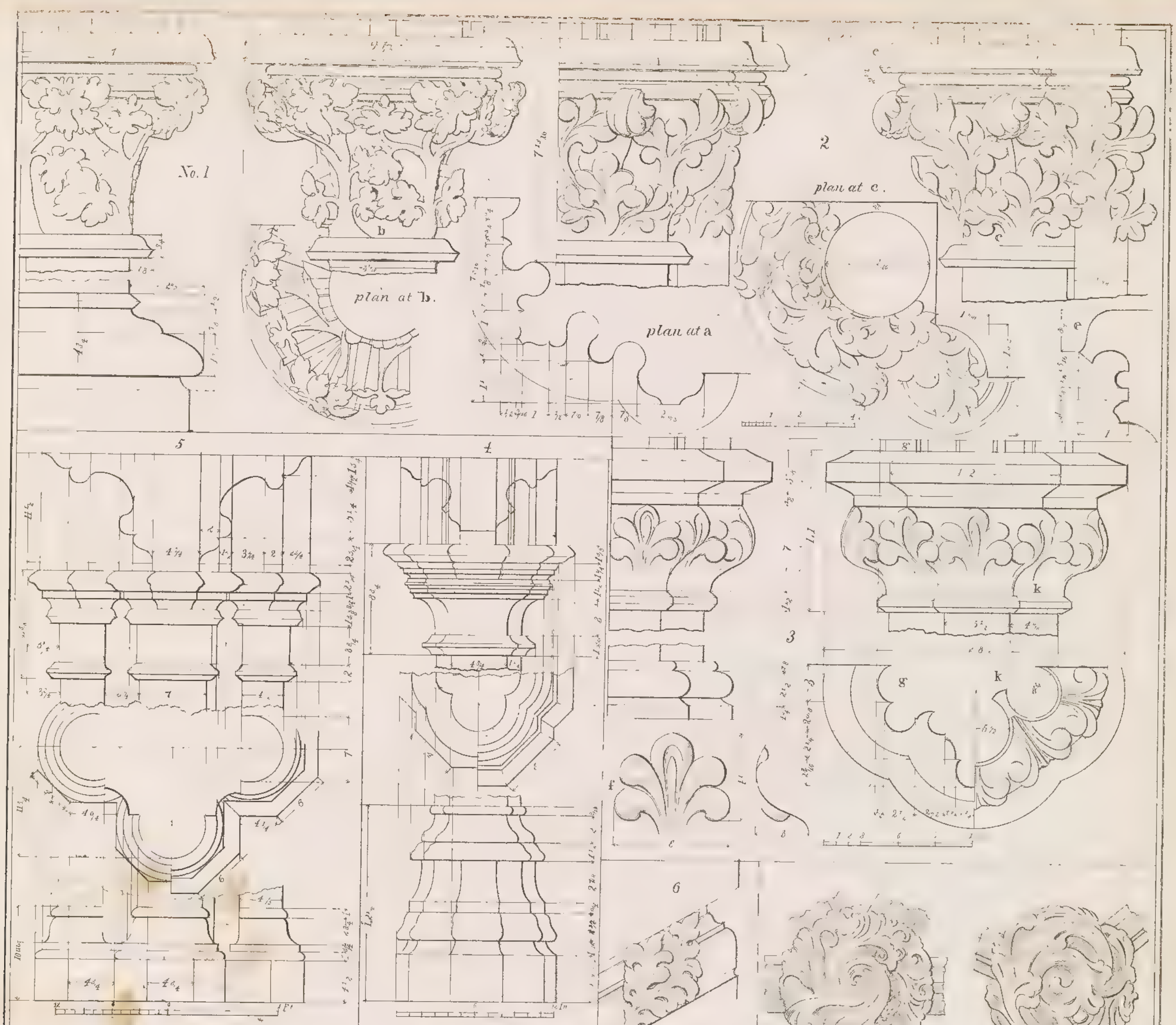
No. 1
plan at b.
plan at a
plan at c.
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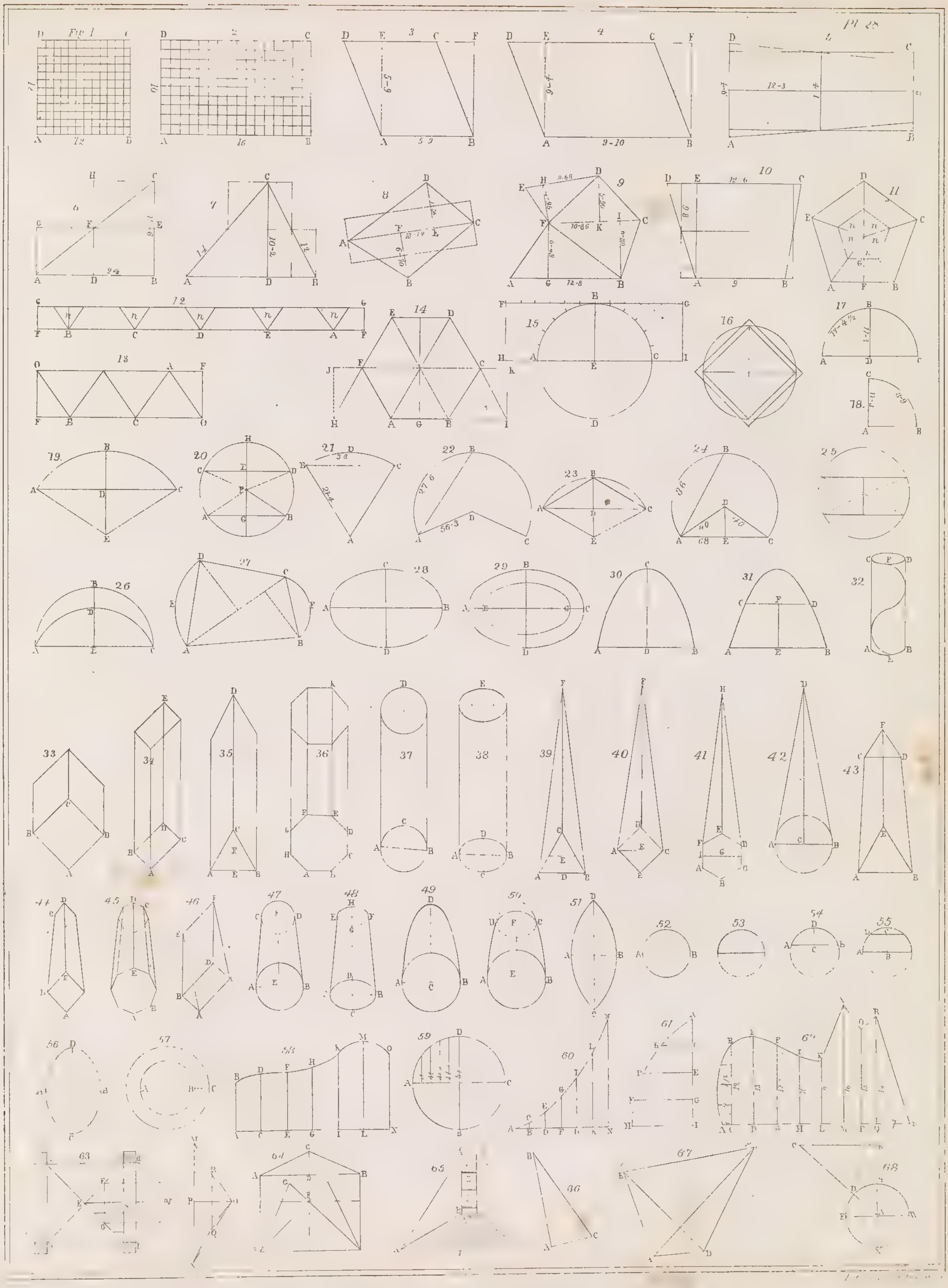
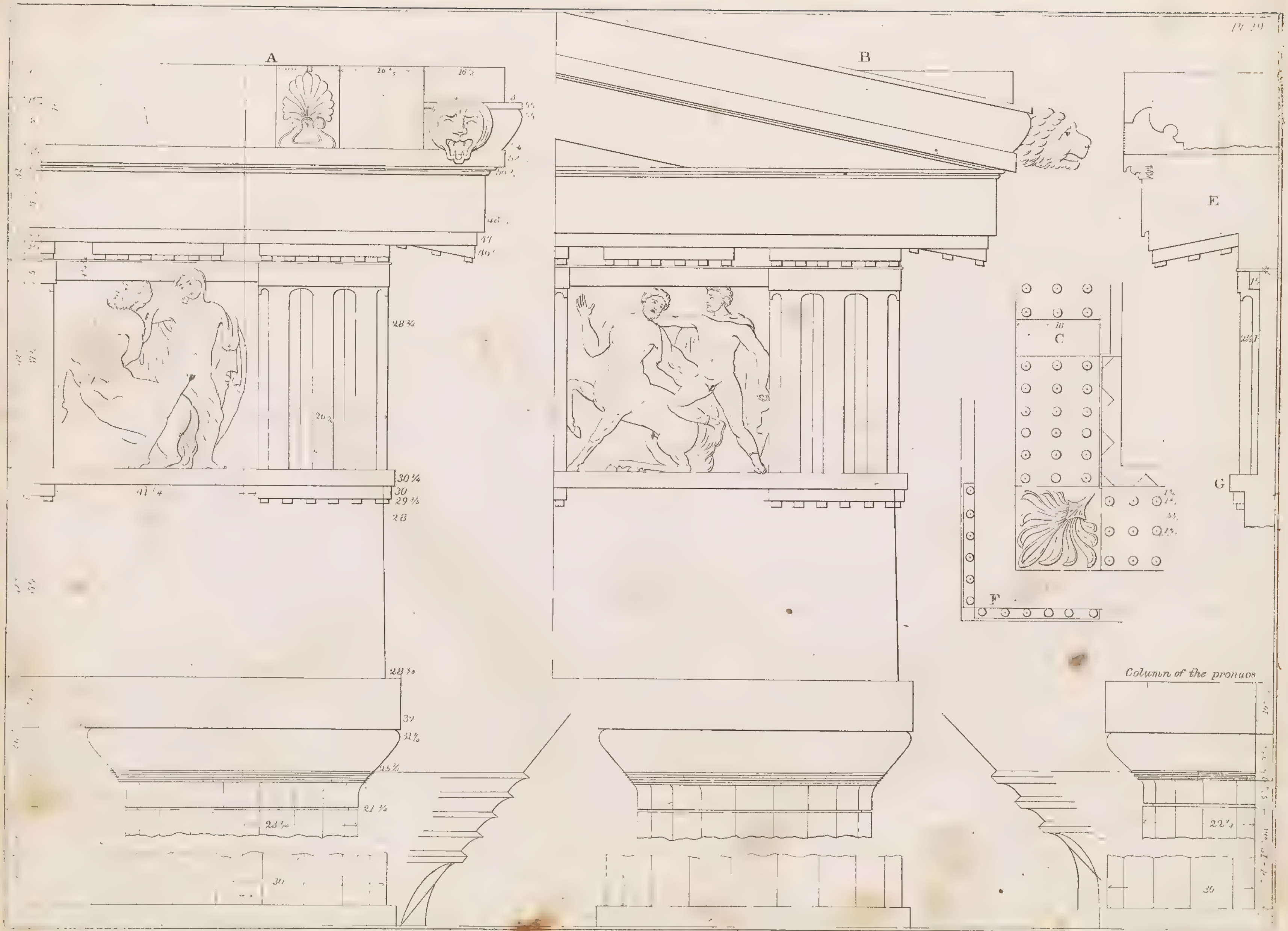

Pl 28


A
B
C
E
F
G
Column of the pronaos
30 ¼
30
29 ⅔
28
28 ¾
31½
39
21 ⅓
22⅔
30

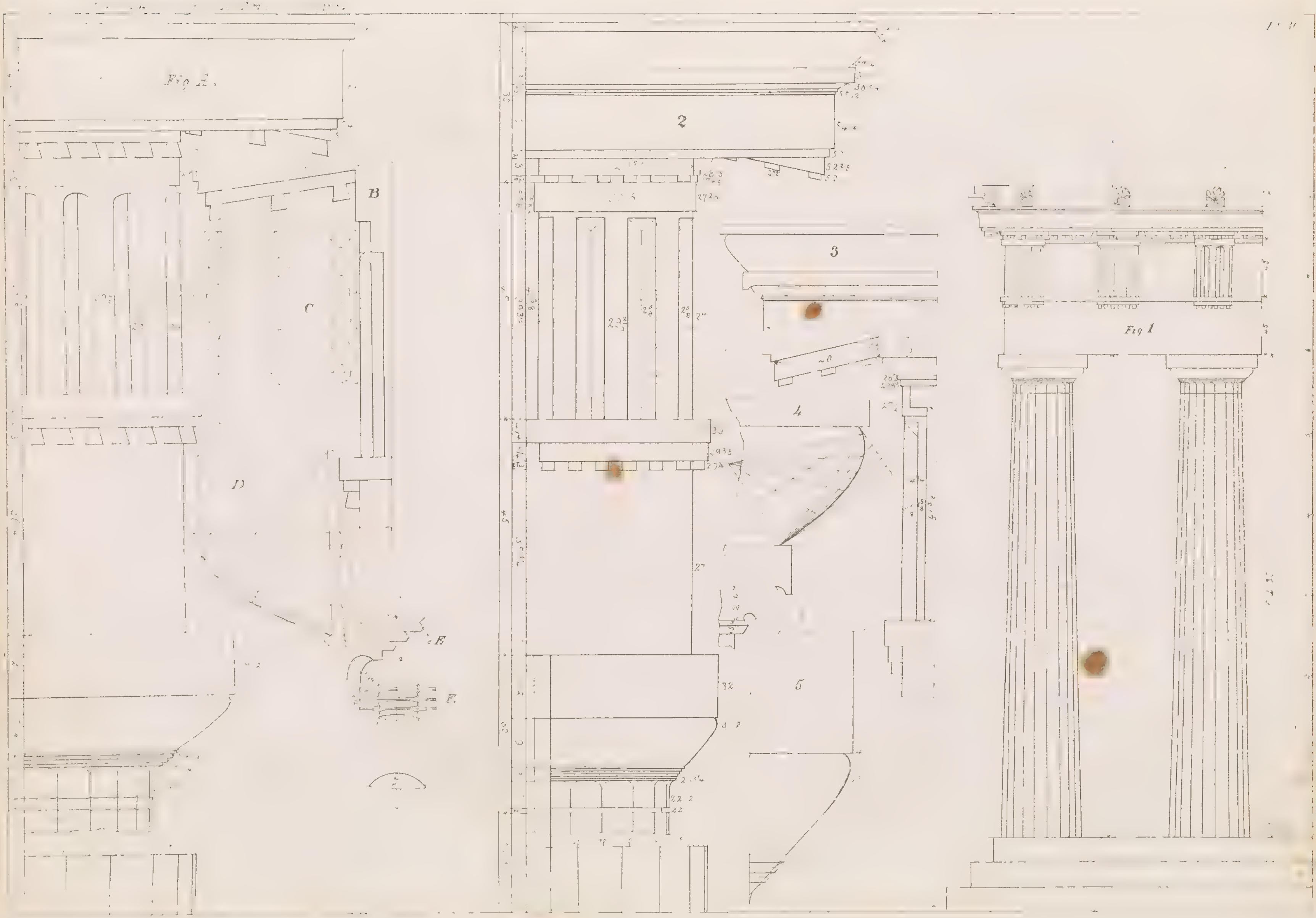


Fig 2.
B
C
D
E
F
2
3
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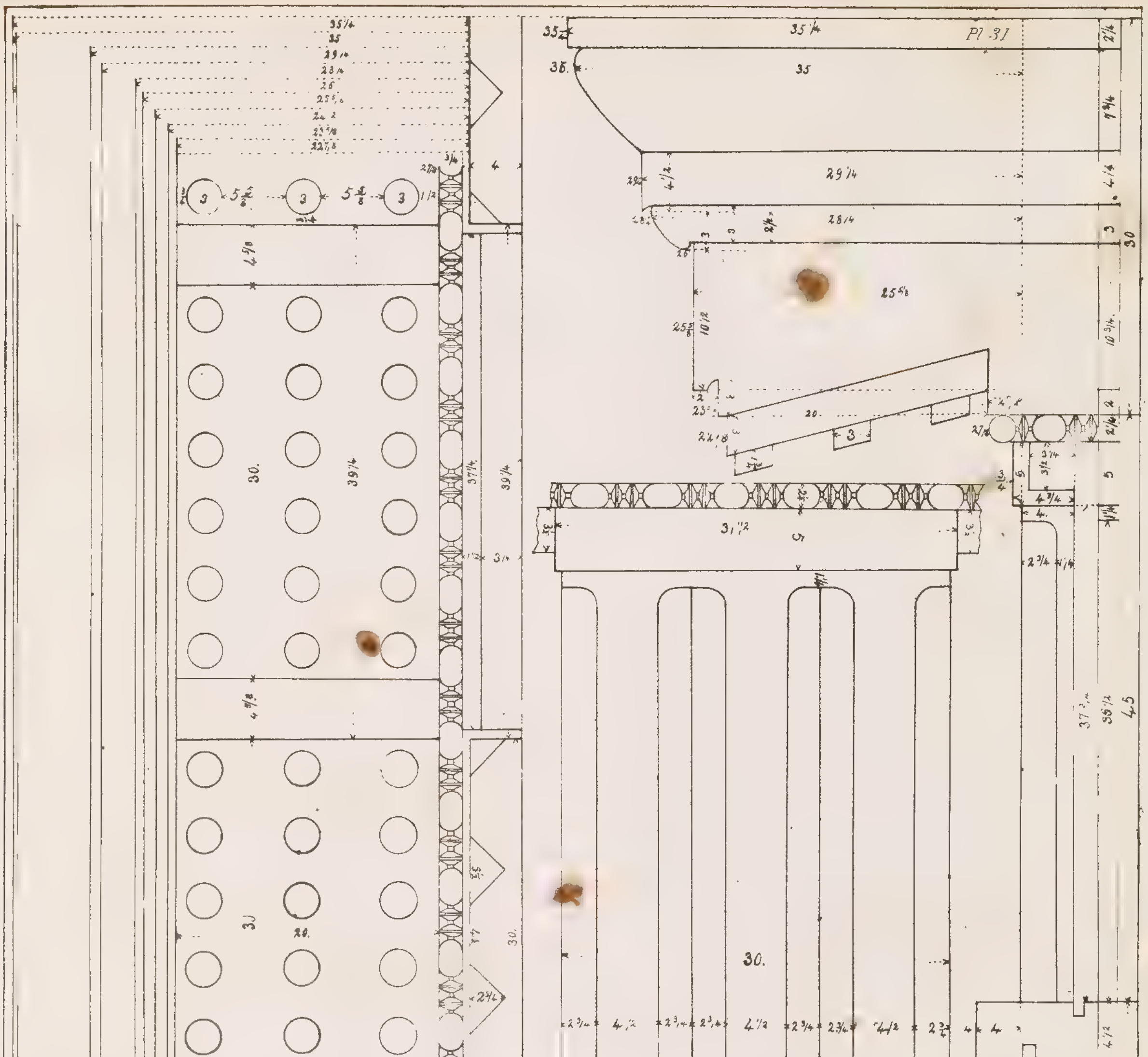
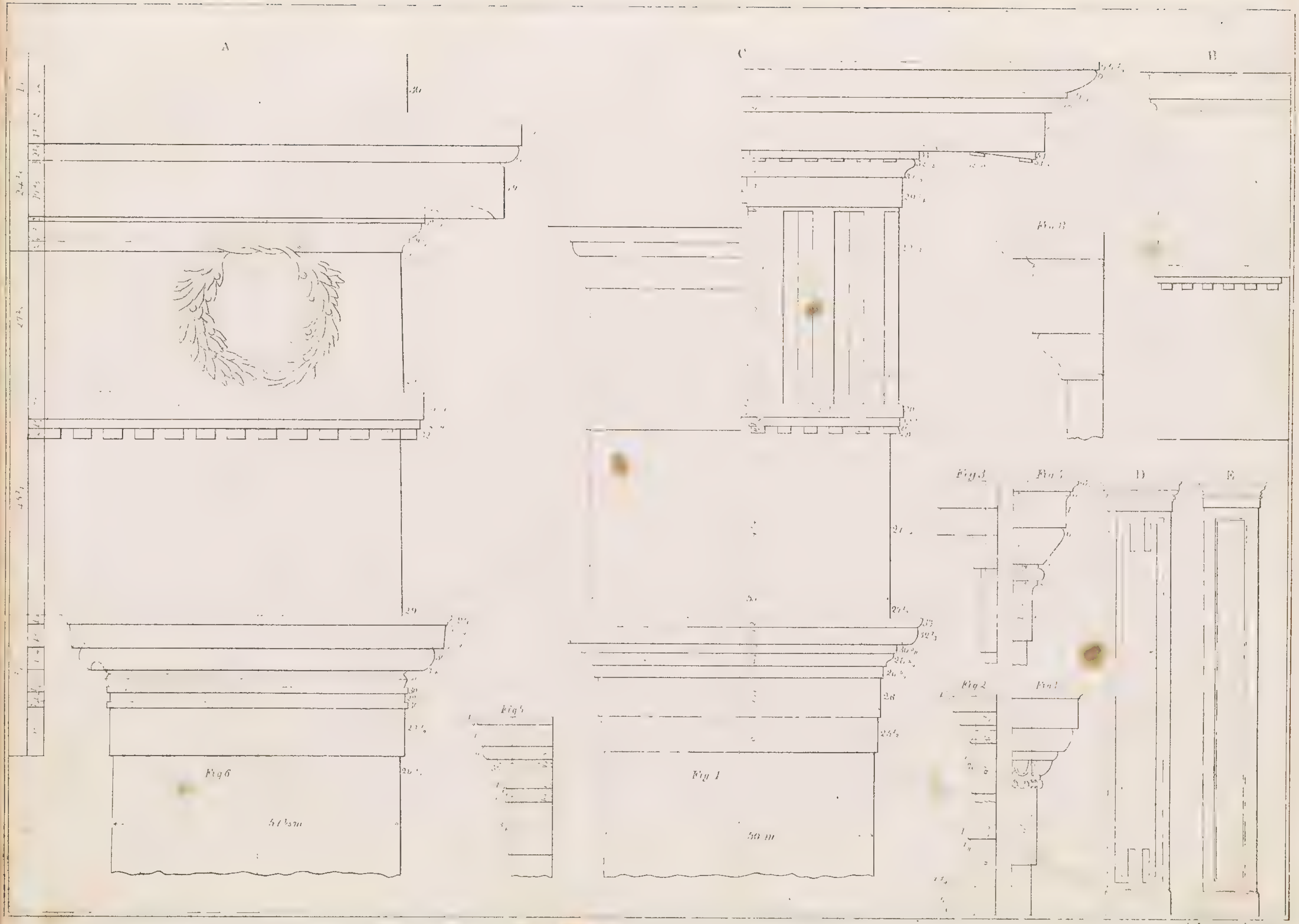
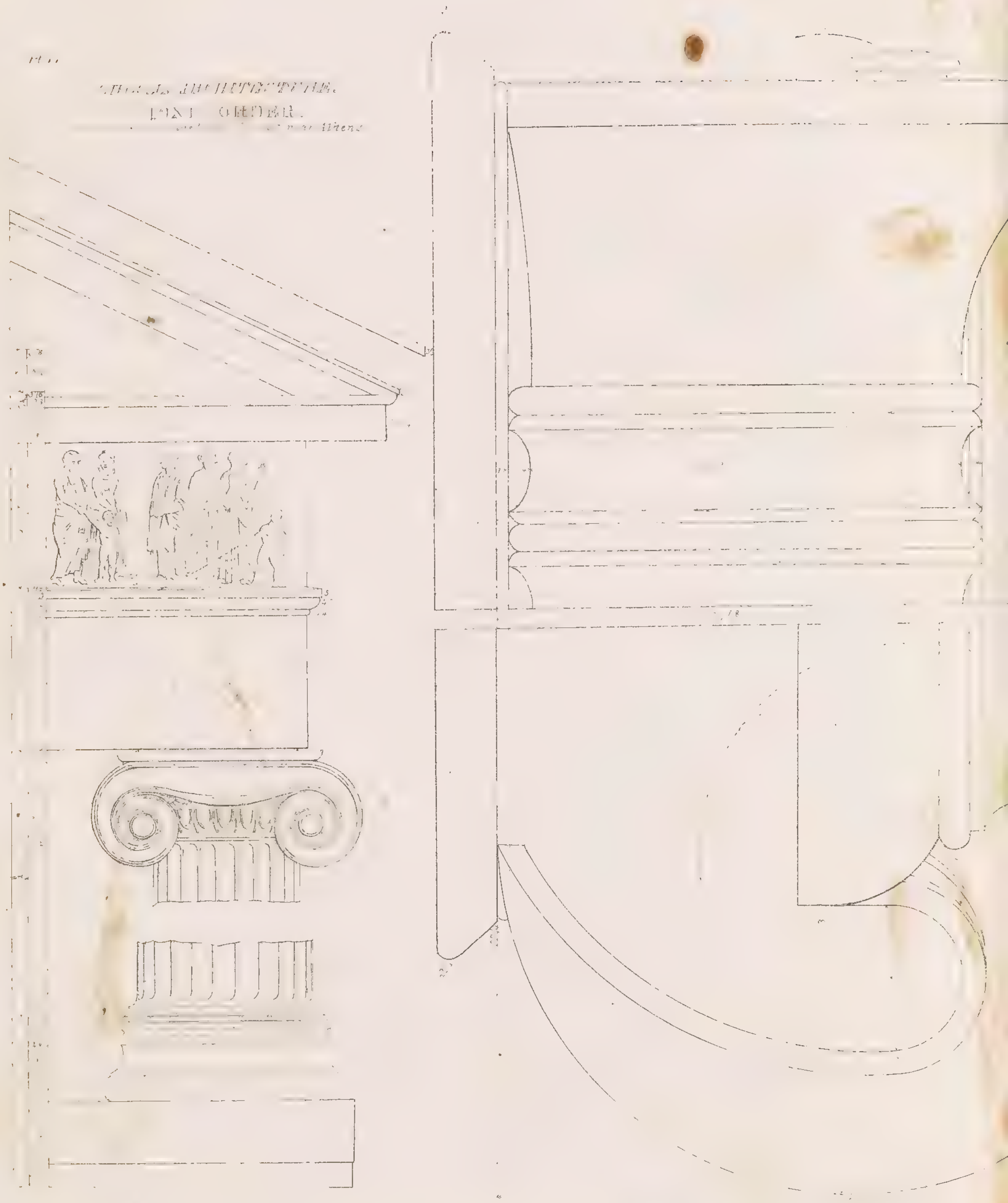
5
Fig 1

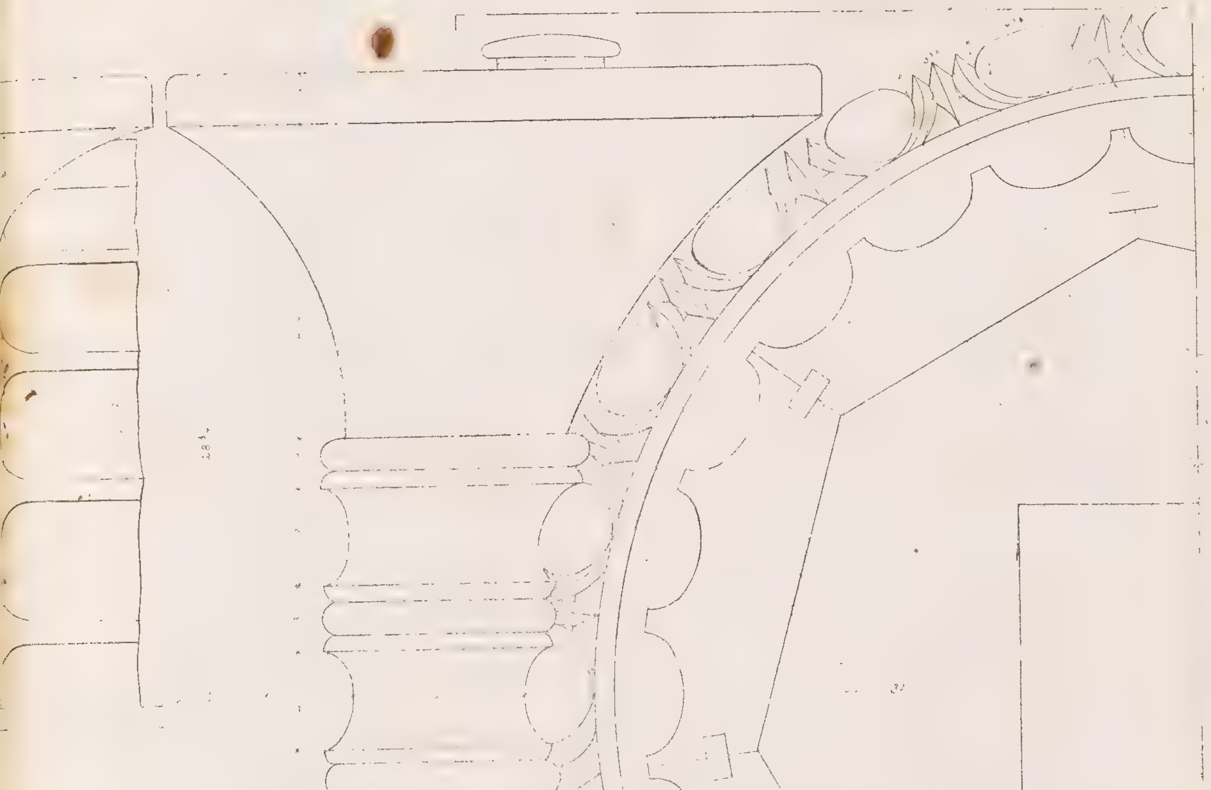

Pl. 31

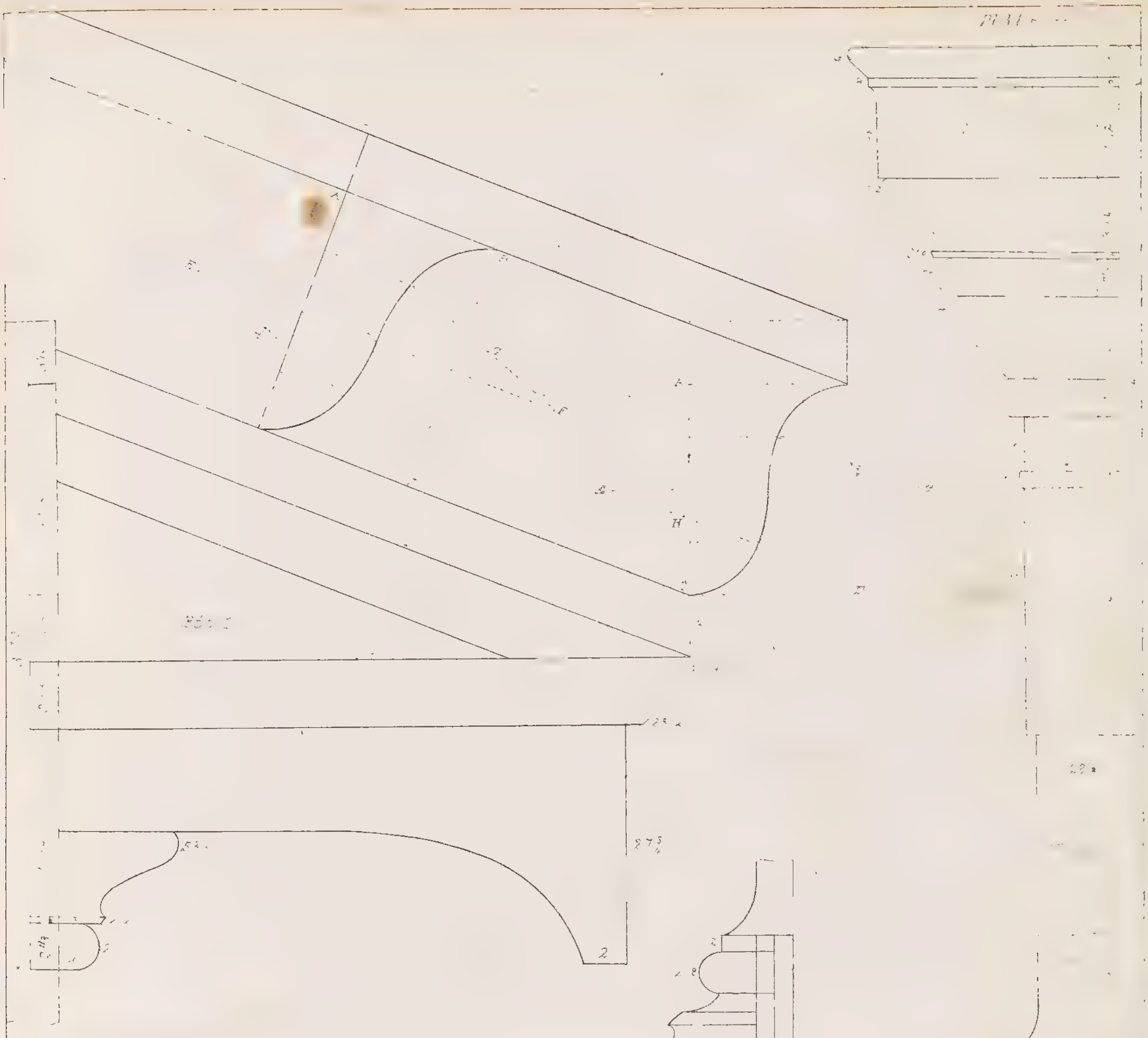

A
C
B
Fig 8
Fig 3
Fig 7
D
E
Fig 2
Fig 4
Fig 5
Fig 6
Fig 1
51⅓m
50 m

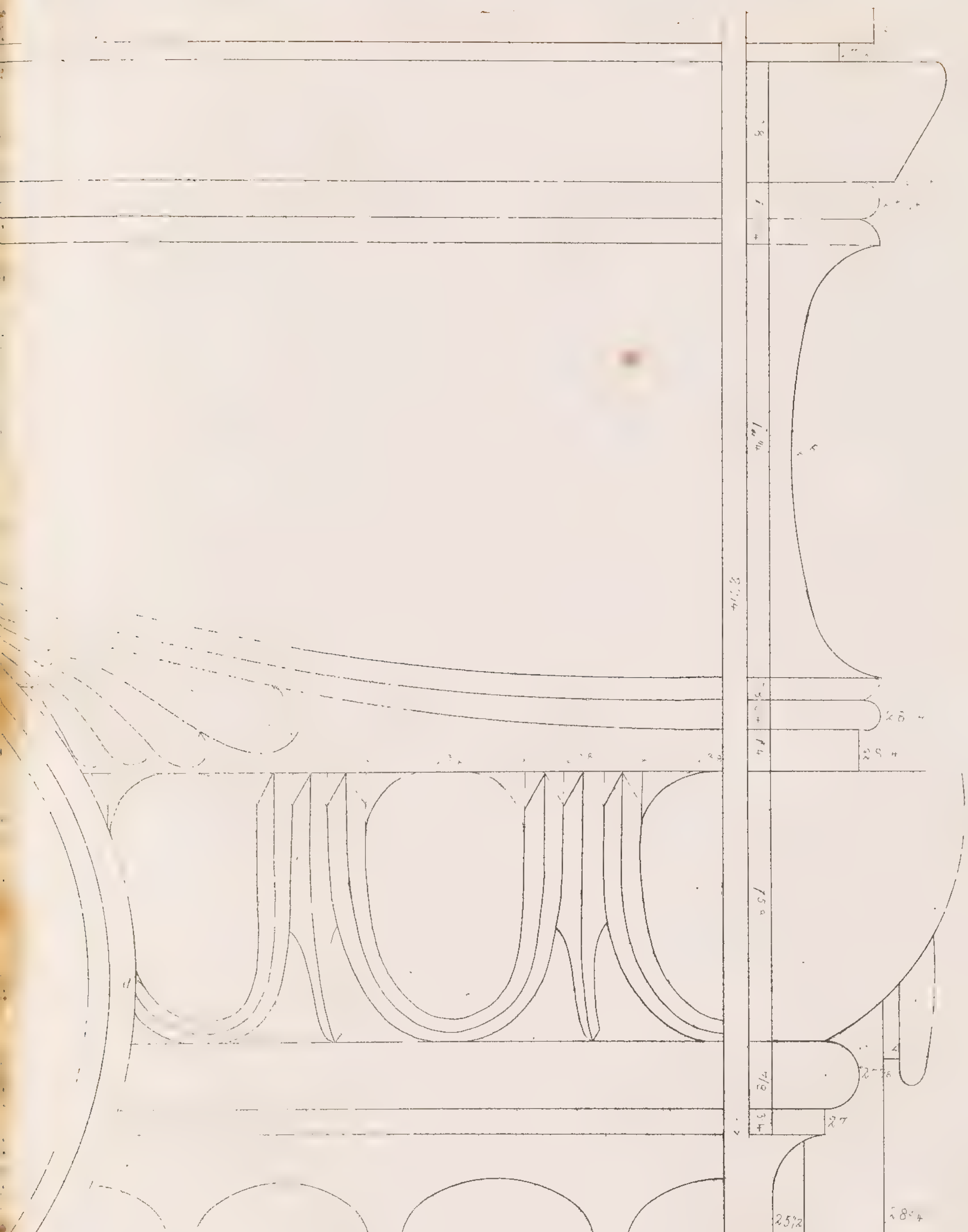
ARCHITECTURE.

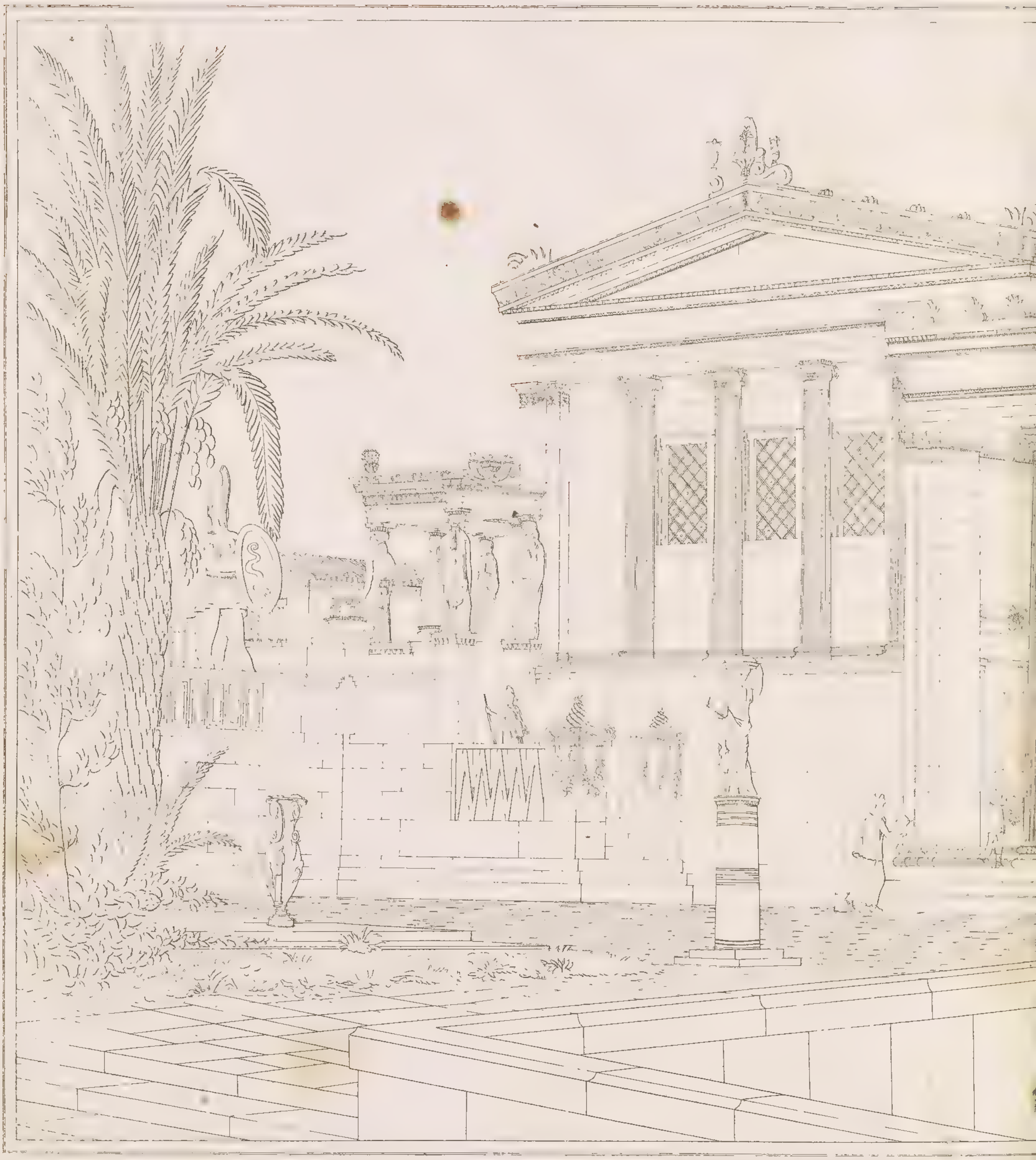
ORDER.







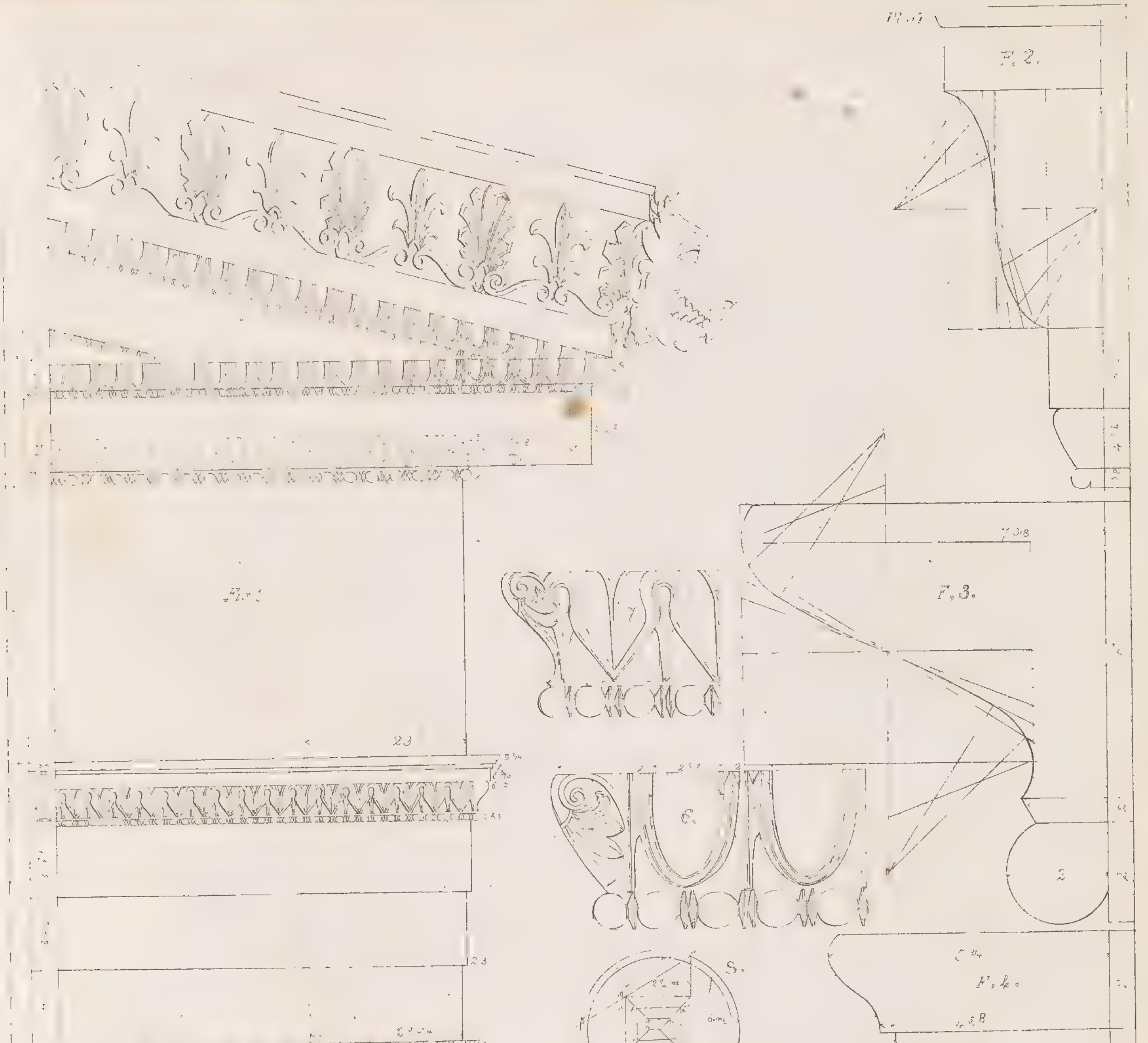




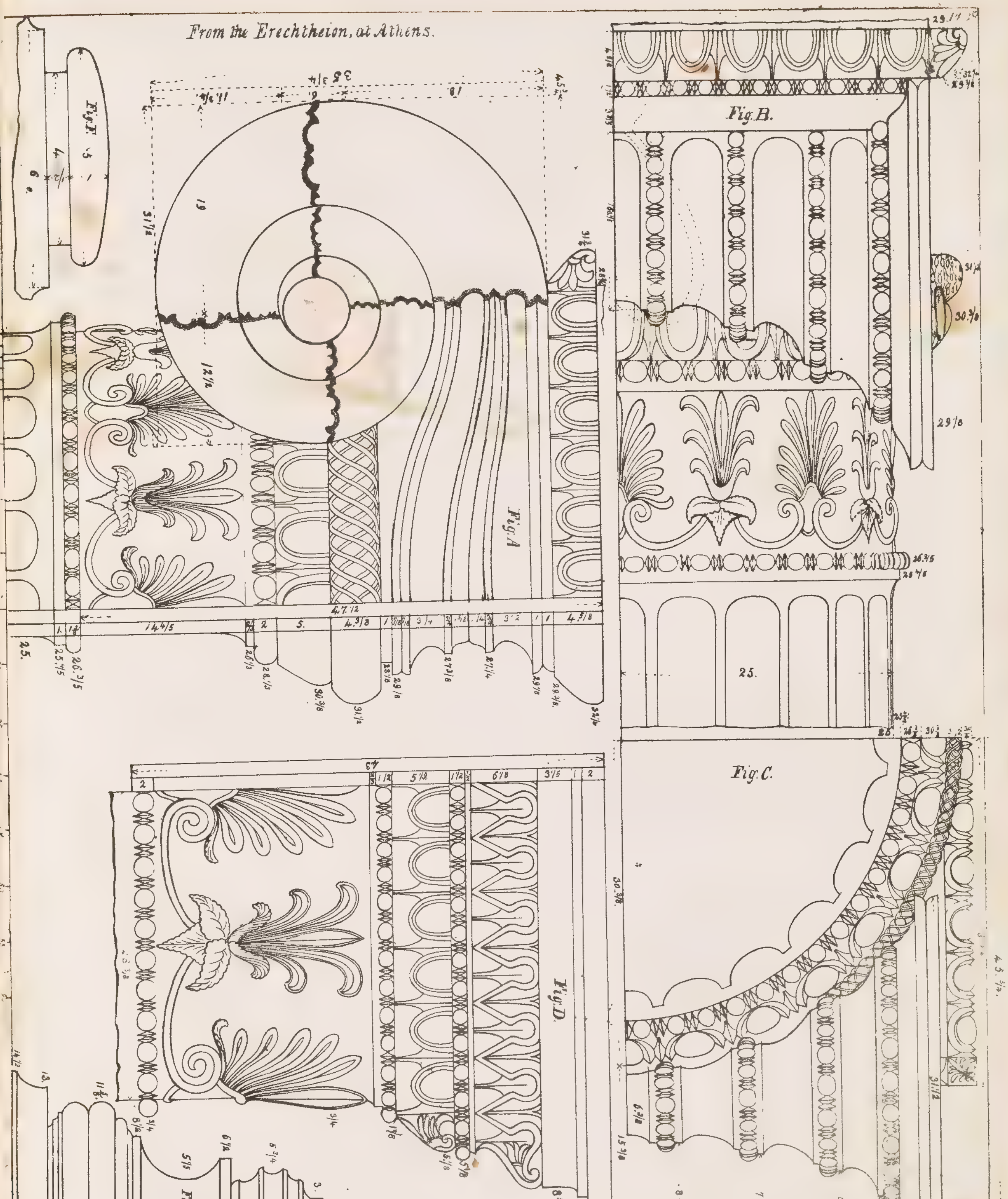
AN IDEAL RESTORATION OF

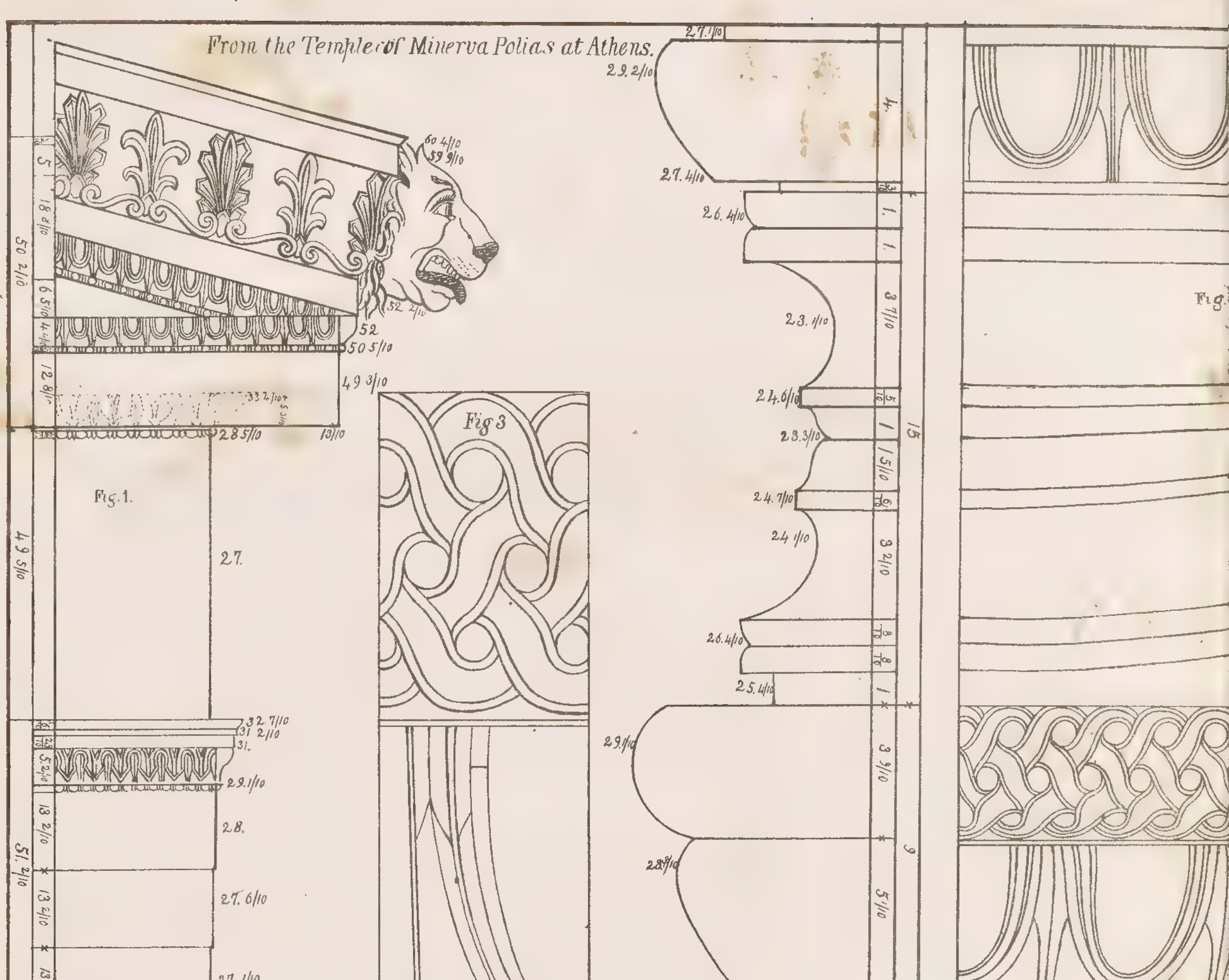
Pl. 16

THE ERECHTHEION, AT ATHENS.

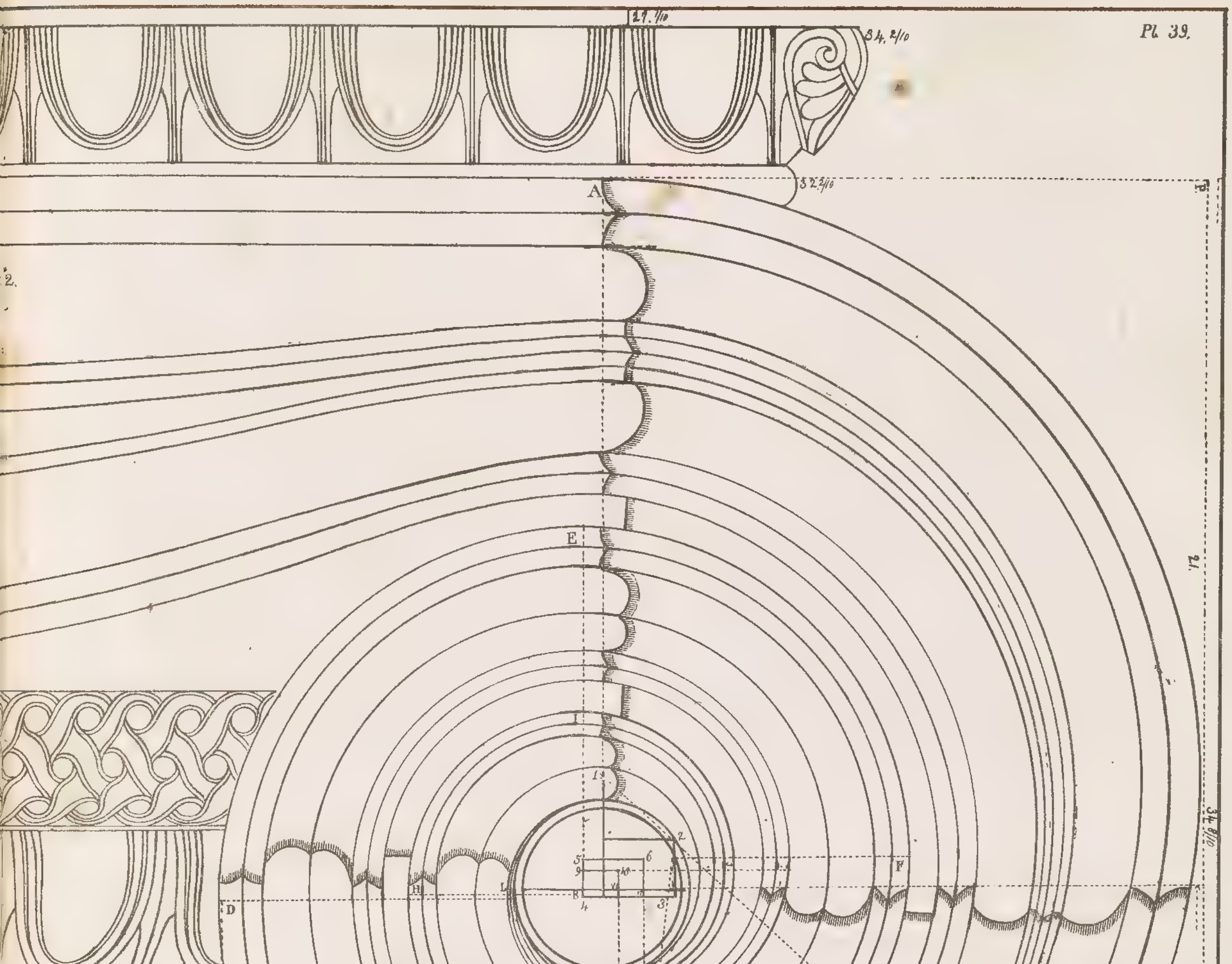

F. 2.
F. 3.
F. 4.
F. 5.
6.
7
8.
9.

From the Erechtheion, at Athens.



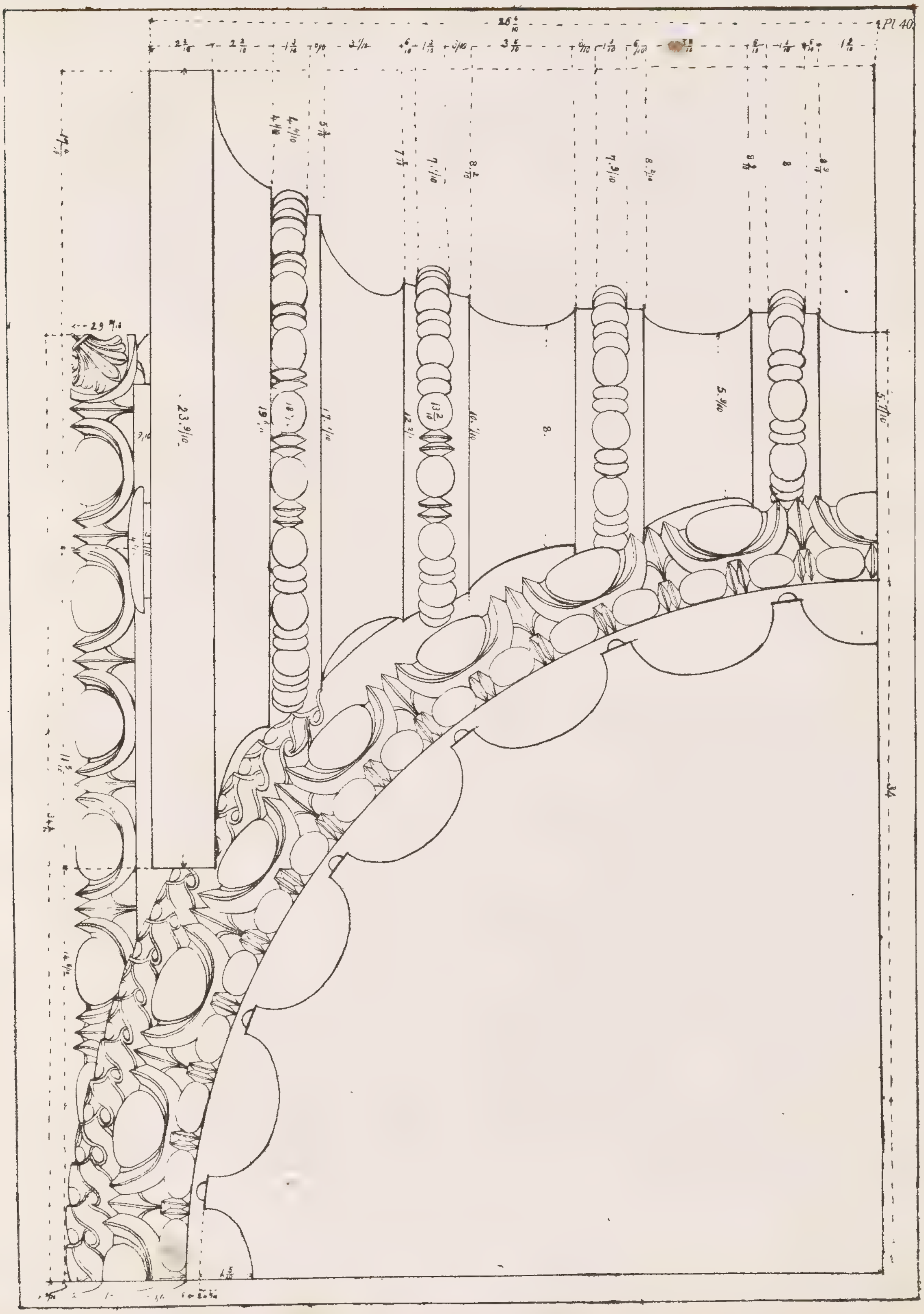

From the Temple of Minerva Polias at Athens.
Fig. 1.
Fig 3
Fig. 4
Fig.
Column. 9, diam. 3. m.
60 4/10
59 9/10
52 2/10
52
50 5/10
49 3/10
33 2/10
28 5/10
13/10
50 2/10
18 8/10
12 8/10
49 5/10
27.
32 7/10
31 2/10
31.
29. 1/10
28.
27. 6/10
27. 1/10
51. 2/10
13 2/10
13 4/10
34. 2/10
15 1/10
44. 2/10
11 8/10
25. 5/10
11 4/10
24 1/10
30.
23. 6/10
36. 7/10
37 4/10
39. 9/10
44. 1/10
48. 2/10
2.4
8
30.
33 6/10
36. 7/10
37. 4/10
35. 6/10
39. 2/10
44 1/10
6 1/10
7 9/10
27. 1/10
29. 2/10
27. 4/10
26. 4/10
23. 1/10
24. 6/10
23. 3/10
24. 7/10
24 1/10
26. 4/10
25. 4/10
29. 1/10
28. 9/10
26. 2/10
26. 3/10
25. 3/10
26. 4/10
24. 7/10
25. 2/10
4.
1.
1.
3 7/10
15
1 5/10
3 2/10
3 9/10
9
5 1/10
1. 2/10
11 2/10
13 5/10
1 5/10
6 2/10
14 8/10
1 1 2 3 4 5

Pl. 39.

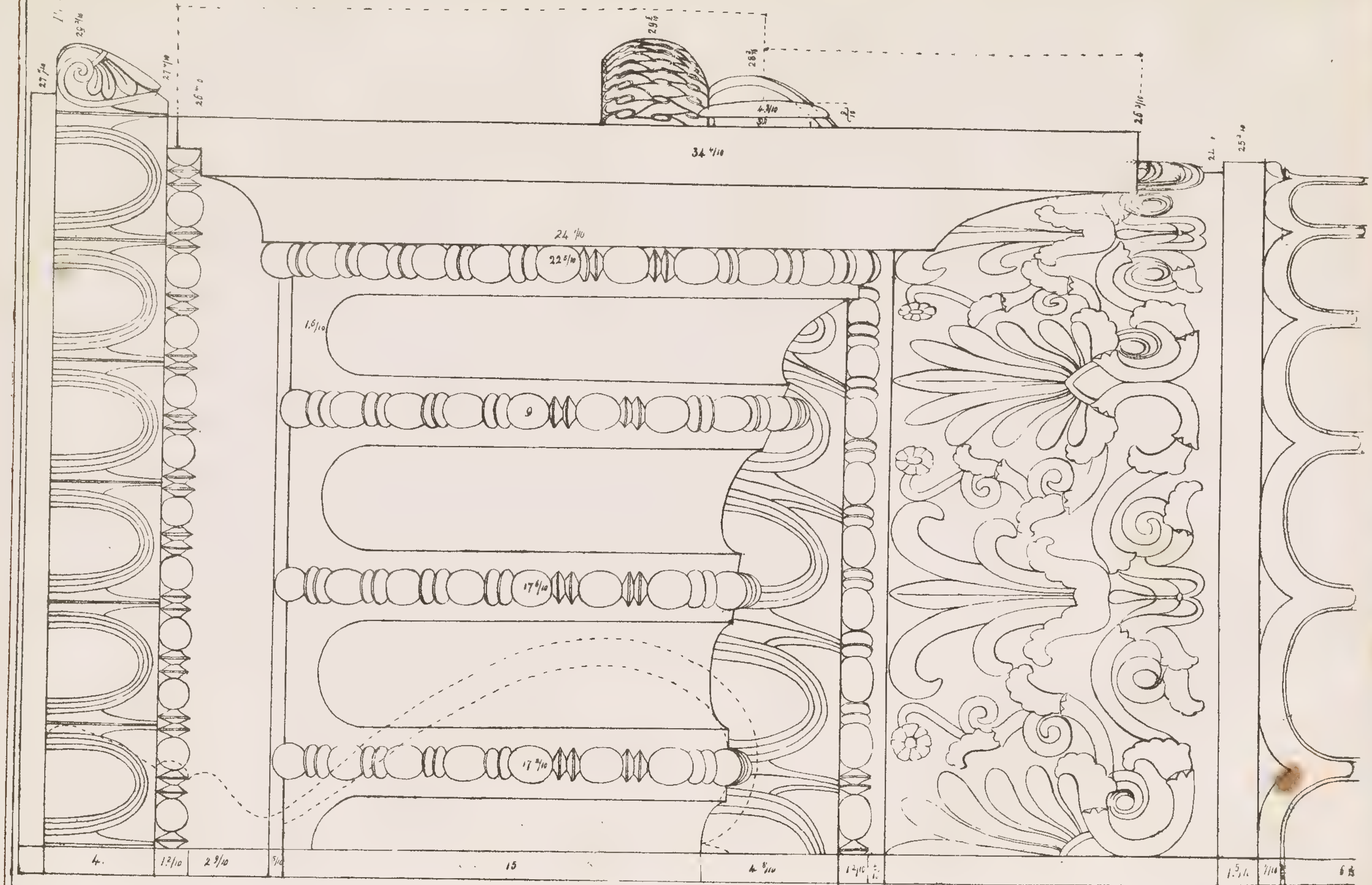


A
E
D
L
F
G
C
B
P
84 2/10
32 2/10
21.
34 8/10
13. 8/10
17 4/10
24 7/10
11 5/10
24 7/10
Fig. 5

Pl. 40

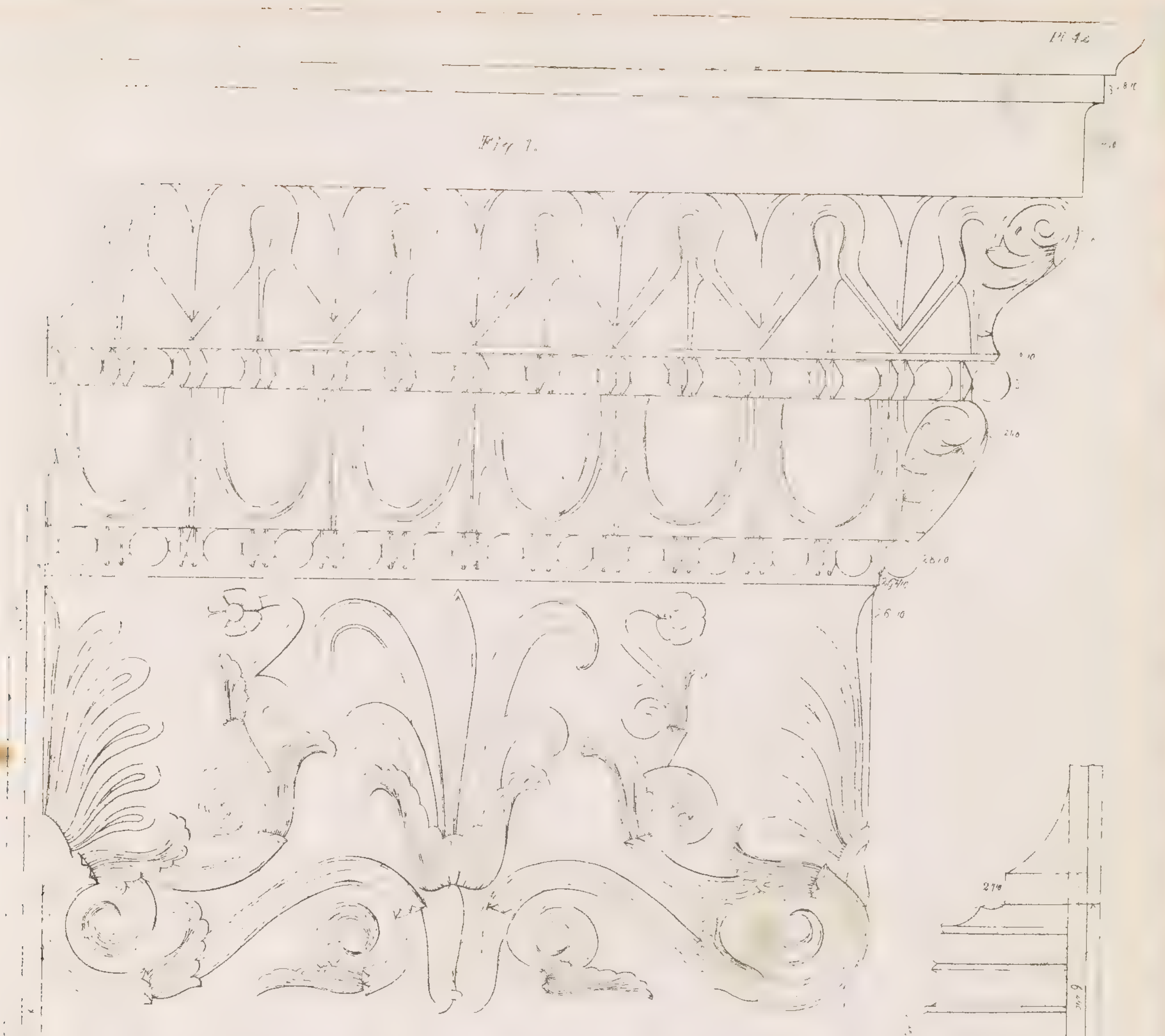


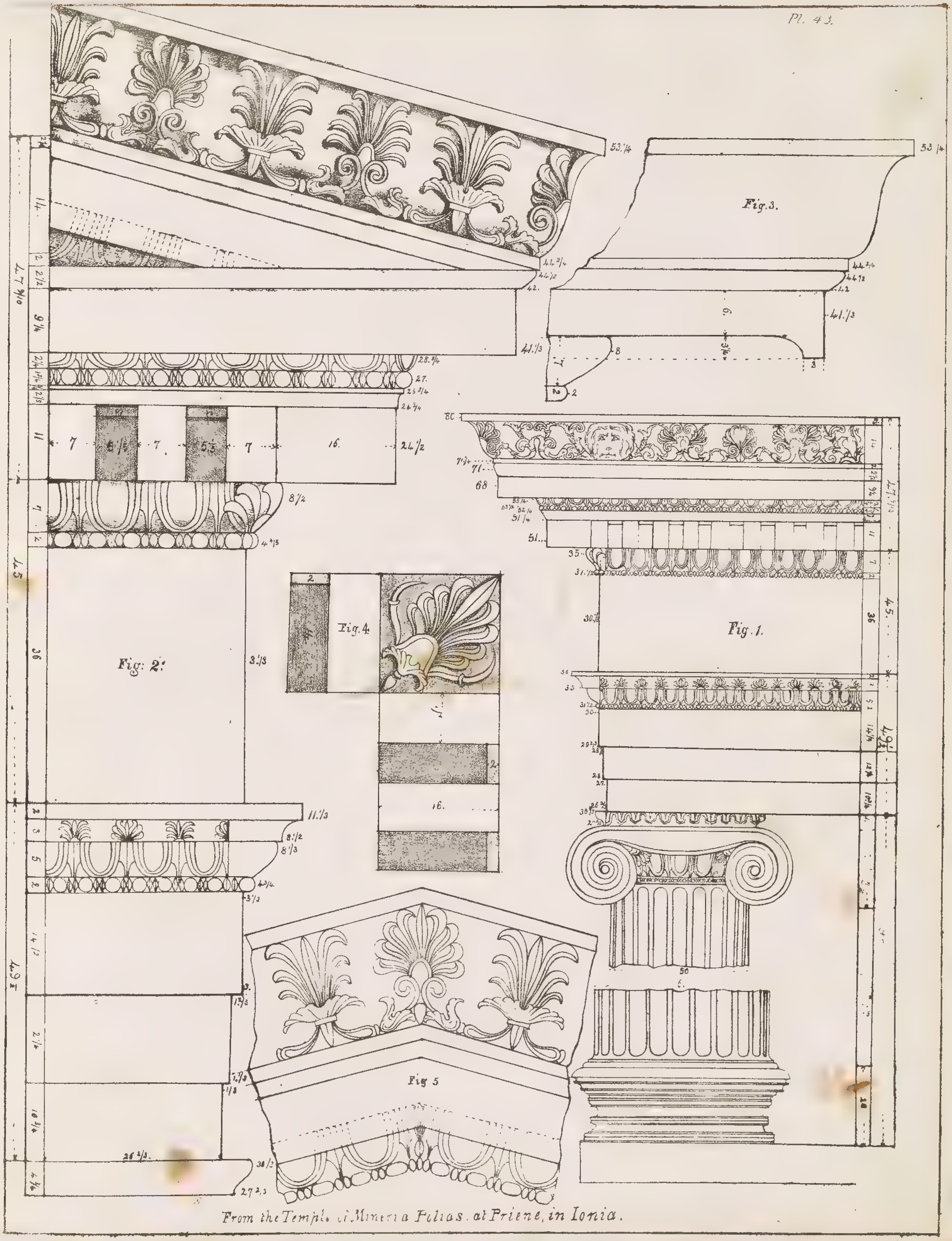
Pl. 41



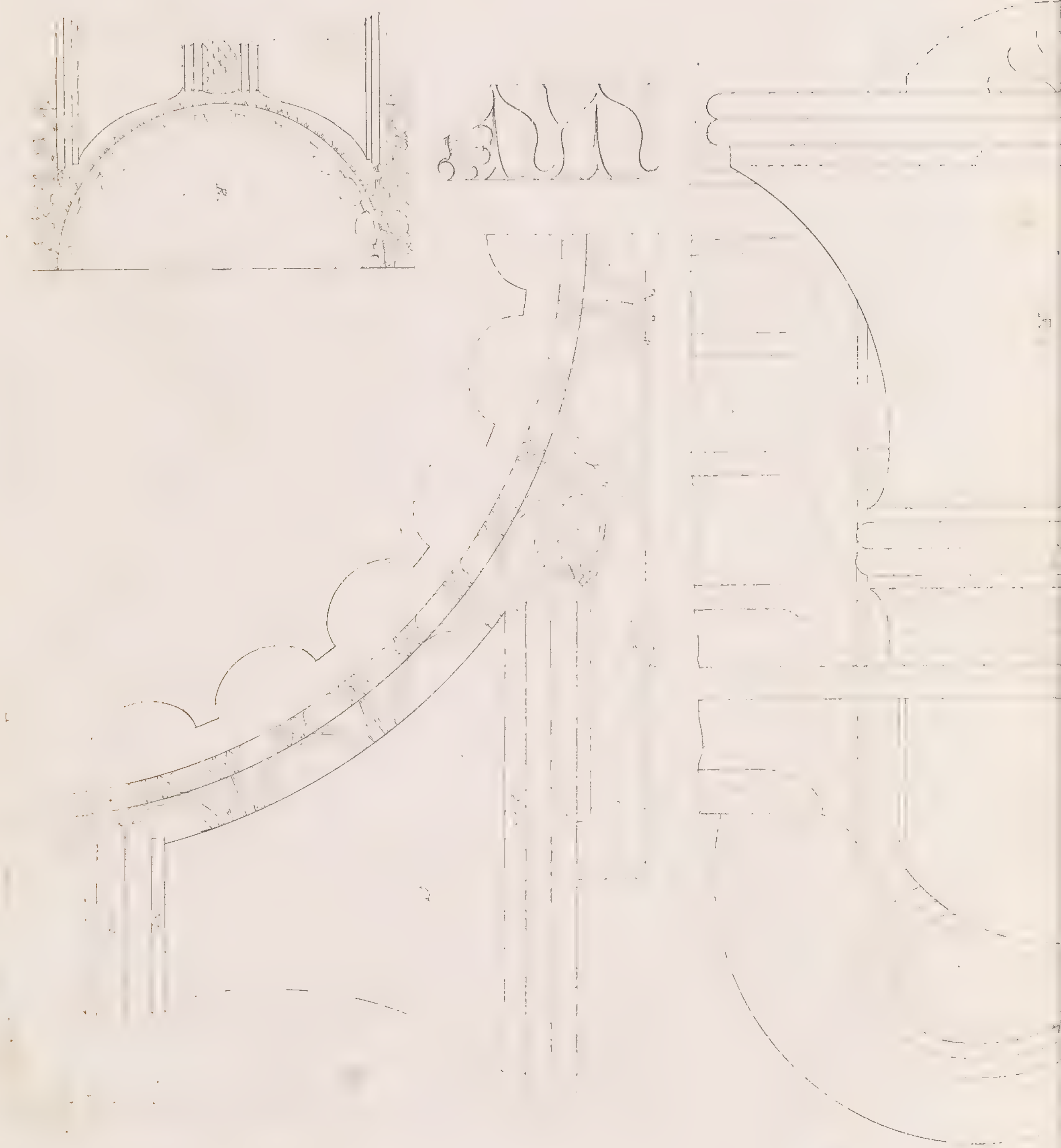
From the Temple of Minerva at Polias

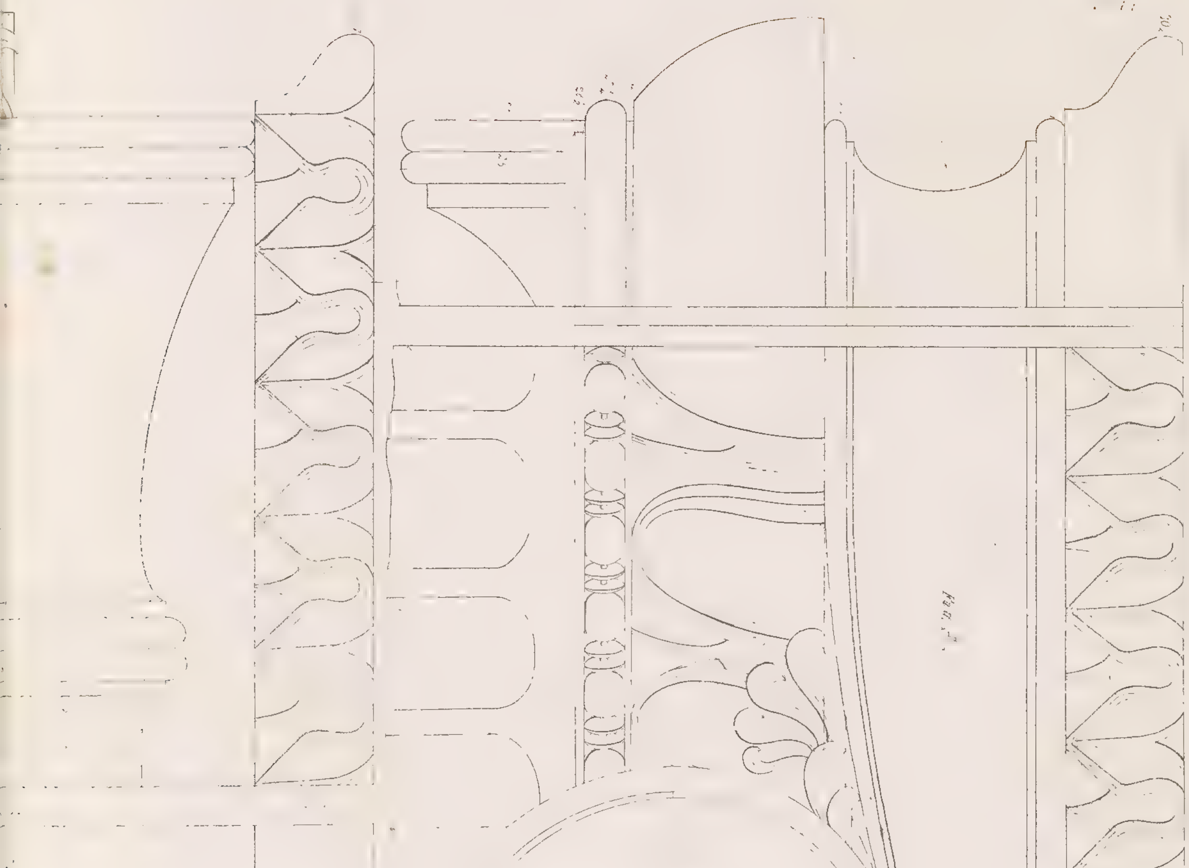
Pl. 42

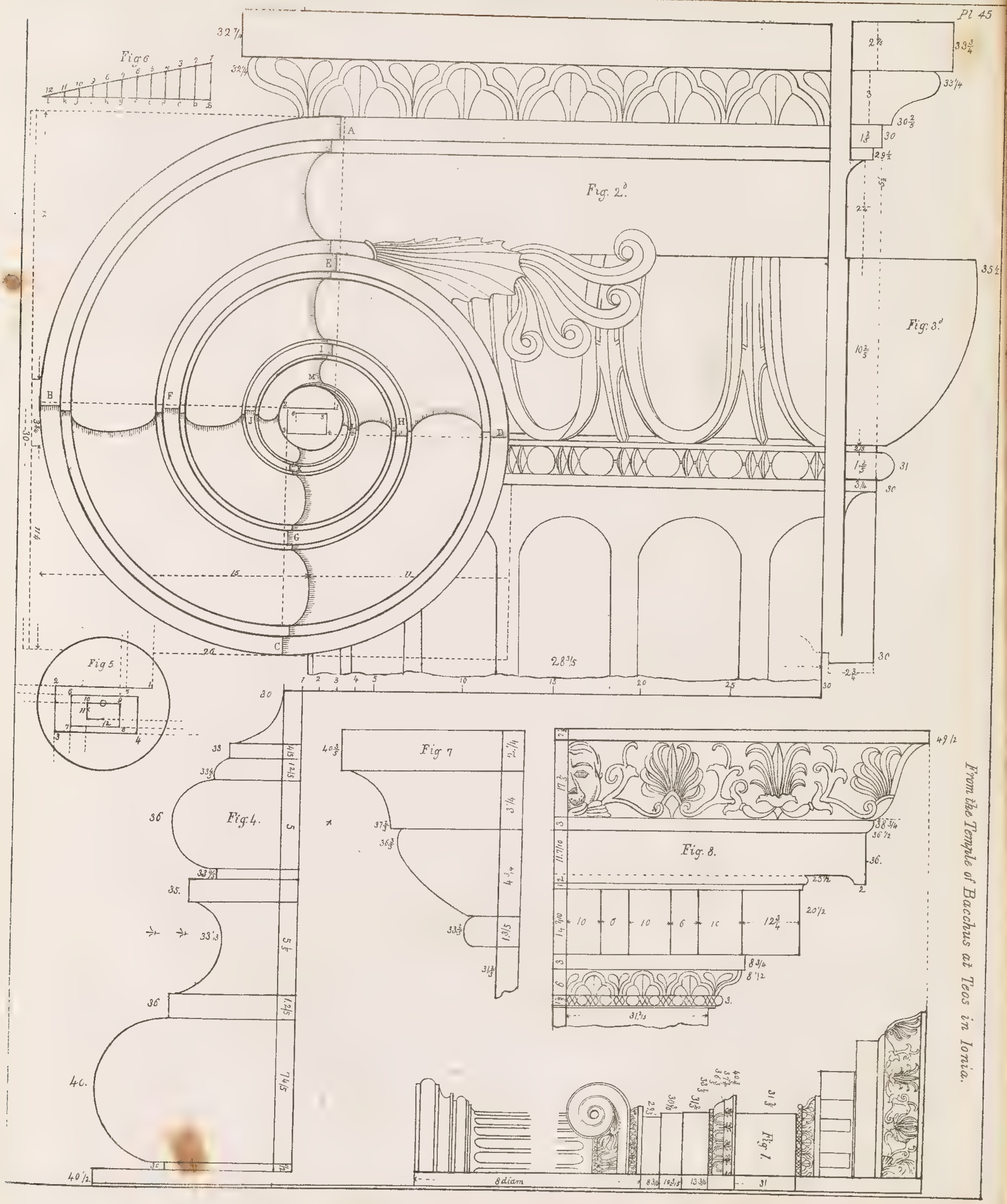




From the Temple of Minerva Polias. at Priene, in Ionia.

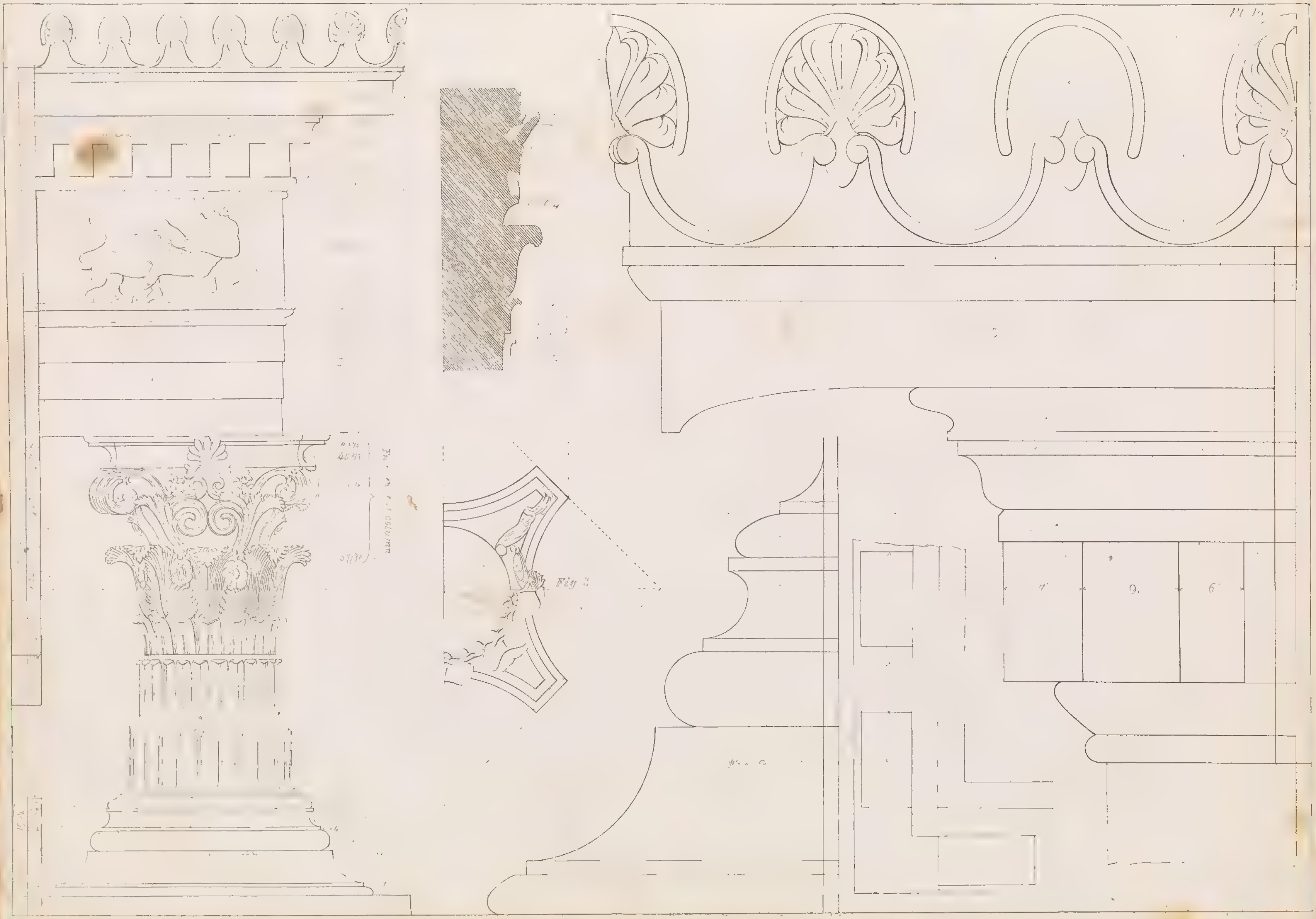




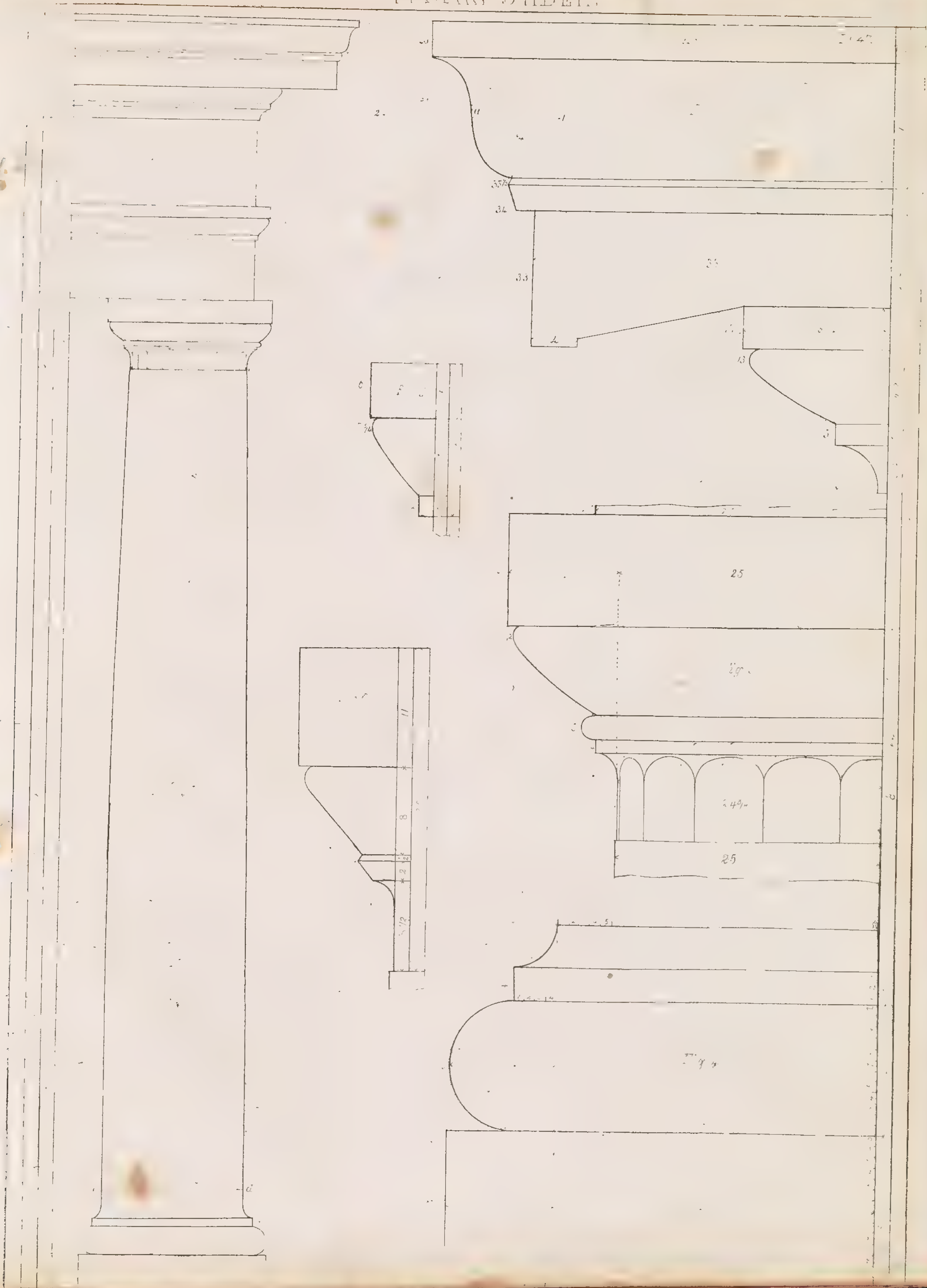


From the Temple of Bacchus at Teos in Ionia.

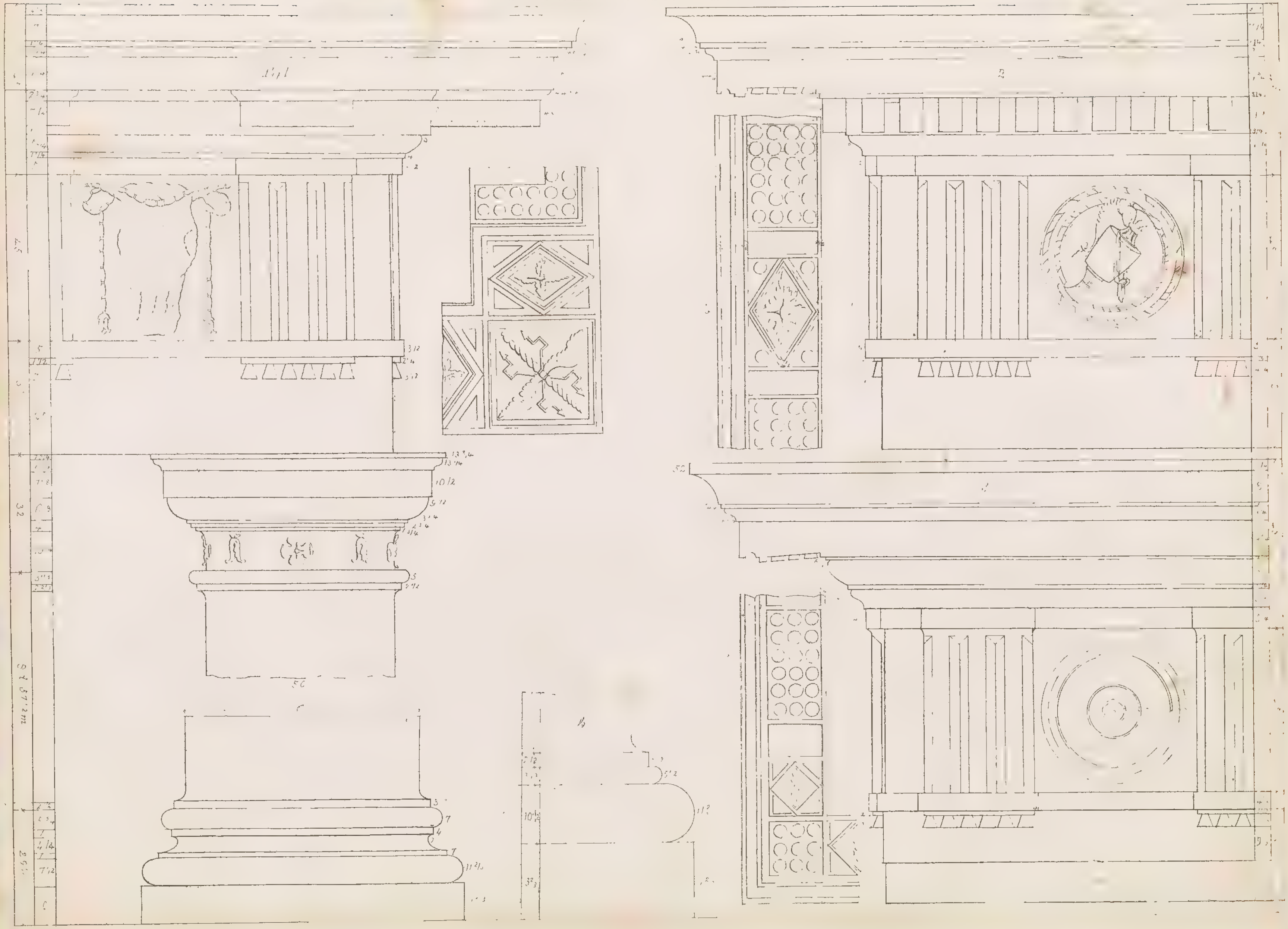
Monument of Lysicrates at Athens



TUSCAN ORDER.



ROMAN DORIC ORDER.



ROMAM CORINTHIAN ORDER.

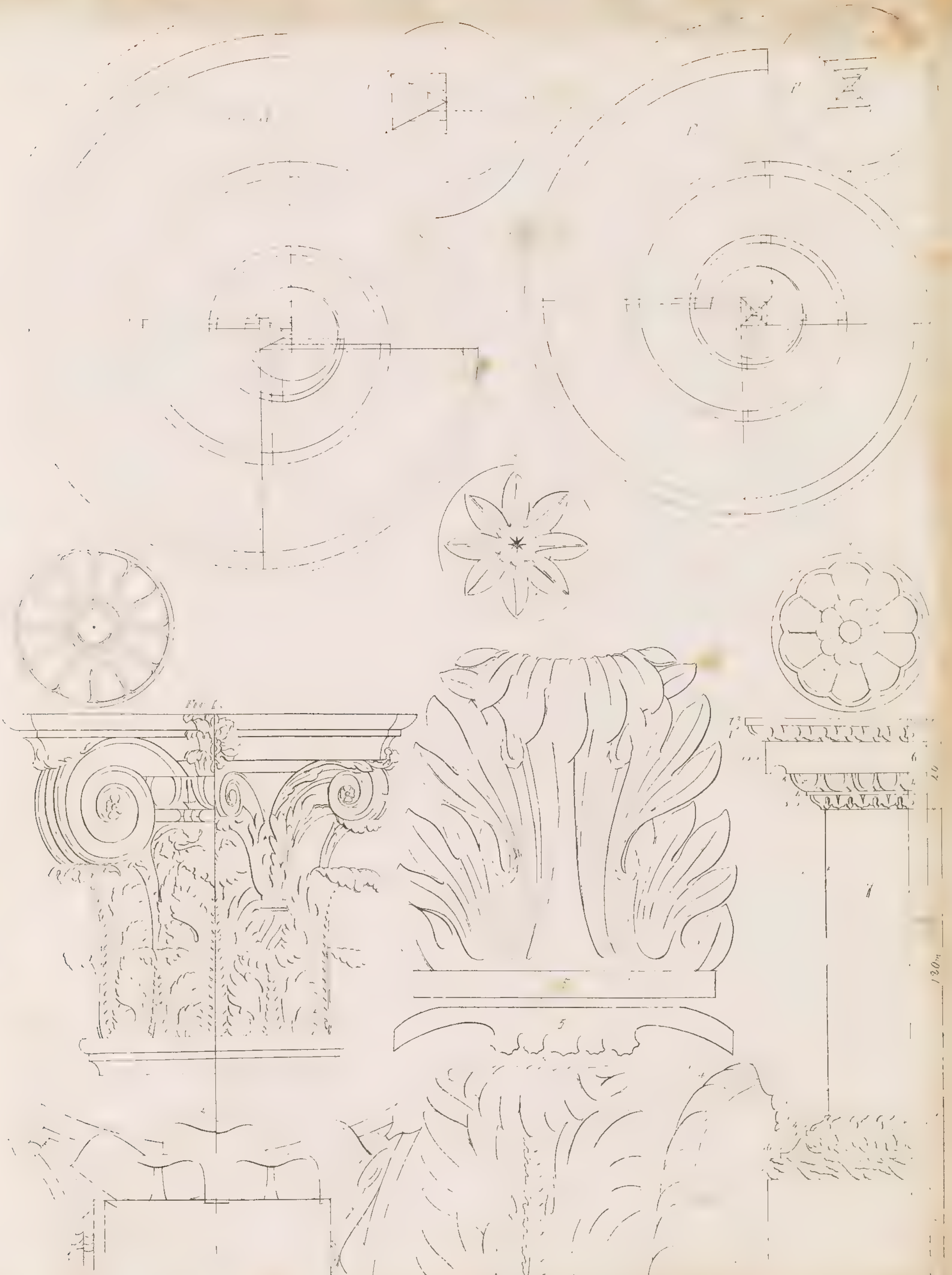
Pl. 49

Plan of the Capital.

Soffit of the Corona.

Angular view of the Capital.

Corinthian Base



50 pls
Coll comp
cpw